Regret and Rejoicing Effects on Mixed Insurance*

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Abstract

This papers examines how regret and rejoicing influence on mixed insurance choice and demand. Unlike expected utility, both regret and rejoicing may explain why some people prefer to hold mixed insurance to term insurance. Over-insurance of mixed insurance is optimal under expected utility framework when its premium is actuarially fair. Since rejoicing weakens mixed insurance demand, there are possibilities that over- and under-insurance is optimal for incorporating rejoicing term.

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1. Introduction

People purchase term insurance to receive compensation for their bereaved family if people die within a certain term. Term insurance can cover mortality risk, but people may not be able to receive benefits from term insurance because they might survive when the policy term ends. In contrast, there is another type of life insurance that can receive benefits both if people die before the policy term ends and survive at the end of the policy term. This type of life insurance is called mixed insurance.

Endowment insurance is a typical example of mixed insurance. According to Dorfman (2008, p. 264), endowment insurance has both "the beneficiary paid if the insured dies before the policy matures" and "the endowment if he or she is alive when the policy matures". Thus, endowment insurance provides benefits in not only death case but also survival case.[2]

It is intuitive that mixed insurance seems to be strange from proper purpose of holding insurance. This intuition is supported by expected utility framework which is a dominant tool in an insurance economics. Since people suffers a mean preserving spread of final wealth by holding mixed insurance instead of term one, risk averse people never want to hold mixed insurance by a classical argument of Rothschild and Stiglitz (1971). Thus, expected utility theory concludes that holding a mixed insurance is irrational.[3]

However, there are some kinds of mixed insurance in an actual life insurance market. Thus, what are the rational reasons to hold mixed insurance is still an open

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[2] For example, according to Life Insurance Fact Book 2014 issued by the life insurance association of Japan (http://www.seiho.or.jp/english/statistics/trend/pdf/2014.pdf), the number of new policies of individual insurance (excluding converted contracts) of endowment insurance accounts for 12.38 percent in fiscal 2013.

[3] Another kind of explanation for insisting irrationality of mixed insurance may conducts through arbitrage opportunities. When mixed insurance can be replicated using other financial instruments like a combination of saving and term insurance, those are priced by excluding arbitrage opportunities. However, perfect replication is difficult in real situations by difference of liquidity. For example, it is difficult to withdraw money from a fixed deposit immediately at time of an accident.

question. In addition, how to decide optimal insurance coverage of mixed insurance is unclear. In order to find the answers from these questions, we attempt to provide a different theoretical ground which calls regret theory.

In section 2, background of this research is described. In section 3, we briefly explain the original regret and rejoicing formation, and introduce the regret formation and the rejoicing formation. In section 4, we consider a choice problem between term and mixed insurance. In section 5, we examine rejoicing effect on mixed insurance demand. Section 6 makes concluding remarks.

2. Background

Regret theory was originally introduced by Bell (1982) and Looms and Sugden (1982). Regret theory is formed by two components, regret and rejoicing by comparing actual wealth to reference one. To avoid confusion, this paper does not call “regret theory”, but “regret and rejoicing formation”. Regret and rejoicing formation is given axiomatic foundations by Sugden (1993) and Quiggin (1994). Since it is difficult to apply their original formation to most insurance problems, Braun and Muermman (2004) proposed a modified version of the original formation and examined regret effects on insurance demand. As they set the reference payoffs as the best wealth, people always feels regret from their actual wealth comparing the best wealth. Their formation was standardized by Wong (2011) and Solnik and Zuo (2012). In accordance with these previous studies, this paper applies rejoicing formation by setting the reference payoffs as the worst wealth.

3. Regret and Rejoicing Formation

A brief explanation of regret theory was introduced by Bell (1982) and Loomes and Sugden (1982). Their formulation included both regret and rejoicing. An agent feels regret when a forgone consequence she had not chosen is preferable than a consequence she had chosen. The rejoicing take place the opposite situation, that is, a consequence she had chosen is preferable than a forgone consequence. Their formulations, however, considered an agent only choose between two alternatives.

