Effects of Corporate Diversification Revisited:
New Evidence from the Property-Liability Insurance industry

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July 15, 2015

Working Paper
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Abstract

In this paper, we reexamine the effect of product diversification on the performance of property-liability insurance companies. A number of previous studies find a diversification penalty when comparing single-line insurers with all other insurers operating in more than one business line. We argue that single-line insurers and insurers that are diversified across multiple lines of business have different investment opportunity sets and cannot be compared. In our analysis, we focus on single-line insurers and insurers that diversified for the first time. Using a treatment effects model and propensity score matching, we do not find any evidence of a negative relationship between diversification and performance.

*JEL classification:* G34, G32; G22

*Keywords:* Diversification, Focus, Organizational Structure, Firm Performance, Insurance Companies
Introduction

The effect of line-of-business diversification on the performance of insurance companies has been the subject of a number of recent studies (see, e.g., Liebenberg and Sommer, 2008; Berry-Stölzle, Hoyt, and Wende, 2013). Theory indicates that diversification is associated with both costs and benefits. Benefits associated with diversification include economies of scope (Teece, 1980), internal capital markets without information asymmetries (Williamson, 1975; Stein, 1977), as well as risk pooling (Cummins, Philips, and Smith, 2001; Cummins and Trainar, 2009, p. 467ff.). On the other hand, diversification may magnify agency costs (Harris, Kriebel, and Raviv, 1982; Rotemberg and Saloner, 1994) and allow inefficient cross-subsidization of poorly performing business units (Rajan, Servaes, and Zingales, 2000). Thus, many papers argue that the net effect of diversification is an empirical question and examine the relative performance of specialized versus diversified firms (see, e.g., Laeven and Levine, 2007; Liebenberg and Sommer, 2008; Schmid and Walter, 2009; Berry-Stölzle, Hoyt, and Wende, 2013). The majority of the papers in this strand of literature document a negative relationship between line-of-business diversification and insurer performance (see, e.g., Laeven and Levine, 2007; Liebenberg and Sommer, 2008; Schmid and Walter, 2009). However, the universe of all diversified insurers may not be the appropriate counterfactual for examining the effect of diversification into a new business line on the performance of a single-line insurer. We argue that single-line insurers and insurers that are diversified across multiple lines of business have different investment opportunity sets and cannot be compared.

The goal of our research is to reexamine the effect of product diversification on the performance of property-liability insurance companies, focusing on changes in firm’s diversification status. We identify insurers that change their diversification status from single-line
to diversify by adding one or more additional business lines, and we compare the performance of these diversifying firm-years to all single-line firm-years. Such a comparison of single-line firms that choose to diversify with single-line firms that choose to stay focused allows us to directly test the effect of line-of-business diversification on insurance companies’ performance.

The argument that specialized firms and diversified firms differ in their ability to exploit market opportunities is not new. Maksimovic and Phillips (2002) develop a neoclassical model of profit maximizing firms with heterogeneous industry-specific productivity. In equilibrium, firms produce in industries in which they have a comparative advantage and firms with lower industry-specific productivity diversify more. In Maksimovic and Phillips’ (2002) model a negative relationship between diversification and firm performance arises endogenously; the negative diversification-performance relationship is driven by differences in industry-specific productivity across firms. Since all firms in the model maximize profit, the negative diversification-performance relationship cannot be interpreted as evidence of value destruction by diversifying firms.

Our baseline test of the diversification-performance relationship is a simple OLS regression of insurers’ return on assets (ROA) on a diversification indicator. In addition, we use two different econometric methods to address firms’ self-selection of a diversification status. First, we employ a two-equation treatment effects model that simultaneously estimates the determinants of insurers’ diversification decision and the effect of diversification on performance. Second, we use three different propensity score matching procedures to generate matched samples of control observations that are similar to diversifying firms based on a wide range of firm characteristics, and we test for differences between the treatment and control groups. In all three tests, we do not find any evidence of a negative relationship between diversification and performance.
We are only aware of one paper that has a similar research focus to ours. Villalonga (2004) examines the value of diversification for non-financial firms focusing on changes in firm’s diversification status. She uses two propensity score matching estimators and a two-equation treatment effects model and does not find any evidence of a diversification penalty. However, to the best of our knowledge no previous insurance specific study has taken such a dynamic perspective. This gap in the literature is all the more important given Santalo and Becerra’s (2008) result that the effect of diversification on firm performance is not homogeneous across industries.

