Health Care Expenditure Shocks and Optimal Annuitization Considerations of Longevity Risk

Abstract

This paper conceptualizes and investigates the interplay of longevity risk and health care expenditure shocks in determining retirees' optimal annuitization decisions. In a life-cycle framework where bequest motives and consumption floors are considered, we examine how longevity risk affects an individual's health state transition process and thus annuity purchase decisions by considering mortality as the end state of such a transition process. Using health transition matrices estimated from the Health and Retirement Survey (HRS) data, we find that retirees have higher annuity demand when considering health shocks, and longevity risk has an even larger impact on their demand when health shocks are taken into account. In addition, the more longevity risk is borne by the poorer health states, the higher the increase in annuity demand. These findings suggest that with longevity risk, annuity is an important hedging instrument for health shocks occurring later in life. A preliminary empirical analysis from a focus group study provides supporting evidence for our predictions.

Keywords: Longevity risk, Annuity demand, health care expenditure shock, health transition matrix. (*JEL*)

1 Introduction

The classical rational choice theory predicts that annuity is attractive to people (Yaari 1965, Davidoff et al. 2005). Along this direction, a natural inference is that with longevity risk, people become more willing to purchase annuity. However, this prediction is not consistent with the observation that relatively few households facing retirement choose to annuitize a substantial portion of their wealth, i.e., the so-called "annuity puzzle." In a rational framework, many important factors affecting annuity demand and general retirement financial decisions have been identified in the recent literature to interpret this puzzle. These factors include preannuitized wealth such as social security and pension plans (Dushi and Webb 2004), uncertain income (Scholz et al. 2006), unfair annuity pricing (Mitchell et al., 1999), bequest motives (Kopczuk and Lupton 2007 and Lockwood 2012), and the minimum consumption level (Ameriks et al. 2011 and Pashchenko 2013). In particular, one such factor is health care expenditure shock that has recently been recognized as of significant relevance in determining retirees' annuitization decisions (Turra and Mitchell 2008, Davidoff 2009, Ameriks et al. 2011, MacMinn and Weber 2011 and Pashchenko 2013).[1]

This strand of literature suggests that to understand the impact of longevity risk on retirees' annuitization under a rational framework, the above important factors should be taken into account. Among these factors, health care expenditure shocks play a key role because longevity risk implies that as people live longer, they are also more likely to face these shocks. This close connection between health and longevity has yet to be considered in current studies of longevity risk and annuity demand.[2] This paper aims to fill this gap by

[1]Apart from the rational framework, behavioral factors have also been proposed to explain the annuity puzzle, such as framing effect (Brown et al. 2008) and reference dependence (Hu and Scott 2007). A comprehensive review of the theory and practice on annuity demand is nicely presented by Benartzi et al. (2011).

[2]The literature connecting long-term care insurance and annuity focuses on the natural hedging between long-term care insurance and annuity since with individuals need long-term care, they become unhealthy and their life expectancy becomes shorter, thereby decreasing expected annuity payments, and visa versa (for example, see Ameriks et al. (2011) and Brown and Warshawsky(2013)). These studies do not consider the interplay between longevity risk and health shocks, in which an important concern is how the pattern of health shocks changes in the context of longevity risk and how that impacts annuitization.

examining the impact of longevity risk on retirees' annuity demand in a life-cycle framework in the meanwhile specifically accounting for health care expenditure shocks they face. While focusing on conceptualizing and modeling the interplay between longevity risk and health shocks, our model also accommodates other relevant factors, including bequest motives and government provided minimum consumption level.

Conceptually, we consider death as the end state in an individual's health status transition process, where different health states (for example, good health, minor health problems, major health problems, etc.) are associated with different health care costs that should be part of the optimization. As individuals move among different health states in this transition process and eventually reach the state of death (which is, of course, irreversible) in their life cycle, mortality rates are naturally associated with the probabilities of other health states. In this vein, mortality improvement implies not only that individuals expect to live longer, but they are also more likely to experience health care expenditure shocks. This possibility has important implications on annuity demand because now retirees not only need more money for their living expenses but also for higher health care costs.[3]

This interplay between longevity and health shocks gives rise to a trade-off between precautionary saving motives that reduce annuity demand and the desire to use annuity as a hedging tool for increased health care shocks and thereby increased annuity demand. This trade-off has been analyzed in a simple two-period model by Davidoff et al. (2005). To characterize the uncertainty of the health care expenditure shocks, they assume that shocks only possibly occur in the second period while annuitization decision was made in the first period. On the one hand, retirees try to precautionarily hedge the health care expenditure shocks by purchasing medical insurance in the first period, reducing the demand for annuity. On the other hand, retirees may purchase more annuity in the first period to hedge against

[3]Yang et al. (2003) find that health care expenditures for elderly people increase substantially with age primarily because mortality rates increase with age and health care expenditures increase as one moves closer to death. The pattern that out-of-pocket medical expenses rise quickly with age is also documented by De Nardi et al. (2010). These evidences suggest that the longer people live, the more prolonged time they spend in relatively unhealthy health states and the higher health care expenditures they incur.

health care expenditure shocks occurring later in the second period. Subsequent studies have provided evidence for the precautionary savings motive in light of unexpected health care expenditure, and find that it makes annuity moderately less attractive (e.g., Turra and Mitchell 2008, Ameriks et al. 2011, and Pashchenko 2013). Similarly, Davidoff (2009) argue that with a high percentage assets in illiquid home properties, long-term care insurance becomes a substitute for annuity. Notably, Pashchenko (2013), while allowing retirees to make annuity purchase at multiple times in the life cycle, recognizes the flip-side of the trade-off that annuity can subsidize health care spending shocks occurring later in life by showing that retirees prefer to annualize more later in life. Our study adds to this strand of literature and provide further supporting evidence for the flip-side of the trade-off. While verifying and formally documenting findings from prior literature (such as adverse selection in annuity purchase, the effects of consumption floor and bequest motives), we find that with longevity risk, retirees consider primarily the hedging role of annuity for health risk later in life and thus purchase more annuity. Because longevity risk is more relevant for older ages and thus has a higher impact on health shocks occurring later in life, the hedging role of annuity for health shocks becomes the dominating effect of the trade-off.

More specifically, we show that the retirees expecting health care expenditure shocks prefer to buy more annuity, and further increase their purchase with longevity risk. The effect of longevity risk on increasing their annuity demand is larger when health shocks are considered than when they are not considered. The more longevity risk is borne by the poorer health states, the higher the increase in annuity demand. This also suggests that previous analysis of longevity risk omitting health shocks underestimates the impact of longevity risk on annuity demand. The predicted positive relationship between health shocks and annuitization is supported by an empirical investigation via a focus group study of two small groups of people (60-70 years of age) from the general public, in which we find a 0.38 correlation between the annutization percentage and the percentage of monthly income expected to spend on out-of-pocket health care expenditure at age 80.[4]

Our study makes two contributions to the longevity risk literature. First, our results suggest that health care expenditure shocks play a key role in analyzing the impact of longevity risk on retirees' annuitization, and thus complements previous studies. As reviewed by Cairns et al. (2008) and Blake et al. (2011, 2013, 2014), studies of longevity risk focus primarily on mortality risk modeling and hedging, while largely omitting the potential role of closely related health shocks. An important implication of our analysis is that when considering health shocks, the scale of mortality risk borne by insurance companies might become significantly different from previous analysis. However, the extent and the significance of the differences are important empirical questions that should be addressed by future research. Another contribution is that to our best knowledge we are the first to propose the conceptualization of the interplay between longevity risk and other health states in a health status transition process. To capture this key interaction, we also propose a practical calibration method based on large scale data bases available to most researchers. More specifically, we estimate the health transition process by using data from the comprehensive and publicly available Health and Retirement Survey (HRS) conducted by the University of Michigan and borrow a health transition matrix structure first proposed by Ameriks et al. (2011) to link it to mortality rates from the Human Mortality Database (HMD) and projected mortality rates based on that. Our analysis provides a simple and extendable framework to accommodate the interplay of health and mortality, which can be used in future studies of longevity risk and annuitization.

The rest of the paper is organized as follows. In section 2, the life-cycle model is introduced and the role of health dynamics is explained. Section 3 estimates the health transition matrix using the HRS data and performs mortality projection using HMD data. Section 4 conceptualizes the health transition process and its connection to mortality rates, and cali-

[4] A brief description of background information on the focus group empirical investigation is provided in Appendix D.

brate these matrices with and without mortality improvement using the HMD mortality and the projected mortality. Section 5 presents the numerical results and elaborates on the interaction between health care expenditure shocks and longevity risk in determining retirees' annuity demand. Section 6 concludes the paper.

2 The Model

In this model, we consider retirees' annuitization with considerations for health care expenditure shocks and longevity risk. We assume a life-cycle (retired) individual lives over the period $n = 0, \ldots, N$, where $n = 0$ and $n = N$ correspond to her retirement age and the maximum age, respectively, with death possibly occurring in any period. In this paper, we assume that the retirement age and the maximum age are 65 and 100 respectively. The death rate q_n in period n evolves in a manner defined subsequently in this section.

Preferences

Assume that a retiree's lifetime utility is time-separable, and in period n she has a constant relative risk aversion utility function over real consumption C_n:

$$u(C_n) = \frac{C_n^{1-\gamma}}{1-\gamma}, \tag{1}$$

where γ is the parameter measuring the extent of risk aversion. He also receives a bequest utility if he dies in period n, denoted by $v(B_n)$. Hence her lifetime utility is

$$E_0 \sum_{n=0}^{N} \beta^n \left(\Pi_{j=0}^{n-1}(1-q_j)\right) \left[(1-q_n)u(C_n) + q_n v(B_n)\right] \tag{2}$$

The retiree leaving a bequest B receives direct utility:

$$v(B) = \frac{\omega}{1-\gamma}\left(\phi + \frac{B}{\omega}\right)^{1-\gamma}, \tag{3}$$

where $\omega > 0$ measures the intensity of the bequest motive and $\phi > 0$ is a shift parameter treating bequests as luxury goods, thus allowing for zero bequests among low-wealth individuals. This form of bequest utility function is also adopted by Ameriks et al. (2011) and Pashchenko (2013).

Health Dynamics and Costs

We follow Ameriks et al. (2011) to model retirees' health dynamics as four distinct states: (1) good health, (2) poor health but requiring no long-term care, (3) requiring long-term care, and (4) death. An individual's health state evolves as a Markov chain with an age-varying one-period transition matrix among the different states.[5] Death is thus the end state of such a health transition process. While death is irreversible, two-way transitions are allowed between the first three states. Following Ameriks et al. (2011), we associate each health state with a deterministic health cost that increases as health deteriorates and ignore costs associated with death.