Braun and Muerrman (2004) treat a multiple alternatives case. The formulation is based on the best payoffs. It should be noted that this formulation is standard in the literatures (Wong (2011), Solnik and Zuo (2012)). The regret formation is represented by

$$U(w) = u(w) - g\big(u(w^{\max}) - u(w)\big). \quad (1)$$

Here, w denotes the actual wealth and $w^{\max}$ the ex post best wealth. u is assumed to be twice differentiable, $u'(x) > 0$ and $u''(x) \leq 0$. and $g: R_+ \to R$ captures regret and is a strictly increasing and strictly convex function with $g(0) = 0$. g is assumed to be twice differentiable, $g'(x) > 0$ and $g''(x) > 0$. g is called regret function and an agent is regret theoretical if she follows the preference representation (1).

We consider another extreme case, that is, the worst payoffs are set to be specific ones to compare actual wealth. In this case, an agent always feels rejoicing. This should be called rejoicing formation. It is represented by

$$U(w) = u(w) + h\left(u(w) - u\big(w^{\min}\big)\right). \quad (2)$$

Here w and u are defined the above and $w^{\min}$ is the ex post worst wealth. Rejoicing is captured by transforming the utility difference actual and the worst wealth trough $h: R_+ \to R$ which is a strictly increasing and strictly concave function with $h(0) = 0$. h is assumed to be twice differentiable. h is called rejoicing function and an agent is rejoicing theoretical if she follows the preference representation (2). This explanation is also applicable to regret function by substituting regret for rejoicing. First, concavity of the rejoicing function signifies that an agent dislikes mean preserving spreads of the utility difference between actual and the best wealth. Second, the total utility is concave by assuming $u'' \leq 0$ and $h'' < 0$, and this guarantees that optimal choices are characterized by the first-order conditions. However, there are few studies that have adopted rejoicing formation for both theoretical and empirical approaches unlike regret formation.

Thus, behavioral properties of rejoicing formation remain important issues for future research.

A rejoicing theoretical agent pays her attention to the worst wealth in her decision makings, whereas a regret theoretical agent does to the best wealth. It seems to be natural that an agent pays her attentions to specific losses that are subjects for insurance products when she chooses them. Accordingly, we believe that it is necessary to examine the rejoicing effect on insurance purchase decision besides the regret effect. Apart from insurance context, we consider that regret formation is more suitable in some cases and for some agents, and that rejoicing formation is in other cases and for other agents. It may be interesting that regret and rejoicing are dealt with simultaneously as in the original regret and rejoicing formation.

In this study, the signs of the second-order derivatives play important roles. We call these their concavity. For example, utility function u is strictly concave (linear) if $u'' < 0$ ($u'' = 0$). This way is applied to a regret function g and to a rejoicing function h. Braun and Muermann (2004) termed a convex regret function, regret aversion. However, this appears to be confusing terminology since an agent with $g' > 0$, always dislike regret even though g is concave. It is even more confusing in the case of rejoicing. So, unlike Braun and Muermann (2004), we do not term functions "averse", but them by their concavity.

4. Choice between Term and Mixed Insurance

4.1 Choice problem

Let us consider an agent who has to choose to hold either term or mixed insurance at the full insurance level. For the sake of comparison, both insurance premiums are set to be actuarially fair.

An agent has initial wealth of $W > 0$ and faces a risky loss. For simplicity, we consider that the risk can be captured by two states: one is called the loss state and the other is the no-loss state. The loss size is denoted by D with $W > D > 0$. The loss probability is $\pi \in (0,1)$ and the no-loss probability is $1 - \pi$. If an agent holds term insurance, they receive coverage D in the loss state and nothing in the no-loss state. If an agent holds mixed insurance, they receive coverage D in the loss state and also receive compensation C in the no-loss state, with $D > C > 0$. Since insurance premiums are actuarially fair, term insurance premium is given by $P_t = \pi D$ and mixed insurance premium by $P_m = \pi D + (1-\pi)C$. If an agent holds term insurance, her final wealth is given by $W_O = W - P_t$ in both states. If an agent holds mixed insurance, her final wealth is given by $W_L = W - P_m$ in the loss state and $W_{NL} = W - P_m + C$ in the no-loss state.[4] It is easy to confirm that the wealth holding mixed insurance is a mean-preserving spread of one with term insurance. As the result, every risk-averse agent prefers to hold term insurance rather to mixed insurance in the expected utility framework. In other words, it is not possible to justify why some people hold mixed insurance.