This paper proceeds as follows. In the next section we discuss related literature and the conceptual background of our research design. This is followed by a description of the data and methodology used to empirically test our hypotheses, and a section containing our results. The final section concludes.

Prior Literature and Conceptual Background

There have been a substantial number of literatures in a relation between diversification and firms’ performance to see if diversification increases or decreases firms’ value. On diversification discount side, Lang and Stulz (1994) and Berger and Ofek (1995) find that diversified firms trade at an average discount relative to single-segment firms; this suggests diversification destroys value of the firms. Moreover, Hoyt and Trieschmann (1991) emphasize on a comparison of risk and return relationship for both Life-Health and Property-Liability, and diversified insurers with Capital Asset Pricing Model and mean-variance approaches to measure the risk-adjusted returns. With accounting measure of profitability as a proxy for market performance, they found that investment in specialized life-health and property-liability insurers perform better than investment in diversified insurers. One possible explanation could be that
larger diversified insurers cannot operate as efficiently as such smaller insurers according to Maksimovic and Phillips (2002) where they claim that conglomerates are naturally different from focused firms. Thus with respect to investment opportunities, our argument in this paper is that if single-line and multiline firms have different investment opportunities then there should be a negative relationship between diversification and firm performance of profit maximizing firms.

Further, Liebenberg and Sommer (2008) find diversification discount of 1 percent of return on assets (ROA) and 2 percent of return on equity (ROE). Additionally, among control variables, they find that both size and capitalization are positively related to accounting performance and mutual insurers are significantly less profitable than stock insurers. They also support that more concentrated firms can charge higher prices and therefore earn higher profits than less concentrated firms. Finally, they assert that unaffiliated insurers consistently outperform aggregated insurer groups which may be due to lower prices, costs of managerial discretion, or other costs associated with conglomeration.

Previous research design including Liebenberg and Sommer (2008) is to compare single-line insurers with all diversified insurers and, hence, implicitly assume that all insurers have the same investment opportunities and can switch back and forth between being single-line and diversified in every year (Hoyt and Trieschmann, 1991; Cummins et al., 2003; Liebenberg and Sommer, 2008). However, our design in this paper is to compare single-line insurers with firm-year observations in which a single-line insurer diversifies for the first time.

Besides, our methodology, Propensity Score Matching, is firstly introduced in Rosenbaum and Rubin (1983). They define the propensity score as conditional probability of assignment to treatment conditional on observed covariates which is a vector of independent variables so observations with the same propensity score have the same distribution of the full vector of
variables X’s which reduces the selection bias. Furthermore, Villalonga (2004) uses Propensity Score Matching to construct matched control firms to focus on diversification effect. She fits the Probit model to estimate the probability that a given firm will diversify by matching diversifying firm with non-diversifying firm with a similar propensity score. Villalonga also uses Heckman (1979) as well and she finds that treatment effect is negative but insignificant. Our result in this paper is similar to Villalonga (2004).

In addition to that, Maksimovic and Phillips (2002) develop a neoclassical model of profit maximizing firms with heterogeneous industry-specific productivity. They claim that in equilibrium, firms produce in industries in which they have a comparative advantage and firms with lower industry-specific productivity diversify more. We follow their studying with respect to the assumption that market actually works and firms are profit maximizing, these profit maximizing firms engage in an activity that only increases value of the firms and make value-maximizing choice of organizational forms and allocating resources across industries. Also, the labor market disciplines managers. If a manager performs poorly then he may get replaced and may end up working for a smaller firm that pays less. In this logic, we assert Liebenberg and Sommer (2008) result should be interpreted differently with appropriate sub-sample and with better methodology, propensity score matching, used by Villalonga (2004) which shows on average diversification does not destroy value of the firms.

**Data and Methodology**

**Sample Selection**

Our initial sample includes all property-liability insurers that report their financials to the NAIC from 1995 to 2004. It has total of 914 unique firms or 6290 firm-year observations which
is the same as Liebenberg and Sommer (2008). Unlike previous research design, our design is to compare single-line insurers with firm-year observations in which a single-line insurer diversifies for the first time. Therefore, we use subgroup of our initial sample where we keep all firm-year observations of single-line firms, all firm-year observations of firm that just diversified, the year the firm becomes multi-line, and drop all firm-year observations of multi-line firms in the years after they diversified. Then, our final sample includes 591 firms or 2621 firm-year observations where 2500 firm-year observations are single-line firms and 121 firm-year observations are diversified firms.