Annuity

An individual enters retirement with wealth $W_0 \geq 0$. At the beginning of retirement, he has an opportunity to buy an immediate annuity. For an initial premium, an immediate annuity provides annual payments that begin immediately and last for life. Given annual payoff Y_n in period n, the actuarially fair price of the immediate annuity beginning right after retirement, i.e. $n = 0$, is equal to

$$A = \sum_{n=1}^{N} R_f^{-n} \Pi_{m=1}^{n} (1 - q_m) Y_n \tag{4}$$

For simplicity, this paper only considers fixed annuity, i.e. the annuity payment in each period is the same. We denote this fixed payment as Y, then $Y_n = Y$. Given retirees' initial

[5]The health dynamics can be alternatively modeled and estimated by use of logit model, in which dependent variable is the current health state and the explanatory variables include previous health state, age, gender and permanent income etc. (Turra and Mitchell 2008, De Nardi, et al. 2010). For our purpose of matching the health dynamics with longevity risk, the method of health transition matrix seems more intuitively appealing and tractable.

wealth W_0, we define the annuitization rate as

$$r = \frac{A}{W_0},$$

and thus the annuity income in each period with annuitization level r becomes

$$Y = \frac{rW_0}{\sum_{n=1}^{N} R_f^{-n} \Pi_{m=1}^{n} (1 - q_m)}.$$

We hence denote Y, as a function of r, as $Y(r)$ in the remainder of this paper.

Investment

In the retirement period n, the individual allocates a fraction w_n of her residual wealth in risky equity with real return $\tilde{R}_n$, and $1 - w_n$ of the wealth in a riskless bond with real return R_n^f. The return $\tilde{R}_n$ is log-normally distributed as $LN(\mu, \sigma^2)$.

Consumption floor and public long-term care

Social security provides a "consumption floor" for retirees in case their wealth plus the annuity income, if any, is insufficient to pay for health care expenses and minimum consumption. In health states 1 and 2, the consumption floor is denoted by C^f. Here we assume that when receiving government transfers in any period, a retiree will give up all her remaining wealth, i.e., her wealth becomes zero at the end of the period. In state 3, the long-term care state, the consumption floor is defined as $C^{PC} < C^f$, capturing the institutional reality of Medicaid. In this case, the retiree cannot afford the cost of long-term care plus the public care level of minimum consumption C^{PC} and thus receive a transfer from government. The assumption $C^{PC} < C^f$ reflects individuals' aversion to publicly provided long-term care, and creates a strong incentive for retirees to maintain sufficient wealth or income to cover the possible cost of private care.

Mortality projection

Substantial improvement in longevity at older ages occurs during the 20th century and becomes an important challenge for individuals in retirement planning. To capture mortality improvement in our model, we adopt the classic Lee-Carter model (Lee and Carter 1992) to calibrate the historical mortality data. In the Lee-Carter framework, an individual's mortality rate $q_{x,t}$ at age x in the year t is modeled as a function of age-specific parameters α_x, β_x, and mortality time-series κ_t with a random factor $\mu_{x,t}$:

$$ln(q_{x,t}) = \alpha_x + \beta_x \kappa_t + \mu_{x,t} \quad (5)$$

where α_x represents the age group shift effect, and β_x denotes the age group's reaction effect to the mortality time-series κ_t. We follow Lee and Carter (1992) to estimate the parameters in the model using a two-stage singular value decomposition. For the detailed implementation procedure of this estimation, the reader can refer to Deng et al. (2013). Note that for an individual who retires at age x during the year t, his death rate q_n in period n equals to $q_{x+n,t+n}$.

The optimization problem

The retiree enters the period n with health state s_n and wealth W_n. The timing of the events is as follows:

1. If $s_n = 4$, a retiree dies and leaves the bequest $B_n = W_n$. Note that the mortality rate in period n corresponds to the probability of being in state 4, the end state of the four-state health status transition, i.e. $q_n = \frac{P\{s_n=4\}}{1-P\{s_n=4\}}$.

2. If $s_n < 4$, the retiree receives her annuity income and pays health care costs, and then chooses the consumption amount and decides the portfolio allocation w_n. She can make any consumption choice C_n that exceeds the consumption floor C^f in health states 1 and 2 or C^{PC} in state 3 and satisfies the budget constraint

$$W_n + Y(r) - H(s_n) - C_n > 0, \quad (6)$$

where $Y(r)$ is the annuity income in period n if the individual has an annuitization level $r \in [0, 1]$, and $Y(r) = 0$ if and only if $r = 0$. $H(s_n)$ represents the individual's health care expenditure in health state s_n. If no consumption $C_n > C^f$ in health states 1 and 2 (C^{PC} in state 3) satisfies equation (6), the retiree will receive a transfer from social security (Medicaid), consume C^f (C^{PC}), and give up all remaining wealth. Letting I_n^G be an indicator variable for social transfer in period n, the retiree's wealth in period $n+1$ is thus given by

$$W_{n+1} = \begin{cases} (W_n + Y(r) - H(s_n) - C_n)(R_n^f + (\tilde{R}_{n+1} - R_n^f)w_n) & \text{if } I_n^G = 0; \\ 0 & \text{if } I_n^G = 1. \end{cases} \tag{7}$$

The retiree maximizes her expected utility from the lifetime consumption (2) subject to the budget constraints (6) and (7). The Bellman equation is

$$U_n(s_n, W_n, r) = \begin{cases} \max_{C_n}[u(C_n) + \beta E_n U_{n+1}(s_{n+1}, W_{n+1}, r)] & \text{if } s_n < 4; \\ v(B_n) & \text{if } s_n = 4. \end{cases} \tag{8}$$

subject to (3), (6) and (7). Note $U_n(s_n, W_n, r)$ in Equation (8) implies that the lifetime utility at the beginning of period n is determined by the initial health state s_n, the wealth level W_n and the annuitization level r that determines the income $Y(r)$ received by retirees if they are alive in Period n.

We solve this problem by standard solution techniques with backward induction adopted in the life cycle literature (Cocoo 2005, Yao and Zhang 2005, Turra and Mitchell 2008, Pashchenko 2013). In period N, the retiree dies surely and her utility in this period comes entirely from her bequest, $U_N(s_N, W_N, r) = v(B_N)$. In every prior period n ($n = 0, \ldots, N-1$), we use the Bellman equation (8) to obtain the optimal choice sequentially. By discretizing the state space, we employ a fine grid for W and the corresponding consumption, and use cubic interpolation to compute continuation values for points that are not on the grid.

In this way, the optimal consumption path over all periods can be calculated for any initial annuitization level r and the resulting lifetime utility at age 65, $U_0(s_0, W_0, r)$, can be obtained. Then we can find the optimal annuitizaton level that leads to the maximal lifetime utility. In Section 5, we will present two sets of results: the one without considering health care expenditure shocks and the one accounting for health shocks. The former case is a simplification of the latter case. In the former case, retirees have only two health states, survival or death, and consequently have no out-of-pocket health care expenditure when they are alive.

3 Estimation of Health and Mortality Dynamics

We first calibrate the health and mortality dynamics. Although we can obtain the mortality rates at each age directly, our model considers mortality as the end state of the health transition process. Therefore, we need to obtain a consistent probability distribution of different health states, including the state of mortality. The ideal dataset for this estimation should be a representative dataset containing health and mortality information for sufficiently large cohort-based samples of retiree at all retirement ages. Unfortunately, such a dataset is not available to us. For our estimation, we use the nationally representative core samples from the Health and Retirement Survey (HRS) conducted by the University of Michigan.[6] This panel survey, designed to provide comprehensive demographic, health, and financial information on individuals from pre-retirement into retirement, has been conducted every 2 years from 1992. A very brief introduction of the HRS survey is provided in the Appendix A and more details are available on the website of HRS. Specifically for our analysis, we use the 2006 core data, 2008 core data and 2008 exit data to estimate retirees' health dynamics. We choose the 2006 and 2008 HRS data in order to use the calibrated values of the out-of-pocket health care expenditure in different health states in Ameriks et al. (2011). Starting

[6] Available from the website of HRS study at http://hrsonline.isr.umich.edu/index.php. We choose to use the HRS database because it provides the most comprehensive publicly available data on individuals' health and retirement related variables. This longitudinal survey data is also suitable for our calibration of retirees' health dynamics over different ages.

with individuals interviewed in 2006, we include all individuals in both the 2006 and 2008 data, including those survived to be included in the 2008 core data and those that have died and thus are included in the 2008 exit data.[7] In this way, we can obtain the empirical distributions of individuals' health states at each age in 2006 and in 2008, and then obtain the estimated two-year health transition matrix between different ages.[8] Here we take two year as one period and partition the overall retirement from the retirement age 65 to the maximum age 100 into 18 periods (i.e., $n = 0, \ldots, 17$). The nth period represents the one from age $65 + 2n$ and age $65 + 2(n + 1)$. For instance, the period 0 represents the period from age 65 to 67. Because of data availability, we can only use the mixed cohorts during a short period of time (i.e., from 2006 to 2008) to carry out this estimation. To the extent that different cohorts of retirees in the data share the same pattern of health dynamics, we can use the estimated health transition matrix to calibrate all retirees' health dynamics in the simulation analysis.[9]

We consider only individuals over age 65 who are in both the 2006 and the 2008 survey data. This leaves us with 11,740 individuals, of which 5,011 are men and 6,724 are women. 5 records were deleted due to gender inconsistency between the 2006 and 2008 data. We follow Ameriks et al. (2011) to model retirees' health dynamics with four distinct states: (1) good health, (2) poor health but requiring no long-term care, (3) long-term care, and (4) death. Because Ameriks et al. (2011) used a proprietary survey to define the four health states while we use the HRS survey data that does not provide such information directly, we need an operational definition of the four states for our estimation. We define the first

[7]The interview of those who died by 2008 is conducted through proxy respondents.

[8]Because HRS survey is conducted every 2 years, we can only obtain the estimated two-year health transition matrix. Therefore, we consider two-years as one period in all of our analysis.

[9]Due to relatively small sample sizes of HRS for each cohort and its relatively short length, direct estimation of health transition matrix by use of the cohort information in HRS data is not feasible. An alternative method is to use period data offered by HRS to estimate the cohort health transition matrix in the same fashion as how the estimation of cohort life table in HMD data is done. Although this method seems promising, the estimation technology suitable for the health transition matrix still need to be developed and is beyond the scope of this paper. As a first step to explore the impact of the interplay between health shocks and longevity risk, we choose instead a simple and feasible approach to calibrate the health transition matrix as described in this paper.

three health states (except the state of death) by aggregating the 10 health categories in Brown and Warshawsky (2013) as follows. State 1, the state of good health, is when the self-reported health status is good to excellent. State 2, poor health but requiring no long-term care, is defined as a state with difficulties in 0-1 activities of daily living (ADLs) but no cognitive impairment, or some major chronic illness, such as heart problems, diabetes, lung disease, and stroke, or when self-reported health is poor to fair. State 3, the long-term care state, is defined as having 2 or more ADLs or cognitive impairment. These health measures are all probed in HRS survey. Under this operational definition, the health states 1, 2 and 3 in our analysis correspond to the category 1, 2-4 and 5-10 of Table 1 of Brown and Warshawsky (2013) respectively. We then obtain the health states of individuals in the 2006 and 2008 data according to this operational definition, and calculate the proportion of individuals transmitting between different health states from 2006 to 2008 to construct the estimated health transition matrix. To remove the impact of HRS's oversampling in blacks and Hispanics, we use the HRS sample weights to adjust the sample population of blacks and Hispanics in the estimation. In this manner, we obtain the HRS-estimated distribution vector of health states 1-4 at age 65 to be the vector $(0.6494, 0.3342, 0.0164, 0)$ for female and $(0.7134, 0.2789, 0.0077, 0)$ for male. We denote this initial distribution vector at age 65 by α and the estimated health transition matrix in period n by $H_E(n)$. The vector of $\alpha\Pi_{n=0}^{m} H_E(n)$ then represents the probability distribution of health status at the end of the period m (or at the beginning of the period $m+1$, i.e., at age $65+2(m+1)$). For instance, $\alpha H_E(0)$ represents the probability distribution of health status at age 67. The fourth element of $\alpha\Pi_{n=0}^{m} H_E(n)$ corresponds to the mortality from age $65+2m$ to age $65+2(m+1)$ (for instance, the fourth element of $\alpha H_E(0)$ represents the mortality from age 65 to 67), since death is the end state (state 4) of the transition process. With the assumption of maximum age 100, we thus obtain a set of mortality variations from age 65 to 100 with two-year increments from the estimated health transition matrix. The corresponding estimated survival probabilities from age 65 to 100 are shown in Figure 1 below.