4.2 Regret formation

An agent follows the regret formation. The best wealth can be achieved by holding term insurance in the loss state, which is given by W_O, and by holding mixed insurance in the no-loss state, which is given by W_{NL}. The agent computes her total utility as follows. If an agent holds term insurance, her total utility is represented by:

$$\begin{aligned}&\pi\big[u(W_O) - g\big(u(W_O) - u(W_O)\big)\big] + (1-\pi)\big[u(W_O) - g\big(u(W_{NL}) - u(W_O)\big)\big]\\&= u(W_O) - (1-\pi)\, g\big(u(W_{NL}) - u(W_O)\big).\end{aligned}$$

The equality comes from $g(0) = 0$. If the agent holds mixed insurance, her total utility is represented as

$$\begin{aligned}&\pi\big[u(W_L) - g\big(u(W_O) - u(W_L)\big)\big] + (1-\pi)\big[u(W_{NL}) - g\big(u(W_{NL}) - u(W_{NL})\big)\big]\\&= \pi u(W_L) + (1-\pi)u(W_{NL}) - \pi g\big(u(W_O) - u(W_L)\big).\end{aligned}$$

The condition that the agent prefers mixed insurance to term insurance can be given as

[4] In the following section, the final wealth holding mixed insurance is expressed $W_L(\alpha) = W - D - \alpha(P_m - D)$ in the loss state and $W_{NL}(\alpha) = W - \alpha(P_m - C)$ in the no-loss state. When $\alpha = 1$, we simply write W_L and W_{NL}.

$$\pi u(W_L) + (1-\pi)u(W_{NL}) - \pi g\big(u(W_O) - u(W_L)\big)$$
$$\geq\ u(W_O) - (1-\pi)\ g\big(u(W_{NL}) - u(W_O)\big). \quad (3)$$

The total utility can be decomposed into two effects, one is captured by utility function u and the other by regret function g. To distinguish between these two effects, the former is designated the risk effect and the latter the regret effect. Since the risk effect is degenerated into the expected utility case, term insurance leads to higher value than mixed insurance in terms of risk effect. However, it is possible that mixed insurance leads to higher value than term insurance in terms regret effect. In this case, the agent prefers to hold mixed insurance rather than term insurance if the regret effect dominates the risk effect. This implies that regret theory may be consistent with why some agents do not want to hold term insurance, but mixed one, even though all agents holds term insurance under expected utility theory. It is difficult to imagine a situation in which an agent holds mixed insurance. Thus, we have to determine specific conditions. To do so, we assume that utility function u is linear, in other words, we ignore the risk effect and focus on the regret effect. In this setting, we make the following result:

Result 1

Utility function is assumed to be linear and that regret function is strictly convex. If the loss probability is more than $1/2$, an agent prefers to hold mixed insurance to term insurance.

Proof

Since $W_O = \pi W_L + (1-\pi)W_{NL}$, condition (3) can be written as

$$\pi g(W_O - W_L) = \pi g\big((1-\pi)C\big) \leq (1-\pi)g(\pi C) = (1-\pi)g(W_{NL} - W_O)$$
$$\Leftrightarrow \frac{g\big((1-\pi)C\big)}{g(\pi C)} \leq \frac{1-\pi}{\pi}.$$

Since $\pi \geq \frac{1}{2} \Leftrightarrow \pi C \geq (1-\pi)C$ for $C \geq 0$, we have

$$\frac{g\big((1-\pi)C\big)}{g(\pi C)} \leq \frac{(1-\pi)C}{(\pi)C} = \frac{1-\pi}{\pi}. (4)$$

by $g'' > 0$ and $g(0) = 0$. □

[Insert Figure 1]

An agent is indifferent to hold either mixed insurance or term insurance when the loss probability is $1/2$. Let us consider that an agent holds mixed insurance. In this case, an impact of regret is composed into two components, one is captured by regret function g and the other is by the loss probability π. When the loss probability π is increasing, the loss probability is, obviously, increasing in regret, but regret function is decreasing in regret since utility difference of its argument, $W_O - W_L = (1-\pi)C$, is decreasing. By convexity of regret function, the latter effect dominates the former one. For the case that an agent holds term insurance, an opposite reasoning can be applied. As a result, an agent prefers to hold mixed insurance to term insurance when the loss probability is more than $1/2$. This result means that even though an agent is slightly risk aversion, that is u'' is slightly negative, the she holds mixed insurance when the loss probability is more than $1/2$.