Additionally, we conduct standard screens where we exclude firms under regulatory scrutiny, firms that report non-positive direct premiums written or total admitted assets, firms with organizational structures other than stock and mutual, firms with fewer than two consecutive years of data, and firms whose reinsurance activity casts doubt on their ability to sustain as a solvent and independent institution.

**Methodology**

Consistent with Liebenberg and Sommer (2008), our multivariate analysis is performed with two ordinary least squares regressions (OLS) as well as Heckman (1979) treatment effect model because firms self-select to diversify or stayed on focused. Our regression analysis focuses on whether diversification is performance enhancing or reducing so we use an indicator variable $MULTLINE$ to denote whether an insurer operates in one line ($MULTLINE = 0$) or multiple lines ($MULTLINE = 1$) in a given year. Our basic regression model that is used to measure the effect of diversification on firms’ performance is defined as following.

First stage:
In this equation, dependent variable is return on asset (ROA) which is a common accounting measure of firms’ performance. In this section, we briefly explain each control variables. First of all, previous studies such as Cummins and Nini (2002) and Browne, Carson, and Hoyt (1999) find a positive relation between firms’ size and performance in the P/L insurance industry so we control for Size which is measured as the natural logarithm of total assets. Moreover, Sommer (1996) finds that safer insurers are able to command higher prices which lead a positive relation between insurer capitalization and performance so CAPASSET, the ratio of policyholder surplus to total assets, is also controlled as well. Besides, to control for risky measures of the firms SDROA, standard deviation of firm, is included. Geographic diversification, GEODIV, is the complement of the Herfindahl index of premiums written across all U.S. states and it is controlled because geographically diversified firms are likely to have less volatile profits due to coinsurance effects. Next, WCONC is a weighted sum of firm exposure to industry concentration across all of the lines in which it operates and it is controlled since there is positive relationship between WCONC and performance of the firms according to Montgomery (1985). PCTLH is percent of life and health business, GROUP and PUBLIC are the dummy variables from Cummins and Sommer.
(1996). Last but not least, \textit{MUTUAL} variable is 1 if mutual, 0 if stock. Description and the summary statistics of these variables used in our regression can be found in Table 1.

We initially estimate equation (1) twice, Ordinary Least Square (OLS) with year dummies and OLS with year, line, state dummies. Endogeneity arises due to omitted variables, measurement error, simultaneity bias, or a combination of these factors according to Wooldridge (2002). Hence, we use a Hausman test for the exogeneity of \textit{MULTLINE} and reject the null hypothesis of exogeneity. Then we use Heckman (1979) treatment effects with maximum likelihood estimator. These three methodologies are conducted in a same manner in Liebenberg and Sommer (2008). Their results and our results are compared in Table 2 and discussed in detail in the following section.

Villalonga (2004) and Campa and Kedia (2002) show that the diversification discount is only the product of sample selection biases; diversified firms trade at a discount prior to diversifying, and when selection bias is corrected for the diversification discount disappears. Therefore, we additionally use propensity score matching method that was used in Villalonga (2004). This method is introduced in Rosenbaum and Rubin (1983) to correct for the selection biases in making estimates for subsample that receive treatment and the other does not receive treatment. The idea of propensity score matching is to use a first stage self-selection model including all explanatory variables and then calculate predicted probability of selection, called propensity scores, to create a matched sample of treated and control observations based on these propensity scores. It will make both treated and controlled sample more alike. Then we to test whether there is significant difference between the treated and control observations to estimate the result as the expectation of the conditional effects over the distribution.
There are several ways to create a matched sample. In this paper, we use three types; nearest neighbor matching, kernel matching, and stratification matching. *Nearest Neighbor Matching* uses for each treated observation \( i \), it selects a control observation \( j \) that has the closest \( x \). *Kernel Matching* uses for each treated \( i \), it matches with several control observations with weights inversely proportional to the distance between treated and control observations. Lastly, *Stratification Matching* uses to compare the outcomes within interval/blocks of propensity scores, for instance, 10 blocks. See APPENDIX for details.