In order to examine the impact of longevity risk, we also need to estimate retirees' mortality improvement. Following the large longevity risk literature, we use 1933-2010 data from the Human Mortality Database (HMD) to project mortality improvement. We adopt the classic Lee-Carter model (Lee and Carter 1992) to implement the projection. Note that we take mortality data in the year 2006 as the starting time and use the projected mortality of an individual at age 65 in the year 2006 to the age 100 until the year 2041 as the measure of longevity risk. The projected survival probabilities are plotted in Figure 1. HMD life-table mortality rates are also shown in this figure as a benchmark. We can see from Figure 1 that compared with the 2006 HMD life-table mortalities and the survival probabilities generated by the HRS-estimated health transition matrix, the projected mortalities exhibit significant improvement, reflecting the impact of longevity risk.[10] Interestingly, females' estimated survival probabilities from HRS data are always lower than the HMD life-table survival probabilities while males' estimated probabilities from HRS data are lower than the HMD ones at younger ages but become higher at older ages. Overall, our estimated HRS survival probabilities are close to the HMD ones although the latter curve is smoother reflecting the larger population used for estimation.[11]

4 Calibration of Health Dynamics with Longevity Risk

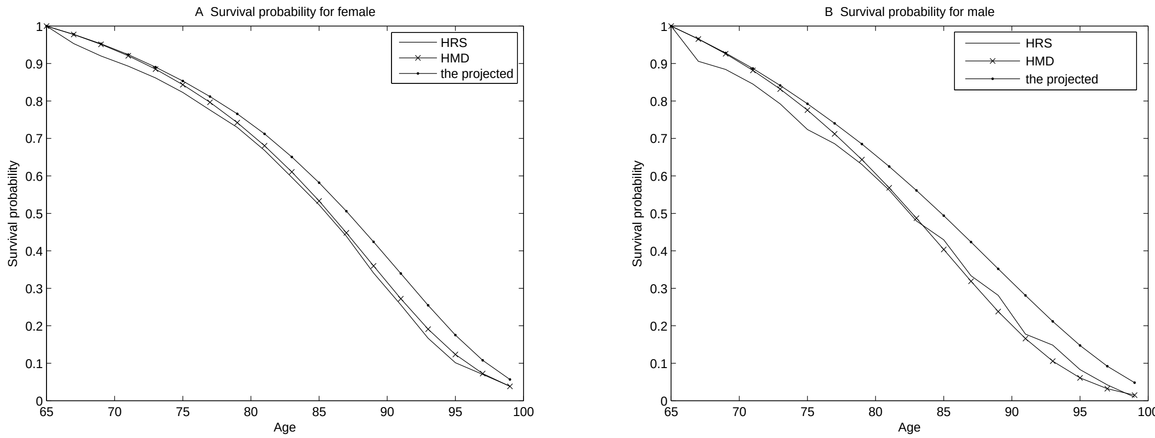
4.1 Adjustment of Health Transition Matrix

In order to examine the effects of longevity risk in individual's optimal decision process trading off annuity purchase and health care expenditure shocks, we need to ensure that the mortality rates as the end state of the health transition matrix are comparable to the

[10] In this figure, the estimated survival probability from HRS data is for a two-year interval. In order to ensure consistency between different cases, we calculate the two-year survival probability for HMD data and the projected mortality. All figures in this paper are shown with two-year intervals.

[11] The HRS mortality curve is less smooth because of the smaller sample sizes available for the estimation of the health transition matrix. We only have 790 and 649 data points for female and male retirees of age 65-66, respectively. Moreover, the number of retirees decreases with age, and retirees of age 97-98 reduce to 28 for female and 12 for male respectively.

Figure 1: Retirees' survival probability



In this figure, the horizontal axis and vertical axis represent the consumer's age and survival probability respectively. Panels A and B describe the survival probability of female and male respectively. In both panels, "HRS","HMD" and "the projected" represent the survival probability estimated from HRS data, HMD life-table data and the projected mortality data, respectively.

projected mortality rates by the Lee-Carter model. As we can see from Figure 1, the mortality rates generated directly by the HRS-estimated health transition matrix are different from the 2006 HMD life-table mortalities that are the basis for the projected mortality rates. Therefore, we need to first make adjustments to these estimated mortality rates.

Because the estimated mortality rates are a coherent part of the health transition matrix, we cannot simply change the mortality rates to match the 2006 HMD life-table mortalities. Instead, we need to adjust the entire health transition matrix so that the mortality rates as the probabilities of state 4 in the transition matrix become identical to the HMD mortalities. Lack of any clear choice of such an adjustment procedure from the previous literature, we resort to two basic principles and make use of a reasonable adjustment structure proposed by Ameriks et al. (2011). The two basic principles are: first, the elements in the matched health transition matrix should be larger than 0 and the sum of all elements in each row equals 1. Second, the derived probabilities of different health states should exhibit a reasonable pattern from younger age to older age that the probability of keeping good health decreases with age and the probability of death increases with age.

Based on these two principles, we employ a period-specific age-adjusted auxiliary matrix $M_0(n)$ to achieve this goal. The vector $\alpha\Pi_{n=0}^{m}[H_E(n)M_0(n)]$ represents the probability distribution of health states at age $65 + 2(m + 1)$. The fourth element of this vector thus corresponds to the mortality from age $65+2m$ to age $65+2(m+1)$, which we need to match with the HMD life-table mortalities over the same period. We borrow the structure of the age-adjusted matrix in Ameriks et al. (2011) to construct M_0 as follows.[12]

$$M_0 = \begin{bmatrix} 1-c_1 & c_1\left(\frac{c_2c_3}{1+c_2+c_2c_3}\right) & c_1\left(\frac{c_2}{1+c_2+c_2c_3}\right) & c_1\left(\frac{1}{1+c_2+c_2c_3}\right) \\ 0 & 1-c_1 & c_1\left(\frac{c_2}{1+c_2}\right) & c_1\left(\frac{1}{1+c_2}\right) \\ 0 & 0 & 1-c_1 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Loosely speaking, the parameter c_1 controls the transition between death and other three health states; c_2 and c_3 not only control how fast the shift of probability mass from death to health states 2 and 1 is relative to that from death to health state 3 respectively, but also affect directly the relative probability weighting between health states 1-3. Here we keep c_2, c_3 unchanged and let $c_2 = 1.3, c_3 = 1.1$ for females and $c_2 = 0.8966, c_3 = 0.5643$ following Ameriks et al. (2011), and change c_1 in each period to obtain the desired mortalities. Therefore, the estimated parameter c_1 varies with the period n.[13] For our matching purpose, the advantage of adopting such a matrix structure and only changing c_1 in such a matrix is that it simultaneously adjusts the probability of all three health states in a uniform way when accounting for the difference between the mortality rate generated by the HRS-estimated health transition matrix and the 2006 HMD life-table mortality. Specifically, note that

[12]As we mentioned above, the HRS-estimated health transition matrix is not smooth enough due to the limited sample size. Therefore, as a preliminary step for the adjustment, we first smooth the original estimated health transition matrix by replacing the outliers in the matrix, such as the zero value of probability in the matrix, by the arithmetical average of the corresponding elements of the matrix in the neighboring ages.

[13]Ameriks et al. (2011) calibrate 12 parameters in the health transition matrix to match eight moments related to long-term care utilization and four moments related to longevity using proprietary survey data, as reported in Table 1 of their paper. We use the calibrated values of c_2 and c_3 for females and males in our analysis.

elements on the diagonal of the health transition matrix represent the probability of staying in one health state from this period to the next, and c_1 changes this probability evenly for each of the health states 1-3 in the matching process. Here c_2 and c_3 ensure the overall probability adjustment maintain the relative proportions for health states 1-3.

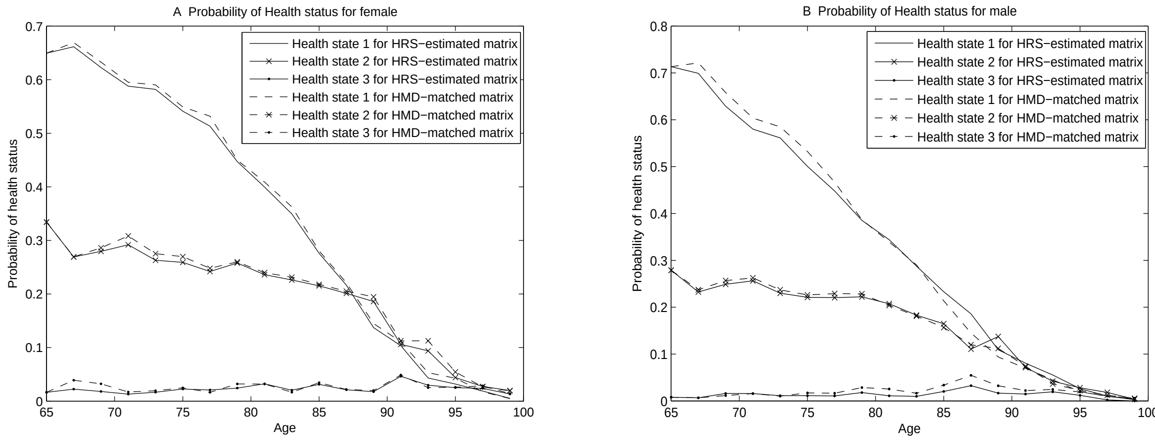
With the matched parameter $c_1(n)$, we can obtain the matched health transition matrix $H_A(n) \equiv H_E(n)M_0(n)$ in period n ($n = 0, \ldots, 17$) and use this matched matrix as the benchmark for later analysis. In the remainder of this paper, this matched matrix is referred to as the HMD-matched health transition matrix. This calculation process is illustrated by an example given in Appendix B.