4.3 Rejoicing formation

We now turn to the argument for the case of rejoicing formation. The argument is similar to the case with regret formation by slight modifications, and the details are almost skipped. The worst wealth is W_L in the loss state and W_O in the no-loss state. The total utility in holding term insurance is given as

$$u(W_O) - \pi h\big(u(W_O) - u(W_L)\big).$$

The total utility in holding mixed insurance is given as

$$\pi u(W_L) + (1-\pi)u(W_{NL}) - (1-\pi)h\big(u(W_{NL}) - u(W_O)\big). \quad (5)$$

As with the regret formation case, a rejoicing theoretical agent may prefer to hold mixed insurance to term insurance. When utility function is linear, we obtain the following result:

Result 2

Utility function is assumed to be linear and that rejoicing function is strictly concave. If the loss probability is less than $1/2$, an agent prefers to hold mixed insurance to term insurance.

An intuition of this result can be obtained by a same reasoning in the regret case. It appears more natural to consider that the loss probability is smaller than $1/2$ in insurance context. We thus adopt the rejoicing formulation for analyzing mixed insurance demand in the following section. However, the analysis is basically applied to the case of regret formation.

5. Full Insurance and Mixed Insurance

5.1 Expected Utility and Term Insurance

We examine how rejoicing influences on mixed insurance demand based on the full insurance theorem by Mossin (1968). Before proceeding with the analysis, we will review the classical result that a risk averse agent decides to buy how amounts of term insurance in the expected utility setting. As in the previous section, the insurance premiums are assumed to be actuarially fair. This assumption makes our analysis keep simple and is common to consider full insurance property.

The objective function is given as

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi u\left(W_L^T(\alpha)\right) + (1-\pi)u\left(W_{NL}^T(\alpha)\right).$$

Here, $W_L^T(\alpha)$ signifies the wealth in the loss state when the agent purchases amount α of term insurance, which is given as $W_L^T(\alpha) = W - D - \alpha(P_t - D)$. $W_{NL}^T(\alpha)$ stands for the wealth in the no-loss state, which is $W_{NL}^T(\alpha) = W - \alpha P_t$. α denotes term insurance demand, whose interval is $[0,1]$. Over-insurance is typically prohibited by law, and so $\alpha > 1$ is not included in the interval. Applying $P_t = \pi D$, the first-order condition (FOC) can be written as

$$\mathrm{V}'(\alpha) = \pi(1-\pi)D\left\{u'\left(W_L^T(\alpha)\right) - u'\left(W_{NL}^T(\alpha)\right)\right\} = 0. \quad (6)$$

The second-order condition (SOC) is also satisfied because $u'' \leq 0$. We omit the argument of the SOC below since it is easily confirmed. According to (6), $u'\left(W_L^T(\alpha)\right)$ has to equal to $u'\left(W_{NL}^T(\alpha)\right)$, in other words, α satisfies the condition $W_L^T(\alpha) =$

$W_{NL}^{T}(\alpha)$. Thus, $\alpha^* = 1$, holds, this means that the full insurance is optimal. This result is reasonable, since there is no risk on final wealth holding term insurance at the full insurance level. It is necessary to note that this full insurance is an interior solution. We have to distinguish between an interior and a corner solution at the full insurance level. For convenience, the former is hereafter termed full insurance and the latter is over-insurance. We should recall that over-insurance, $\alpha > 1$, is intrinsically prohibited.

5.2 Rejoicing Formation and Mixed Insurance

Let us now consider that an agent is regret theoretical. The agent chooses an optimal mixed insurance level, α, to maximize their total utility:

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi\left[u\left(W_L(\alpha)\right) + h\left\{u\left(W_L(\alpha)\right) - u\left(W_L(0)\right)\right\}\right]$$
$$+(1-\pi)\left[u\left(W_{NL}(\alpha)\right) + h\left\{u\left(W_{NL}(\alpha)\right) - u\left(W_{NL}(1)\right)\right\}\right].$$