**Results**

Basic summary statistics and description of variables used in this paper are presented in Table 1 where we compare mean and standard deviation of all variables with those in Liebenberg and Sommer (2008). This contrast is meaningful since their research design is to compare single-line insurers with all diversified insurers. Such an approach implicitly assumes that all insurers have the same investment opportunities and can switch back and forth between being single-line and multi-line in any given year. Therefore we design differently in this paper in a way that it only catches newly diversified firms. It means to compare single-line insurers with firm-year observations in which a single-line insurer diversifies for the first time and then drop afterwards. Although the basic summary statistics are very similar in Table 1, the diversification effect result from different methodology is different as we expected.

Result for the effect of diversification status on return on asset (ROA) using two ordinary least square methods and Heckman treatment effect method appear in Table 2. First ordinary least square (OLS) in the first column is conducted with year dummy and second OLS in the second column is conducted with year, line, and states dummies. Then, third column indicates Heckman
(1979) treatment effect results. In this treatment effect model, we initially compute equation (1) with two instrumental variables, age and reinsurance, then estimate equation (2) by using predicted values for MULTLINE obtained in the first-stage. Note that last column represents Liebenberg and Sommer (2008) Heckman treatment effect result for a comparison purpose. All the coefficient estimates on MULTLINE are still negative but significance disappeared which is not consistent with Liebenberg and Sommer (2008) result. Other control variables are almost exactly the same as their estimations though.

Furthermore, table 3 shows the effect of diversification on the return on assets with propensity score matching method. Again, all the coefficient estimates on MULTLINE are still negative but significance disappeared which is consistent with Villaongla (2004) where she also finds insignificant negative coefficients of diversification estimators in her study and concludes that diversification does not destroy firms’ value on average. We can also conclude that there is no negative relationship between diversification and firms’ performance with newly diversified sub-sample firms and with propensity score matching.

**Conclusion**

Liebenberg and Sommer (2008) previously found the diversification discount effect on value of the firm and we were not persuaded because profit maximizing firms should be only engaged in an activity that increases value of the firm and should make value-maximizing choice of organizational forms and allocating resources across industries if market actually works. In this paper, we revisit Liebenberg and Sommer (2008) with more proper sample, newly diversified and single-line firms, and with advanced methodology, propensity score matching used by Villalonga (2004). By matching newly diversifying and single-segment firms on their propensity score-the
predicted values from a probit model of the propensity to diversify, we did not find diversification
discount effect on firm value. Therefore we conclude that diversification does not destroy value
of the firms on average.
References


Table 1. Variable Definition Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Liebenberg and Sommer (2008)</th>
<th>Our sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.02</td>
<td>(0.05)</td>
</tr>
<tr>
<td>ROA</td>
<td>Net income/total admitted assets</td>
<td>0.03</td>
<td>(0.10)</td>
</tr>
<tr>
<td>LINES</td>
<td>Number of lines in which firm has positive direct premium written</td>
<td>5.91</td>
<td>(4.63)</td>
</tr>
<tr>
<td>MULTLINE</td>
<td>Multline= 1 if LINES &gt; 1, 0 otherwise</td>
<td>0.79</td>
<td>(0.41)</td>
</tr>
<tr>
<td>SIZE</td>
<td>Natural logarithm of total admitted assets</td>
<td>17.64</td>
<td>(2.19)</td>
</tr>
<tr>
<td>CAPASSET</td>
<td>Policyholder surplus/total admitted assets</td>
<td>0.49</td>
<td>(0.44)</td>
</tr>
<tr>
<td>GEODIV</td>
<td>1-Herfindahl index of DPW across 57 geographic area</td>
<td>0.33</td>
<td>(0.37)</td>
</tr>
<tr>
<td>WCONC</td>
<td>Weighted sum of market share per line multiplied by line specific Herfindahl</td>
<td>0.05</td>
<td>(0.02)</td>
</tr>
<tr>
<td>PCTLH</td>
<td>Percentage of premiums from life–health insurance</td>
<td>0.44</td>
<td>(2.16)</td>
</tr>
<tr>
<td>MUTUAL</td>
<td>MUTUAL = 1 if firm is a mutual, 0 otherwise</td>
<td>0.48</td>
<td>(0.50)</td>
</tr>
<tr>
<td>GROUP</td>
<td>GROUP = 1 if firm is a group, 0 otherwise</td>
<td>0.34</td>
<td>(0.47)</td>
</tr>
<tr>
<td>PUBLIC</td>
<td>PUBLIC = 1 if firm is publicly traded, 0</td>
<td>0.08</td>
<td>(0.28)</td>
</tr>
<tr>
<td>SDROA</td>
<td>Standard deviation of ROA</td>
<td>0.03</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