Figure 2 shows the probabilities of health states 1-3 at each age based on the HMD-matched health transition matrix and compares them with those from the HRS-estimated matrix. Intuitively, the probabilities of health states 1 and 2 generally decrease with age while the probabilities of health state 3 increase slightly. Therefore, probabilities of healthier states are much higher than the probabilities of the long-term care state at younger ages and all probabilities become more indistinguishable at very old age (e.g., age 95 and above). Overall, the probabilities of the three health states are all close to each other in the HRS-estimated case and the HMD-matched case. For females, the probabilities from the HMD-matched matrix are larger than the ones from the HRS-estimated matrix, whereas for males, the probabilities from the HMD-matched matrix are larger than ones from the HRS-estimated matrix at younger age but are smaller at older age. These patterns are consistent with the differences between the HMD mortality and the HRS-estimated mortality, as shown in Figure 1.

4.2 Health Transition Matrix with Mortality Improvement

Now we turn to considering the interplay between mortality improvement and health dynamics. The large literature of health care suggests that with medical technology development comes improved health and hence mortality improvement (Najman and Levine, 1981, Bunker

Figure 2: The probability of health states 1-3 at different ages



In this figure, the horizontal axis and vertical axis represent the age and the probabilities of health states 1-3 respectively. Panels A and B describe the probability of health states for female and male, respectively.

et al. 1994 and Rice and Fineman 2004). However, this improvement may also result in prolonged time spent in relatively unhealthy health states.[14] Consequently, longevity risk not only entails retirement living expense concerns but also concerns for possibly increasing health care expenditure shocks. As we aim at studying the tradeoff between the risk of longevity on annuity purchase and that on health care expenditure shocks, we need to estimate a longevity-adjusted health transition matrix reflecting the overall effect of longevity risk on the probabilities of each of the three health states and the mortality rates. Again, note that in our model framework, mortality is the end state of a coherent health transition process. Therefore, any changes in one element (e.g., the probability of health state 4, or death rate) result in an updated transition process that has to be taken into account in the model. To explore this question, we consider three basic scenarios and use three auxiliary matrices to capture different possible types of adjustment for longevity risk. The first scenario is that the reduced mortality rate is allocated to increase the probability of having good health (i.e., health state 1). This scenario is characterized by the auxiliary matrix M_1 and

[14]Yang et al. (2003) find that health care expenditures for elderly people increase substantially with age primarily because mortality rates increase with age and health care expenditures increase as one moves closer to death. The pattern that out-of-pocket medical expenses rise quickly with age is also documented by De Nardi et al. (2010). These evidences suggest that the longer people live, the more prolonged time they spend in relatively unhealthy health states and the higher health care expenditures they incur.

the mortality improvement allocation is captured by parameter d_1. Specifically, we adjust the value of d_1 such that the derived mortality rate in the distribution vector of health status at the end of the period m (i.e., the mortality rate from age $65 + 2m$ to age $65 + 2(m + 1)$), $\alpha\Pi_{n=1}^{m}[H_A(n)M_1(n)]$, is equal to the projected mortality ($m = 0, \ldots, 17$). The corresponding longevity-adjusted probabilities of the three health states for females and males are shown in Panel A of Figure 3 and 4 respectively. In a similar fashion, we consider the second and the third scenarios as the probability transfer from the death state (health state 4) to health state 2 and health state 3, captured by the matrix M_2 and M_3 and parameters d_2 and d_3, respectively. The resulting longevity-adjusted probabilities of health states for females and males are shown in Panels B and C of Figure 3 and Figure 4, respectively. The probabilities from the HMD-matched transition matrix of different health states are reproduced in all panels of both figures for comparison purposes. An example calculation for each of the three scenarios is illustrated in Appendix C.

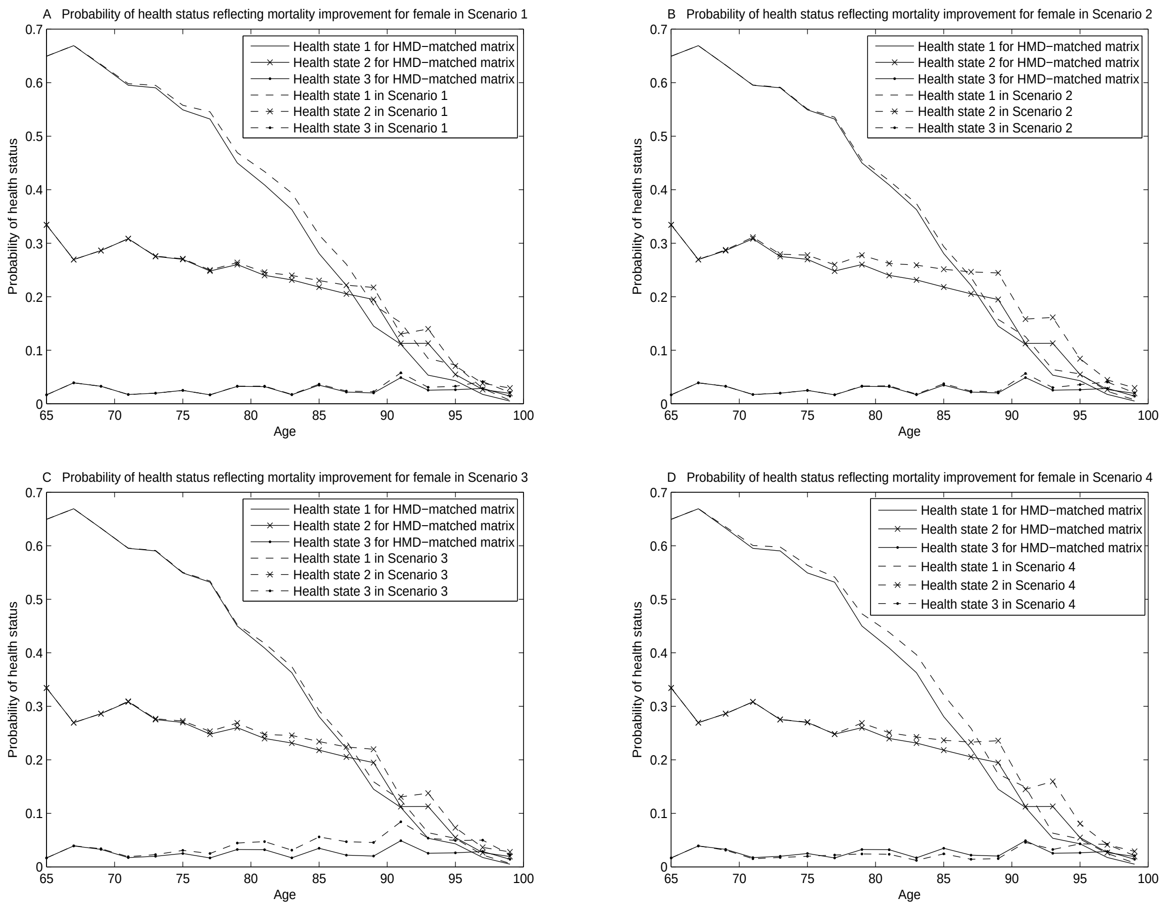
$$M_1 = \begin{bmatrix} 1+d_1 & 0 & 0 & -d_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, M_2 = \begin{bmatrix} 1 & d_2 & 0 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} M_3 = \begin{bmatrix} 1 & 0 & d_3 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where d_1, d_2, d_3 are the parameters to be estimated.

As a robustness check, we also consider a fourth scenario by adopting the transition matrix of Ameriks et al. (2011), defined as auxiliary matrix M_4, to account for the mortality improvement,

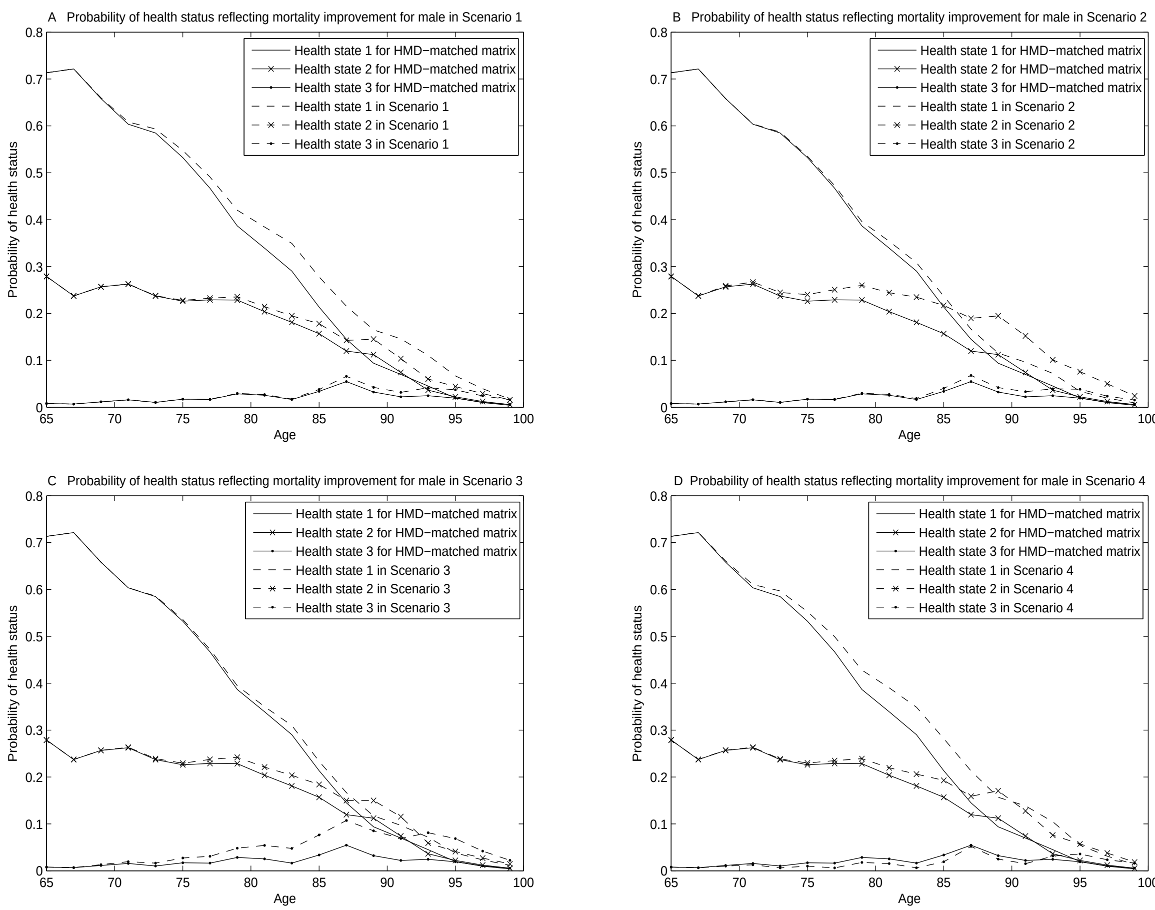
$$M_4 = \begin{bmatrix} 1-d_4 & d_4\left(\frac{d_5 d_6}{1+d_5+d_5 d_6}\right) & d_4\left(\frac{d_5}{1+d_5+d_5 d_6}\right) & d_4\left(\frac{1}{1+d_5+d_5 d_6}\right) \\ 0 & 1-d_4 & d_4\left(\frac{d_5}{1+d_5}\right) & d_4\left(\frac{1}{1+d_5}\right) \\ 0 & 0 & 1-d_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Figure 3: The probability of health states 1-3 for females with mortality improvement



In this figure, the horizontal axis and vertical axis represent the age and the probabilities of health states 1-3 respectively. Panels A, B, C and D describe the probability of health states for females in Scenarios 1-4 respectively.