Here, $W_L(\alpha)$ represents the wealth in the loss state when the agent purchases amount α of mixed insurance, which is given as $W_L(\alpha) = w - D - \alpha(P_m - D)$. $W_{NL}(\alpha)$ stands for the wealth in the no-loss state, which is $W_{NL}(\alpha) = w - \alpha(P_m - C)$. α denotes mixed insurance demand. Applying $P_m = \pi D + (1-\pi)C$, the FOC can be written as

$$V'(\alpha^*) = \pi(1-\pi)(D-C)\left[u'\left(W_L(\alpha^*)\right)\left(1 + h'\left\{u\left(W_L(\alpha^*)\right) - u\left(W_L(1)\right)\right\}\right)\right.$$
$$\left. - u'\left(W_{NL}(\alpha^*)\right)\left(1 + h'\left\{u\left(W_{NL}(\alpha^*)\right) - u\left(W_{NL}(1)\right)\right\}\right)\right] = 0. \quad (7)$$

To examine mixed insurance demand at the full hedging level, we determine the sign of (7) at $\alpha^* = 1$:

$$V'(1) = \pi(1-\pi)(D-C)\left[u'(W_L)\left(1 + h'(UD_L)\right) - u'(W_{NL})\left(1 + h'(0)\right)\right] \quad (8)$$

For notational ease, we denote $W_L(1) = W_L$ ($W_{NL}(1) = W_{NL}$) and $UD_L = u(W_L) - u\left(W_L(0)\right)$. The sign of (8) coincides with

$$u'(W_L)\left(1 + h'(UD_L)\right) - u'(W_{NL})\left(1 + h'(0)\right), \quad (9)$$

since all the other terms of (8) are strictly positive. Since $W_L < W_{NL}$ and u' is decreasing, $u'(W_L) \geq u'(W_{NL})$. Since $UD_L > 0$ and h' is decreasing, $h'(L) \leq h'(0)$. From this observation, we obtain that the sign of (9)—and thus that of (8)—is indeterminate. This means that it is possible that optimal mixed insurance demand is either over- or under-insurance. Full insurance is occurred on a knife-edge case.

5.3 Decomposition of Utility and Rejoicing Effect

To examine what factors determine mixed insurance demand under rejoicing formation, we consider here two special cases: one is no rejoicing function, and the other is linear utility function. The first case is no rejoicing effect and is degenerated to the expected utility case. The latter case is no risk effect. In the expected utility case, the objective function is written as

$$\max_{\alpha\in[0.1]} V(\alpha) = \pi u\big(W_L(\alpha)\big) + (1-\pi)u\big(W_{NL}(\alpha)\big).$$

The FOC at $\alpha = 1$ is written as

$$V'(1) = \pi(1-\pi)(D-C)\{u'(W_L) - u'(W_{NL})\} \geq 0. \quad (10)$$

Since the all terms are strictly positive, (10) is positive. This means that over-insurance is optimal. An intuition for this result is as follows. Unlike term insurance, the wealth holding mixed insurance remains some risk at the full insurance level. Thus, the agent eagers to hold the mixed insurance beyond the full insurance level so as to decrease in risk of the wealth. We summarize this argument into the following:

Result 3

If an agent has concave utility function and no rejoicing, that is, risk averse under expected utility representation, optimal mixed insurance holding is over-insurance under actuarially fair premium.

Unlike the term insurance, an agent cannot exclude risk even though she holds the full insurance level of the mixed insurance. This gives her a motive to hold the mixed insurance beyond the full insurance level. As a result, optimal mixed insurance holding is over-insurance.

Next, we next consider another special case where the utility function is linear. The total utility is written as

$$\max_{\alpha\in[0,1]} V(\alpha) = \pi\left[W_L(\alpha) + h\big(\alpha(D - P_m)\big)\right] + (1-\pi)\left[W_{NL}(\alpha) + h\big((1-\alpha)(P_m - C)\big)\right]. \quad (11)$$

Here, the utility difference is calculated as $W_L(\alpha) - W_L(0) = \alpha(D - P_m)$ and $W_{NL}(\alpha) - W_{NL}(1) = (1-\alpha)(P_m - C)$. The FOC (11) at $\alpha = 1$ is given as

$$V'(1) = \pi(1-\pi)(D - C)\{h'(D - P_m) - h'(0)\}. \quad (12)$$

Since $h'(D - P_m) < h'(0)$ and the other terms are strictly positive, (12) is strictly negative. This means that under-insurance is optimal. An intuition for this result is as follows. While an agent feels maximum rejoicing in the loss state, she does not feel any rejoicing in the no-loss state at the full insurance level. The total utility can be increasing by marginal decrease in mixed insurance holding from the full insurance level since gaining rejoicing in the no-loss state is more than losing it in the loss state by the concavity of rejoicing function. We summarize this argument into the following:

Result 4

If an agent has linear utility function and concave rejoicing function, optimal mixed insurance holding is under-insurance under actuarially fair premium.