N: 6290

Note: Initial sample includes all property-liability insurers that report their financials to the NAIC from 1995 to 2004 in total of 914 unique firms or 6290 firm-year observations which is the same as Liebenberg and Sommer (2008). Subgroup of our initial sample includes all single-line firms and all firm-year observations of firm that just diversified, the year the firm becomes multi-line, and drop all firm-year observations of multi-line firms in the years after they diversified. Our final sample includes 591 firms or 2621 firm-year observations where 2500 firm-year observations are single-line firms and 121 firm-year observations are diversified firms.
Table 2. Effect of diversification on the return on assets based on OLS and treatment effects model

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>OLS with year dummies</th>
<th>OLS with year, state, and line dummies</th>
<th>Heckman self-election, ML estimator</th>
<th>Heckman self-selection, Liebenberg and Sommer (2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MULTLINE</td>
<td>-0.014 (0.008)</td>
<td>-0.010 (0.011)</td>
<td>-0.028 (0.029)</td>
<td>-0.061 (0.008)***</td>
</tr>
<tr>
<td>SIZE</td>
<td>0.009 (0.001)***</td>
<td>0.008 (0.001)***</td>
<td>0.009 (0.001)***</td>
<td>0.006 (0.001)***</td>
</tr>
<tr>
<td>CAPASSET</td>
<td>0.083 (0.009)***</td>
<td>0.082 (0.008)***</td>
<td>0.085 (0.008)***</td>
<td>0.085 (0.004)***</td>
</tr>
<tr>
<td>GEODIV</td>
<td>0.007 (0.006)</td>
<td>-0.001 (0.007)</td>
<td>-0.001 (0.006)**</td>
<td>-0.006 (0.002)***</td>
</tr>
<tr>
<td>WCONC</td>
<td>0.086 (0.056)</td>
<td>0.081 (0.057)</td>
<td>0.082 (0.056)</td>
<td>0.082 (0.032)**</td>
</tr>
<tr>
<td>PCTLH</td>
<td>0.006 (0.001)***</td>
<td>0.006 (0.001)***</td>
<td>0.006 (0.001)***</td>
<td>-0.001 (0.000)**</td>
</tr>
<tr>
<td>MUTUAL</td>
<td>-0.015 (0.004)***</td>
<td>-0.015 (0.004)***</td>
<td>-0.016 (0.004)**</td>
<td>-0.010 (0.002)***</td>
</tr>
<tr>
<td>PUBLIC</td>
<td>-0.001 (0.013)</td>
<td>-0.003 (0.013)</td>
<td>-0.000 (0.002)</td>
<td>0.001 (0.003)</td>
</tr>
<tr>
<td>GROUP</td>
<td>-0.035 (0.006)***</td>
<td>-0.035 (0.005)***</td>
<td>-0.034 (0.006)**</td>
<td>-0.012 (0.002)***</td>
</tr>
<tr>
<td>SDROA</td>
<td>0.131 (0.037)***</td>
<td>0.129 (0.036)***</td>
<td>0.127 (0.037)**</td>
<td>0.053 (0.022)**</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.085 (0.017)***</td>
<td>-0.081 (0.018)***</td>
<td>-0.084 (0.016)***</td>
<td>-0.069 (0.012)***</td>
</tr>
</tbody>
</table>

| N           | 2,621                 | 2,621                                 | 2,621                              | 6,290                                               |
| R²          | 0.14                  | 0.15                                 | 0.18                               | 0.16                                                |

Note: The dependent variable is ROA. First OLS is an ordinary least squares regression model with year dummies. Second OLS is with year, state and line dummies. HECKMAN is a self-selection effects regression with maximum likelihood method. MULTLINE is equal to one for diversified insurers, and zero otherwise. SIZE is equal to the natural logarithm of total admitted assets. CAPASSET is the ratio of policyholder surplus to total admitted assets. GEODIV is equal to one minus the Herfindahl index of premiums across 57 geographic areas. WCONC is the weighted sum of firm market share per line multiplied by each line’s Herfindahl index. PCTLH is the percentage of premiums attributable to life–health insurance. MUTUAL is equal to one if the ultimate ownership form is mutual, zero otherwise. GROUP is equal to one for aggregated groups, zero otherwise. PUBLIC is equal to one if the insurer is publicly traded, zero otherwise. SDROA5 is the standard deviation of ROA. Standard errors (in parentheses) for OLS models are corrected for clustering at the insurer level. Our final sample includes 591 firms or 2621 firm-year observations where 2500 firm-year observations are single-line firms and 121 firm-year observations are diversified firms. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively.
Table 3. Effect of diversification on the return on assets based on propensity score matching estimators