Figure 4: The probability of health states 1-3 for males with mortality improvement



In this figure, the horizontal axis and vertical axis represent the age and the probabilities of health states 1-3 respectively. Panels A, B, C and D describe the probability of health states for males in Scenarios 1-4 respectively.

Note that M_4 has the same structure as M_0 as shown in Equation (9) although M_4 is used here to capture the impact of mortality improvement on the health state transition process whereas M_0 was previously used to ensure that the HRS-estimated health transition matrix is consistent with the HMD life-table mortality rates. Again, we keep d_5 and d_6 unchanged (let $d_5 = 1.3$, $d_6 = 1.1$ for females and $d_5 = 0.8966$, $d_6 = 0.5643$ as calibrated in Ameriks et al. (2011)) and adjust d_4 such that the derived mortality rate in the distribution vector of health states at the the end of the period m, $\alpha\Pi_{n=0}^{m}[H_A(n)M_4(n)]$, is equal to the projected mortality.[15] The resulting probabilities of the three health states in scenario 4 for females and males are reported in Panel D of Figure 3 and 4, respectively. Again, the health state probabilities from the HMD-matched transition matrix are reproduced in Panel D for comparison purposes.

Females (Figure 3) and males (Figure 4) exhibit the same pattern in all scenarios so we discuss them together here. In the first scenario where the reduced mortality rate is initially allocated to state 1, the probability that retirees live with good health (health state 1) becomes significantly larger with mortality improvements. However, we can also observe that the probability of being in health state 2 also increases moderately but the probability of health state 3 only increases slightly and only at older ages. This result represents the transmission effect from one state to another due to the two-way transitions among health states 1, 2 and 3 across many time periods. This transmission effect among states and across time periods is illustrated and further explained by the example presented in Appendix C. Indeed, after the first period health state transition, the reduced mortality rate is only transferred to the the probability of health state 1-3, respectively in Scenarios 1-3. However, as retirees age and go through subsequent periods of health transitions, not only will the probability of the initial corresponding health state in each scenario change,

[15]Here we adopt this specific form of auxiliary matrix M_4 as a possible alternative scenario in which all three health states change simultaneously to accommodate the mortality improvement. One, of course, can use another structure for this auxiliary matrix or use another set of values for d_5 and d_6 in M_4 to illustrate the possible interplay between health shocks and longevity risk. As we explain further below, we believe that important insights from the different scenarios remain the same.

the probabilities of other health states will change as well in every scenario. We observe this similar dynamics for scenarios 2-4. In the second scenario where the reduced mortality rate is initially allocated to state 2, the probability of health state 2 increases the most while the probabilities of health states 1 or 3 only increase slightly. In the third scenario, the probability of health state 3 increases the most whereas the probability of health state 1 increases the least. In the fourth scenario where the reduced mortality rate was allocated to all the other three states simultaneously, the probabilities of both health states 1 and 2 increase moderately whereas the probability of health state 3 decreases a bit at younger ages and increases a little at older ages. The pattern of probability change in health states 1-3 for Scenario 4 is not only driven by the transmission from the death state to these health states due to mortality improvement, but also by the transmission between these three health states (recall the two-fold role of d_5 and d_6). Although we only examine four typical cases to illustrate how longevity risk might influence the health transition matrix and the probabilities of each health state, the transmission effects described above suggest that these four scenarios are not the polar cases. While by definition the overall impact of mortality improvement is positive on the likelihood of being in one or more of the other three health states, through the four cases, we are able to capture different patterns in allocating the mortality rate improvement to each of the three different health states.

5 Retirees' Optimal Annuitization

To identify the effects of health shocks with longevity risk on retirees' annualization decisions, we first consider the baseline case without any considerations for health care expenditure shocks, and then add in the health shocks to compare retirees' optimal annuitization under each case. In both cases, we explore the impact of longevity risk by comparing retirees' optimal annuitization under the HMD life-table mortality with that under the projected mortality.

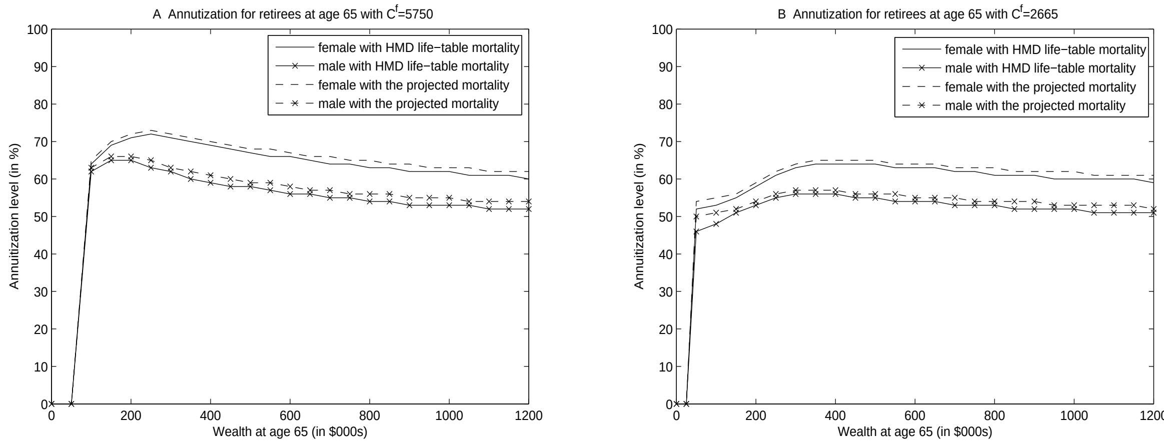
5.1 Parameter Setting and Numerical Solution

We follow the life cycle literature to set values for common parameters . As in Turra and Mitchell (2008), Ameriks et al. (2011) and Pashchenko (2013), we set β, the time preference discount factor, to be 0.97, and γ, the constant relative risk aversion coefficient, to be 3. We follow Ameriks et al. (2011) to set the benchmark state-dependent health costs to be: $1000 in state 1, $10000 in state 2, $50,000 in state 3, and 0 in state 4 (we ignore costs associated with death).

For investment allocation, we follow Cocoo(2005) and Yao and Zhang (2005) and assume that the real risk-free rate $R_f = 2.0\%$. As in Yao and Zhang (2005), the risk premium is set at $\mu = 4.0\%$ and the standard deviation of the risky asset is $\sigma = 15.7\%$. Following Ameriks et al. (2011), the consumption floor for retirees in health states 1 or 2 is set at $C^f = \$5,750$ and the consumption floor for retirees in health state 3 is $C^{PC} = \$2,200$.

Under the above parameter setting, the optimization problem of annuitization is solved with grid searching method in two steps. First, we discretize annuitization level into a grid of 100 over the interval [0,1], and given any of these discretized annuitization level, we use the Bellman equation (8) and the numerical solution method described in Section 2 to obtain the optimal consumption path over the entire retirement period and calculate the resulting lifetime utility at age 65. In this procedure, as a state variable in every period, the wealth level W_n is discretized into a grid of 24 over the interval [0,120,000]. With the discretized wealth level in each period, we can get the corresponding optimal consumption and the lifetime utility from this period, and further use cubic interpolation to compute the lifetime utility values in each period for points that are not on the grid in the backward induction. Next, we choose the annuitziaton level that leads to the maximal lifetime utility at age 65 over all discretized annuitization values as the optimal annuitization for retirees.

Figure 5: Optimal annuitization for retirees without health shocks



In this figure, the horizontal axis and vertical axis represent the retirees' wealth level and annualization level respectively. Panels A and B describe the respective annuitization level of retirees under two different consumption floor assumptions.

5.2 Optimal Annuitization without Health Shocks

This subsection analyzes the baseline case where retirees do not factor in health care expenditure shocks in their optimal decision making process. In this case, they can have only two health states, survival or death, and consequently have no out-of-pocket health care expenditure when they are alive. With this simplification, we eliminate any effects of the health shocks and can thus focus on the impact of other important factors on retirees' optimal annuitization, including longevity risk, bequest motive and the consumption floor. Note that in this case, there is only one consumption floor as the differentiation between long-term care state and other health states is no longer relevant. Figure 5 illustrates optimal annualization decisions for females and males in this baseline case.

In this figure, the solid curve and the dash curve represent retirees' optimal annuitization under HMD life-table mortality and the projected mortality improvement, respectively. Not surprisingly, both females and males purchase more annuity when there is longevity risk. Females purchase more annuity than males because of longer life expectancy. However, retirees with different initial wealth do not fully annuitize their wealth, resulting mainly from their bequest motives.[16]

Interestingly, comparing Panel A with Panel B of Figure 5 in which the different level of consumption floor is set, we can see that the impact of consumption floor is two-fold. One the one hand, retirees with low wealth prefer not to purchase annuity because of the government guaranteed consumption floor, and the higher the consumption floor, the more likely they are to do so. Indeed, knowing that they will receive government transfer when their beginning wealth plus income in any period is lower than a threshold, retirees with low initial wealth prefer not to annuitize their wealth such that they can consume more early in their retirement and still maintain the minimum consumption level determined by the consumption floor later in life, and thus achieve higher utility. On the other hand, the lower consumption floor provides a higher incentive for retirees with high wealth to maintain sufficient wealth to retain the private consumption option. As a result, given a lower consumption floor, retirees with high wealth prefer to purchase less annuity.

5.3 Optimal Annuitization with Health Shocks

Now we consider the effect of health care expenditure shocks in retirees' optimal decision making process and further investigate health-dependent longevity risk. As described previously, retirees live in three health states: the state of good health, the state of poor health but requiring no long-term care, and the state of long-term care. Therefore, under the parameter setting of Subsection 5.1, we obtain the optimal annuitization with the HMD-matched

[16] Although this paper does not attempt to explain the "annuity puzzle," our results are not inconsistent with the low ratio of annuitization well documented in the literature. The annuitization rate obtained within our framework under the proposed parameter setting is not too high when additional factors are taken into account. We made a few simplifying assumptions in our main model framework in order to keep our analysis trackable as we primarily model the health shocks with longevity risk. For example, we assume that all wealth can be annuitized. However, illiquid assets such as houses often account for a large proportion of retirees' total assets, and yet cannot easily be annuitized. This factor effectively decreases the predicted annualization (out of retirees' total wealth) to be more in line with what is observed in the marketplace (Pashchenko 2013). Additionally, the optimal annualization in our main model framework represents the percentage of wealth transformed into an annual income after retirement. The actual annuity purchase thus would be significantly less than what our model predicts if retirees have preannuitized wealth from social security and/or pension funds, which may also account for a large part of retirees' total wealth (Pashchenko 2013).