<table>
<thead>
<tr>
<th></th>
<th>Nearest Neighbor Matching</th>
<th>Kernel Matching</th>
<th>Stratification Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROA</td>
<td>-0.0146</td>
<td>-0.0133</td>
<td>-0.0129</td>
</tr>
<tr>
<td></td>
<td>(-0.91)</td>
<td>(-1.21)</td>
<td>(-1.96)</td>
</tr>
<tr>
<td>N</td>
<td>2621</td>
<td>2621</td>
<td>2621</td>
</tr>
</tbody>
</table>

Note: The dependent variable is ROA. Nearest Neighbor Matching uses for each treated observation i, it selects a control observation j that has the closest x. Kernel Matching uses for each i, it matches with several control observations with weights inversely proportional to the distance between treated and control observations. Lastly, Stratification Matching uses to compare the outcomes within interval/blocks of propensity scores, for instance, 10 blocks in this paper. Our final sample includes 591 firms or 2621 firm-year observations where 2500 firm-year observations are single-line firms and 121 firm-year observations are diversified firms. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively.
APPENDIX

Propensity Score Matching

The propensity score is defined by Rosenbaum and Rubin (1983) as the conditional probability of receiving a treatment given pretreatment characteristics:

$$p(X) \equiv \Pr(D = 1 | X) = E(D | X)$$

where $D = \{0, 1\}$ is the indicator of exposure to treatment and $X$ is the multi-dimensional vector of pretreatment characteristics.

Rosenbaum and Rubin (1983) show that if the exposure to treatment is random within cells defined by $X$, it is also random within cells defined by the values of the one-dimensional variable $p(X)$. As a result, given a population of units denoted by $i$, if the propensity score $p(X_i)$ is known, then the Average effect of Treatment on the Treated (ATT) can be estimated as follows:

$$\tau = E\{Y_1i - Y_0i | Di = 1\} = E\{E\{Y_1i - Y_0i | Di = 1, p(X_i)\} | Di = 1\}$$

where the outer expectation is over the distribution of $(p(X_i) | Di = 1)$ and $Y_1i$ and $Y_0i$ are the potential outcomes in the two counterfactual situations of treatment and no treatment.

Three types of Propensity Score Matching

(1) Nearest Neighbor Matching

Let $\tau$ be the set of treated units and $C$ the set of control units, and let $Y_\tau i$ and $Y_C j$ be the observed outcomes of the treated and control units, respectively.

$$C(i) = \min_i \| p_i - p_j \|$$

Denote by $C(i)$ the set of control units matched to the treated unit $i$ with an estimated value of the propensity score of $p_i$. that is a singleton set unless there are multiple nearest neighbors.

(2) Kernel Matching

The kernel matching estimator is given by

$$\tau^K = \frac{1}{NT} \sum_{i \in T} \left\{ Y_i^T - \frac{\sum_{j \in C} Y^C_j G\left( \frac{p_i - p_j}{hn} \right)}{\sum_{k \in C} G\left( \frac{p_k - p_i}{hn} \right)} \right\}$$

where $G(*)$ is a kernel function and $hn$ is a bandwidth parameter. Under standard conditions on the bandwidth and kernel,
(3) Stratification Matching

This method is based on the same stratification procedure used for estimating the propensity score. Note that by construction in each block defined by this procedure, the covariates are balanced and the assignment to treatment can be considered random. Hence, letting \( q \) index the blocks defined over intervals of the propensity score, within each block the program computes

\[
\tau_q^S = \frac{\sum_{i \in I(q)} Y_i^T}{N_q^T} - \frac{\sum_{j \in I(q)} Y_j^C}{N_q^C}
\]

where \( I(q) \) is the set of units in block \( q \) and \( N_T^q \) and \( N_C^q \) are the numbers of treated and control units in block \( q \)