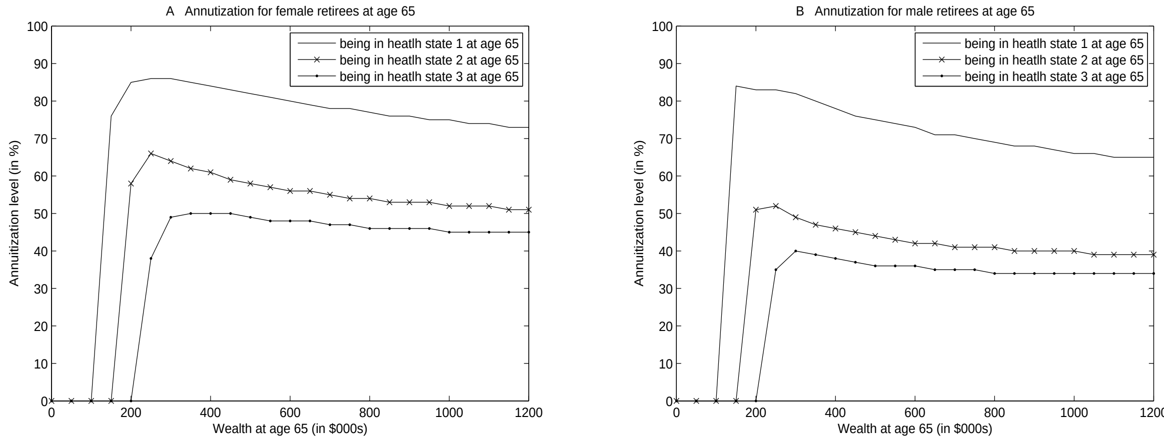
health transition matrix H_A and under the longevity-adjusted health transition matrix H_AM_i $(i = 1, 2, 3, 4)$. The results are shown in Figures 6, 8 and 9.

Figure 6 shows the optimal annuitization of the retirees in each of the three health states at different wealth levels at age 65, given the HMD-matched health transition matrix. Females and males are presented in Panels A and B respectively. For simplicity, we assume that there is no discrimination in annuity pricing with respect to the insured' health state, which seems to be consistent with current practice and the prior literature.[17] Consequently, adverse selection occurs, as can be observed in Figure 6. The retirees in the good health (health state 1) at age 65, whether female or male, have the highest optimal annuitization while the retirees needing long-term care (health state 3) annuitize the least. This is true at any initial wealth level above the respective minimum threshold when retirees start to purchase annuity. This leads to adverse selection when healthier people purchase more annuity and acquire higher annuity income on average, and is consistent with prior literature (Finkelstein and Poterba 2002, 2004). Interestingly, we can also see from the figure that the less healthy an individual is initially, the more unlikely she will purchase annuity with low wealth. This pattern reflects the impact of the consumption floors.

Figure 7 compares optimal annuitization with and without health care considerations and illustrates the impact of health care expenditure shocks on retirees' annuitization decisions. The dashed line represents the retirees' weighted average annuitization over three possible health states 1-3 while the solid line reproduces what was shown in Figure 5. Note that longevity risk is not considered for the moment. Figure 7 shows that both female and male retirees with wealth level above the minimum threshold determined by the consumption floors, annuitize more when they take health shocks into account. Recall that retirees have to make a trade-off in deciding annuitization while managing health care expenditure shocks that can occur in the future. On the one hand, annuity may decrease the precautionary

[17] Finkelstein and Poterba (2002) point out: " In the UK market, most firms do not collect any information on the annuitant beyond age and gender."

Figure 6: Optimal annuitization for retirees with health shocks

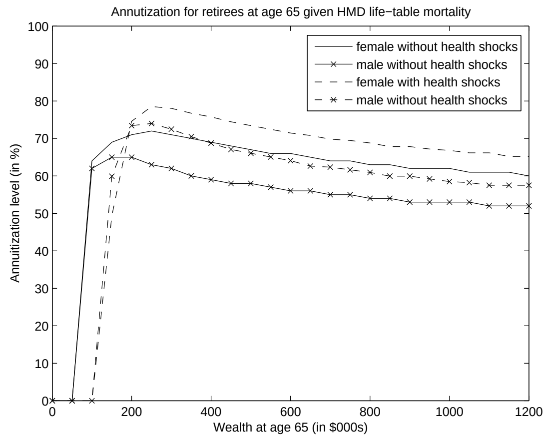


In this figure, the horizontal axis and vertical axis represent the retirees' wealth level and annualization level respectively. Panels A and B describe optimal annualization for females and males respectively.

savings available for retirees to pay for costs associated with health shocks at younger ages, whereas on the other hand, annuity can serve as an instrument to hedge against the health care expenditure shocks occurring at older ages. Our results suggest that the more important feature of annuity at play here is to hedge against health care costs later in life. This figure again demonstrates the effect of the consumption floors: compared to when there were not health shocks, retirees with low wealth levels become even more unwilling to purchase annuity in order to consume more *and* pay for health costs early in their retirement.

This paper is most interested in how health-dependent longevity risk affects retirees' optimal annuitization. We obtain the optimal annuitization decisions given each of the four sets of longevity-adjusted health transition matrices, corresponding to Scenarios 1 to 4 described in Section 4. Optimal annuitization for retirees in health states 1 to 3 at age 65 with different initial wealth levels are presented in Panels A to C, respectively, of Figure 8 for females and 9 for males. For comparison purposes, the benchmark cases where longevity risk is not considered (as shown in Figure 6) are also plotted in each panel of Figures 8 and 9. Note that for ease of presentation, each of the curve in the figures has been truncated by the wealth level from the left to show only the part of the curve where individuals have positive annuity demand.

Figure 7: Retirees' optimal annualization with and without health shocks

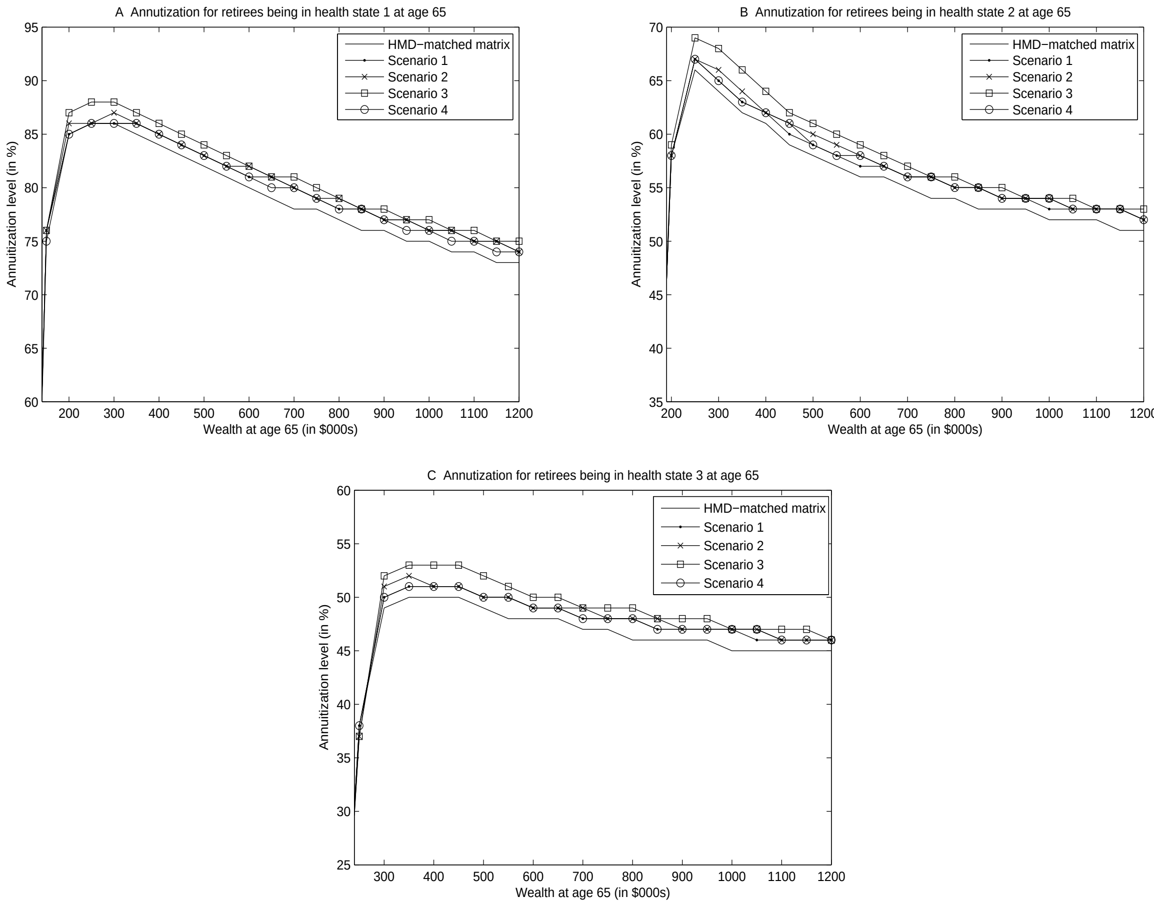


In this figure, the horizontal axis and vertical axis represent the retirees' wealth level and the annualization level respectively.

Figures 8 and 9 show that with health-dependent mortality improvement, retirees prefer to purchase more annuity and the increase in annuity demand may be larger than when not considering health shocks. As shown in both figures, there is a clear pattern that compared to benchmark case with no longevity risk, the annuity demand is higher in all four scenarios and the increase in demand becomes larger as we move from Scenario 1 to 3. For female retirees, annuity demand in Scenario 1 is 1-2 percent higher than the baseline case, and the increase in Scenarios 2 and 3 are around 2-3 and 3-4 percent, respectively. For male retirees, the increase in Scenarios 2 and 3 are even larger. Note that as the allocation pattern of reduced mortality moves from what is in Scenario 1 to 2 and 3, retirees are more likely to become unhealthy with longevity improvement, thus facing higher future health care expenditure shocks and needing more money to pay for the associated costs. To the extent that annuity serves as an hedging instrument for these health care costs, retirees will prefer to purchase more annuity. This result indicates that when mortality improvement implies that retirees expect to live longer, but more likely in poor health, they have higher demand for annuities.

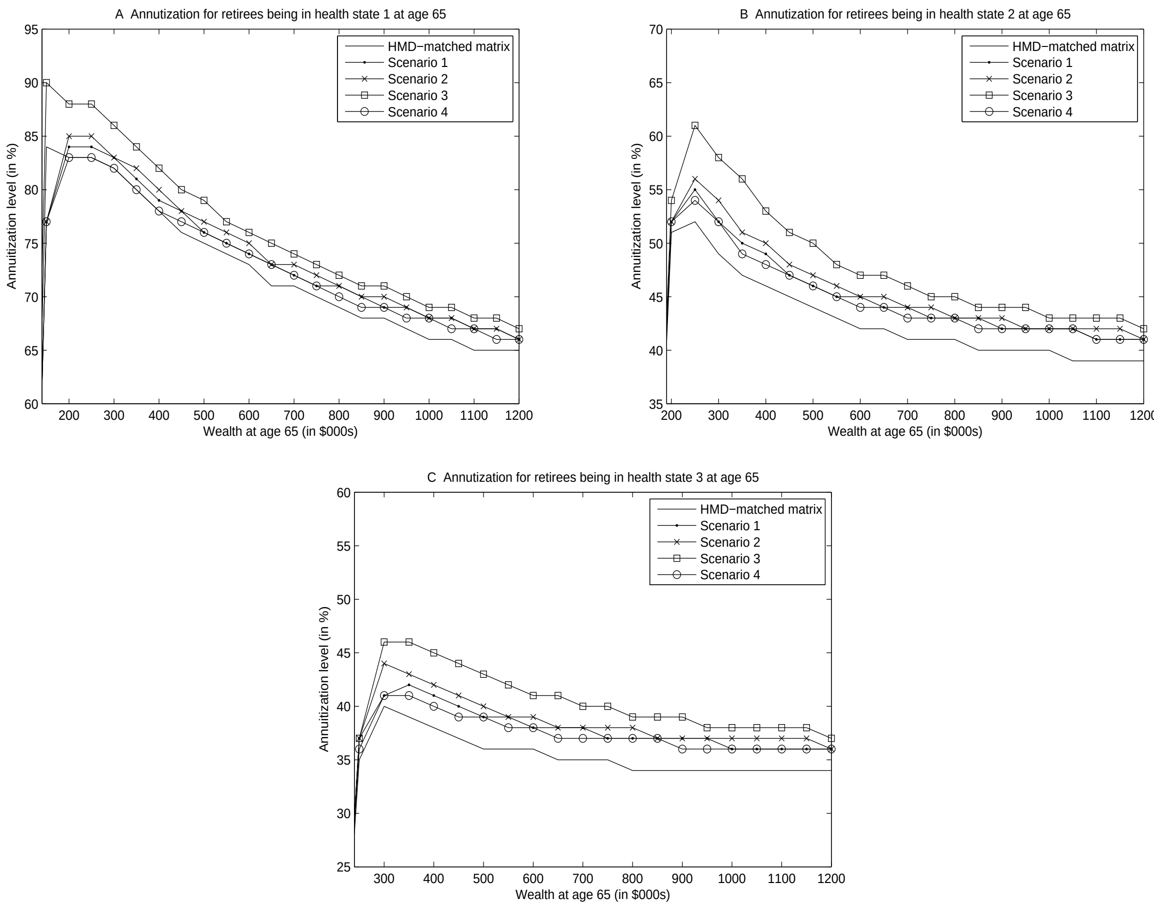
In addition, compared with the impact of longevity risk without health care costs considerations (1 percent increase in annuity demand as shown in Figure 5), the impact of longevity risk is economically more significant when health care shocks are taken into ac-

Figure 8: Optimal annualization for females with mortality improvement



In this figure, the horizontal axis and vertical axis represent the retirees' wealth level and the annualization level respectively. Panels A, B and C describe optimal annuitization for females with initial health states 1-3 at age 65 respectively.

Figure 9: Optimal annualization for males with mortality improvement



In this figure, the horizontal axis and vertical axis represent the retirees' wealth level and the annualization level respectively. Panels A, B and C describe optimal annuitization for males with initial health states 1-3 at age 65 respectively.

count in retirees' optimization process. This result suggests that retirees' annuity demand is significantly affected by the health-dependent mortality improvement, and thus analysis of the impact of longevity risk on annuity demand should not ignore the closely related health shocks. Scenario 4 demonstrates the robustness of our analysis. Recall that the health care expenditure shocks in Scenario 4 result in higher impact on health states 1 and 2, as was shown in Panel D of Figure 3 and 4. As a result, we find that the optimal annuitization in this scenario lies between the benchmark case with no mortality improvement and Scenario 3. However, recall that even Scenario 3, the scenario that results in the largest increase in annuity demand in our analysis, does not represent an extreme case. There are still scenarios in practice representing other ways to capture how mortality improvement may affect the health state transition matrix. These scenarios may lead to an even larger increase in annuity demand resulting from longevity risk and the health transition process. One plausible scenario is to decrease the probabilities of health states 1 and 2, and increase correspondingly the probability of health state 3, representing what was documented in the health care literature that as people live longer, they are also more likely to incur higher health care expenditure shocks. The goal of this paper is to make a first attempt at conceptualizing the interplay of longevity risk and the health status transition process in determining optimal annuitization. While we propose a set of typical longevity-adjusted health transition matrices to illustrate our framework and derive insights, it is not our goal to empirically verify these matrices. Exactly how and to what extent health transition matrix should change in the context of longevity risk under each practical situation warrants further empirical research.

Lastly, similar to Figure 7, we can see from Panels A to C of Figure 8 and 9, the less healthy a retiree was initially, the minimum wealth level required for her to purchase annuity becomes larger. Indeed, retirees with poor health realize that they will face higher health care expenditure on average and their low wealth cannot pay for all the associated costs in the entire retirement periods. As a strategic response, they just decide not to annuitize their wealth and consume it at younger ages and pay for health care costs, and then enjoy government transfer for consumption and public care subsidy at older ages.

We perform various sensitivity analysis using alternative values of the risk aversion parameter γ and the discount factor β. In these alternative settings, our main conclusions on the impact of longevity risk and health care expenditure shocks on optimal annuitization are robust.

6 Discussions and Conclusion

This paper studies the interaction between longevity risk and health care expenditure shocks in determining retirees' annuity demand. In a life-cycle framework where bequest motives and consumption floors are considered, we examine how longevity risk affects an individual's health state transition process and thus the associated out-of-pocket health care expenditure by considering mortality as the end state of such a transition process. Using health transition matrices estimated from the Health and Retirement Survey (HRS) data, we find that retirees have higher annuity demand when considering health shocks, and longevity risk has an even larger impact on their demand when health shocks are taken into account. In addition, the more longevity risk is borne by the poorer health states, the higher the increase in annuity demand.

Our findings suggest that annuity is an important hedging instrument for health shocks in the context of longevity risk, and thus contribute to resolving competing theoretical predictions regarding the interactions of annuity purchase, precautionary savings, and health shocks. Our results also have practical implications for annuity product design and public policy making. For instance, we show that with longevity risk, health shocks may become larger and more annuities are required in late life. Consequently, the deferred annuity products aiming to pay at older ages or growing annuities might be more attractive for consumers. Our results also suggest that public policy makers should take into account the negative effects of consumption floors on annuity demand.

In addition, this paper contributes to the longevity risk literature by proposing a conceptualization of the interplay between longevity risk and the health status transition process, both of which play a key role in annuity purchase and other retirement financial decisions. Our analysis provides a simple and extendable framework to accommodate the interplay of health and longevity, which can be used in future studies of longevity risk and annuitization. To capture this key interaction, we also propose a practical calibration method based on large scale data bases available to most researchers.

As a first step toward understanding the interaction between health shocks and longevity risk, we offer a calibration of health dynamics with longevity risk by use of a Markov chain method in this paper. However, the proposed calibration has several limitations and warrants further methodological and empirical research. First, our estimation of the health transition matrix is based on the HRS data and thus cannot capture fully the inherent cohort effects of the health transition matrix. Second, we characterize some typical scenarios by illustrating how the pattern of the probability distribution of individuals' health status at each age might change with longevity risk. The actual pattern of health dynamics with longevity risk is a separate research question that should be explored by future empirical research. Moreover, our life-cycle model can be extended to incorporate into the analysis additional factors that may affect annuity demand, including perannuitized wealth considerations, illiquid assets such as houses, unfair annuity pricing and increasing health care expenditure, which is left for future research.

To strengthen our understanding of how retirees make annuity purchase decisions, we conduct an empirical investigation via a focus group study of two small groups of people (60-70 years of age) from the general public, followed by a post-study survey. Preliminary findings from the focus group study are consistent with the insights from our proposed life-cycle framework. First, individuals' willingness to annuitize their wealth is low on average and is very heterogeneous. The percentage of wealth that individuals in the focus groups is willing to annuitize ranges from 5% to 50%. Second, health care expenditure shocks seem to be a relevant consideration for these individuals in purchasing annuities although their expectations about future health care shocks vary widely. When asked what percentage of their monthly income they believed they have to spend on out-of-pocket health care costs (personally, not covered by Medicare and/or insurance) at age 80, responses ranged from 2% to 90% with a mean of 20.4% and a standard deviation of 18.9%. Third and most importantly, the correlation between the annutization ratio and the percentage of monthly income expected to spend on out-of-pocket health care expenditure at age 80 is 0.38. This result is consistent with our main finding that annuity is an instrument to hedge against health care shocks at older ages.

Moreover, except for the factors identified in the rational choice theory, we also find other behavior factors that are important in determining individuals' annuitization decisions from the focus group study. First of all, the focus group participants were very unknowledgeable about annuities, and possibly because of the lack of knowledge, the participants appeared very vulnerable to public relations efforts or any information from a credible source on the topic. This result suggests that behavioral factors may play an important role in affecting people's annuity purchase, such as framing effect (Brown et al. 2008) and reference dependence (Hu and Scott 2007). More details of the focus group study findings along with additional empirical analysis to explain individuals' annualization decisions in the context of longevity risk are part of ongoing research in a separate paper (Ai et al. 2015).

Appendix

A. A brief introduction of HRS data

The Health and Retirement Survey (HRS) conducted by the University of Michigan is a national longitudinal survey of the economic, health, marital, and family status, as well as public and private support systems, of older Americans. The HRS data includes a nationally representative core sample as well as additional samples of blacks, Hispanics and

Florida residents. The HRS data provides comprehensive information on the demographics of household members, their health, health services and insurance, housing, employment, assets and incomes, wills and life insurance, and social security for permitted respondents. In particular, the HRS database contains detailed health information, including information on physical health, cognition, functional limitations and helpers, physical measures and disability. The HRS data consists of different level of information suitable for different purposes of studies, including the household level, respondent level, sibling level, household member and child level, helper level, transfer-to-child Level and transfer-from-child level.

The HRS core data was collected in 1992, 1994, and 1996 and the Assets and Health Dynamics of the Oldest Old (AHEAD) data was collected in 1993 and 1995. These surveys were merged in 1998 to represent the U.S. population over age 50 in 1998. Subsequently, the HRS survey has been conducted every 2 years. Two new cohorts were added in 1998: the Children of the Depression (1924-1930) and the War Babies (1942-1947). A fourth cohort, Early Baby Boomers (1948-1953), was added in 2004 and a fifth cohort, Mid Boomers (1954-1959), was added in 2010.

B. An example calculation of the HMD-matched health transition matrix.

We illustrate the calculation of the matched health transition matrix via the following example. Given the female's first period estimated matrix (i.e., period 0, or the period from age 65 to 67)

$$H_E(0) = \begin{bmatrix} 0.87707163 & 0.10589806 & 0.00851516 & 0.00851516 \\ 0.26510328 & 0.58496775 & 0.03099470 & 0.11893427 \\ 0.20634070 & 0.29945456 & 0.39682965 & 0.09737509 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

We adjust c_1 in $M_0(0)$ such that the probability distribution of health status at age 67, $\alpha H_E(0)M_0(0)$ gives rise to a mortality rate over age 65-66 of 0.0224 (obtained from the HMD life-tables by calculating the death rate from age 65 to 67) in its fourth element.

Therefore, the age-adjusted matrix $M_0(0)$ is

$$M_0(0) = \begin{bmatrix} 1.0115 & -0.0044 & -0.0040 & -0.0031 \\ 0 & 1.0115 & -0.0065 & -0.0050 \\ 0 & 0 & 1.0115 & -0.0115 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the resulting HMD-matched health transition matrix

$$H_A(0) = \begin{bmatrix} 0.8798 & 0.1052 & 0.0074 & 0.0076 \\ 0.2659 & 0.5864 & 0.1000 & 0.0476 \\ 0.2070 & 0.3001 & 0.3973 & 0.0956 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The probability distribution of health status at age 67 becomes $(0.6691, 0.2694, 0.0391, 0.0224)$.

C. An example calculation of the longevity-adjusted health transition matrix in the Scenarios 1-3.

Considering health dynamics for females at age 65, the health transition matrices adjusted for mortality improvement allocation as described by Scenario 1-3 are presented below. Note that HA(0) was calculated in Appendix B. Note that all the auxiliary matrices M_i ($i = 1, 2, 3, 4$) are calculated in the same fashion as illustrated in Appendix B, i.e., by adjusting parameter c_1 so that $\alpha\Pi_{n=0}^{m}H_A(n)$ produces a probability distribution vector of health states, the fourth element of which now equals to the projected mortality rate for the period m

$(m = 0, \ldots, 17)$.

$$H_A(0) * M_1(0) = \begin{bmatrix} 0.8799 & 0.1052 & 0.0074 & 0.0075 \\ 0.2660 & 0.5864 & 0.1000 & 0.0476 \\ 0.2070 & 0.3001 & 0.3973 & 0.0956 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_A(0) * M_2(0) = \begin{bmatrix} 0.8798 & 0.1054 & 0.0074 & 0.0075 \\ 0.2659 & 0.5865 & 0.1000 & 0.0476 \\ 0.2070 & 0.3002 & 0.3973 & 0.0956 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_A(0) * M_3(0) = \begin{bmatrix} 0.8798 & 0.1052 & 0.0076 & 0.0075 \\ 0.2659 & 0.5864 & 0.1001 & 0.0476 \\ 0.2070 & 0.3001 & 0.3973 & 0.0956 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_A(0) * M_4(0) = \begin{bmatrix} 0.8801 & 0.1051 & 0.0073 & 0.0075 \\ 0.2660 & 0.5866 & 0.0999 & 0.0475 \\ 0.2070 & 0.3002 & 0.3974 & 0.0954 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

With the common distribution of initial health status for females at age 65, $\alpha = (0.6493, 0.3342, 0.0164, 0)$, the new distribution vectors of health states for females at age 67 become for the HMD-matched case and scenarios 1-3

$$\alpha H_A(0) = (0.6691, 0.2694, 0.0391, 0.0224),$$

$$\alpha H_A(0) M_1(0) = (0.6692, 0.2694, 0.0391, 0.0223),$$

$$\alpha H_A(0) M_2(0) = (0.6691, 0.2695, 0.0391, 0.0223),$$

$$\alpha H_A(0) M_3(0) = (0.6691, 0.2694, 0.0392, 0.0223).$$

In scenarios 1-3, the mortality rate is $0.0223 < 0.0224$, reflecting mortality improvement. Note that as a result of this first period health state transition, the reduced mortality rate 0.0001 is only transferred to the the probability of health state 1-3, respectively in Scenarios 1-3. The probabilities of other health states are not impacted. This is implied by how we construct the one period auxiliary matrix M_i ($i = 1, 2, 3, 4$). However, it is important to note that as retirees age and go through subsequent periods of health transitions, not only will the probability of the initial corresponding health state in each scenario change (as have been shown above), the probabilities of other health states will change as well in every scenario. This is easily seen by considering the rules of matrix operations and an example of the calculation results is given below for the distribution of health states for these females when they are at age 71 (after three periods of transition)

$$\alpha\Pi_{n=0}^{2}H_A(n) = (0.5952, 0.3083, 0.0171, 0.0794),$$

$$\alpha\Pi_{n=0}^{2}[H_A(n)M_1(n)] = (0.5980, 0.3085, 0.0171, 0.0763),$$

$$\alpha\Pi_{n=0}^{2}[H_A(n)M_2(n)] = (0.5955, 0.3110, 0.0171, 0.0763),$$

$$\alpha\Pi_{n=0}^{2}[H_A(n)M_3(n)] = (0.5955, 0.3093, 0.0188, 0.0763).$$

We now can see that compared with the HMD-matched case, the probabilities of both health states 1 and 2 increase in Scenario 1 and 2, and the probabilities of health states 1-3 all increase in Scenario 3. Similar results can be found by calculating these vectors for later ages. These results reflect the transmission effect permeating through the health status transition process due to two-way transitions among health states 1, 2 and 3 in an individual's life cycle.

D. A brief description on back ground information of focus group empirical investigation

A focus group is a technique commonly used in industry and marketing whereby a small group of people (8-12) gather to discuss a given topic. It is led by a moderator who asks predeveloped questions and probes but does not interfere with the participants' opinions. For this research, we conducted two different focus groups on the same evening with people who were both retired and not retired. The individuals were recruited by a professional research firm that specializes in focus groups (over 30 years in business) and were led by a professional focus group moderator (over 10 years experience). At no time were the participants altered to the word "annuity" during recruiting or the focus group discussion process, until the concept was introduced into the discussion by the moderator. This was critical to this study and closely monitored by the researchers. The purpose of the group discussion was stated to be "retirement".

After the focus groups were completed, participants filled out a short survey on their backgrounds. There were a total of nineteen focus group participants aged 60-70 years, with nine being female. Eight were 65 or older and eleven were younger than 65. Fifteen were married currently and seventeen have children. Seven participants are still employed full-time, six identified themselves as retired only, and six participants said they had some combination of employed and retired (including part-time employment).

References

Ai, J., P. L. Brockett, L. L. Golden and W. Zhu, 2015, Competition for Customer's Resources: The Plight of Annuities in Longevity Risk Protection, *Working Paper*.

Ameriks, J., A. Caplin, S. Laufer and S. Van Nieuwerburgh, 2011, The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives, *Journal of Finance*, 66(2): 519-561.

Benartzi, S., A. Previtero and R. H. Thaler, 2011, Annualization Puzzles, *Journal of Economic Perspectives*, 25(4): 143-164.

Blake, D., C. Courbage, R. MacMinn and M. Sherris, 2011, Longevity Risk and Capital

Markets: The 2010-2011 Update, *Geneva Papers on Risk and Insurance-Issues and Practice*, 36: 489-500.

Blake, D., R. MacMinn, J. S. Li and M. Hardy, 2014, Longevity Risk and Capital Markets: The 2012-2013 Update, *North American Actuarial Journal*, 18(1): 1-13.

Blake, D., R. MacMinn and R. Maurer, 2013, Longevity Risk and Capital Markets: The 2011-2012 Update, *Journal of Risk and Insurance*, 80(3): 495-500.

Brown, J. and M. Warshawsky, 2013, The Life Care Annuity: A New Empirical Examination of An Insurance Innovation that Addresses Problems in the Markets for Life Annuities and Long-term Care Insurance, *Journal of Risk and Insurance*, 80(3): 677-703.

Brockett, P. L. and L. L. Golden, 2007, Biological and psychobehavioral correlates of credit scores and automobile insurance losses: toward an explication of why credit scoring works, *Journal of Risk and Insurance*, 74: 23-63.

Brown, J.R., J. R. Kling, S. Mullainathan and M. V. Wrobel, 2008, Why Don't People Insure Late-Life Consumption? A Framing Explanation of the under-Annuitization Puzzle, *American Economic Review*, 98(2):304-309.

Bunker, J.P., H.S. Frazier and F. Mosteller, 1994, Improving Health: Measuring the Effects of Medical Care, *Milbank Quarterly*, 72(2): 225-258.

Cocoo, J. F., 2005, Portfolio Choice in the Presence of Housing, *Review of Financial Studies*, 18(2): 535-567.

Davidoff, T., 2009, Housing, Health, and Annuities, *Journal of Risk and Insurance*, 76(1): 31-52.

Davidoff, T., J. Brown, and P. Diamond, 2005, Annuities and Individual Welfare, *American Economic Review*, 95(5): 1573-1590.

De Nardi, M., E. French, J. B. Jones, 2010, Why do the elderly save? The role of medical expenses, *Journal of Political Economy*, 118(1): 39-75.

Dushi, I. and A. Webb, 2004, Household annuitization decisions: simulations and empirical analysis, *Journal of Pension Economics and Finance*, 3(2): 109-143.

Finkelstein, A. and J. Poterba, 2002, Selection Effects in the United Kindom Individual Annuities Market, *Economic Journal*, 112: 28-50.

Finkelstein, A. and J. Poterba, 2004, Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market, *Journal of Political Economy*, 2004, 112(1): 183-208.

Hu, W., and J. S. Scott, 2007, Behavioral Obstacles in the Annuity Market, *Financial Analysts Journal*, 63(6): 71-82.

Kopczuch, W., and J. Lupton, 2007, To Leave or Not to Leave: The Distribution of Bequest Motives, *Review of Economic Studies*, 74: 207-35.

Lockwood, L., 2012, Bequest motives and the annuity puzzle, *Review of Economic Dynamics*, 15(2): 226-243.

MacMinn, R. and F. Weber, 2011, The Annuity Puzzle, *Working Paper*.

Mitchell, O. S., J. Poterba, M. Warshawsky, and J. R. Brown, 1999, New Evidence on the Money's Worth of Individual Annuities, *American Economic Review*, 89: 1299-1318.

Najman, J. M., and S. Levine, 1981, Evaluating the impact of medical care and technologies on the quality of life: A review and critique, *Social Science and Medicine. Part F: Medical and Social Ethics*, 15(2-3): 107-115.

Pashchenko, S., 2013, Accounting for non-annuitization, *Journal of Public Economics*, 98: 53-67.

Rice, D. P. and N. Fineman, 2004, Economic Implications of Increased Longevity in the United States, *Annual Review of Public Health*, 25: 03.1-03.17.

Scholz, J. K., A. Seshadri, and S. Khitatrakun, 2006, Are Americans Saving 'Optimally' for Retirement? *Journal of Political Economy*, 114: 607-643.

Sinclair, S., Smetters, K., 2004. Health shocks and the demand for annuities. Technical Paper Series, 2004-9. Congressional Budget Office.

Turra, C. M., Mitchell, O. S., 2008, The impact of health status and out-of-pocket medical expenditures on annuity valuation, In: Ameriks, J., Mitchell, O. (Eds.), *Recalibrating Retirement SPending and Saving*, Oxford University Press, 227-250.

Yang, Z., E. C. Norton, and S. C. Stearns, 2003, Longevity and Health Care Expenditures The Real Reasons Older People Spend More, *Journals of Gerontology: Series B*, 58(1): S2-S10.

Yao, R. and H. H. Zhang, 2005, Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints, *Review of Financial Studies*, 18(1): 197-239.