At the full insurance level, an agent gains rejoicing in the loss state, while she does not gain one in the no-loss state. For a marginal decrease of mixed insurance holding, she loses some rejoicing in the loss state, but gains one in the no-loss state. The latter dominates the former because of the concavity of rejoicing function. As the result, the optimal mixed insurance holding is under-insurance.

From the results of the two special cases, the following intuition is taken: risk effect propels over-insurance and rejoicing effect drives under-insurance, and that optimal insurance is determined by their relative strength. We will now confirm this intuition formally.

5.4. Risk and Rejoicing Effects on Mixed Insurance Demand

Let us consider utility function $\hat{u}$ and rejoicing function $\hat{h}$ such that full insurance is optimal, *i.e.*

$V'(1) = \pi(1-\pi)(D-C)$

$$\left[\hat{u}'(W_L)\left(1+\hat{h}'(UD_L)\right) - \hat{u}'(W_{NL})\left(1+\hat{h}'(0)\right)\right] = 0. \quad (13)$$

This can be rewritten as

$$\frac{\hat{u}'(W_L)}{\hat{u}'(W_{NL})} = \frac{1+\hat{h}'(0)}{1+\hat{h}'(UD_L)}.$$

The relative strength of utility function u and rejoicing function h is characterized by their degrees of concavity as showing following propositions.

Result 5

Let us consider rejoicing function $\hat{h}$. Over- (under-) insurance is optimal if utility function u is more (less) concave than $\hat{u}$ in the sense that

$$-\frac{u''(x)}{u'(x)} \geq -\frac{\hat{u}''(x)}{\hat{u}'(x)}.$$

Proof:

A more (less) concave relation is equivalent to that $u'(x)/\hat{u}'(x)$ is decreasing (increasing) in x. Thus, for x, y with $x > y$

$$\frac{u'(y)}{\hat{u}'(y)} \leq \frac{u'(x)}{\hat{u}'(x)} \Leftrightarrow \frac{u'(y)}{u'(x)} \leq \frac{\hat{u}'(y)}{\hat{u}'(x)}$$

Setting $x = W_{NL}$ and $y = W_L$, we have[5]

$$\frac{u'(W_L)}{u'(W_{NL})} \geq \frac{\hat{u}'(W_L)}{\hat{u}'(W_{NL})} = \frac{1+\hat{h}'(0)}{1+\hat{h}'(UD_L)}$$

$$\Leftrightarrow u'(W_L)\left(1+\hat{h}'(UD_L)\right) - u'(W_{NL})\left(1+\hat{h}'(0)\right) \geq 0.$$

This indicates that over- (under-) insurance is optimal if an agent has a more (less) concave utility function u than $\hat{u}$. □

The form of the degree of concavity defined the above, is same for the absolute Arrow-Pratt aversion in the expected utility framework.

[5] Utility function is preserved under affine transformation, so that we can set $UD_L = u(W_L) - u\big(W_L(0)\big) = \hat{u}(W_L) - \hat{u}\big(W_L(0)\big)$.

A similar argument can be applied to rejoicing function. We consider that rejoicing function h is more (less) concave than $\hat{h}$ in the sense that[6]

$$-\frac{h''(x)}{1+h'(x)} \geq -\frac{\hat{h}''(x)}{1+\hat{h}'(x)}.$$

By a similar argument of the above, we have

$$\frac{\hat{u}'(W_L)}{\hat{u}'(W_{NL})} = \frac{1+\hat{h}'(0)}{1+\hat{h}'(UD_L)} \leq \frac{1+h'(0)}{1+h'(UD_L)}$$

$$\Leftrightarrow \hat{u}'(W_L)\big(1+h'(UD_L)\big) - \hat{u}'(W_{NL})\big(1+h'(0)\big) \leq 0.$$

This indicates that under- (over-) insurance is optimal if an agent has a more (less) concave rejoicing function h than $\hat{h}$. We summarize this argument into the following.

Result 6

Let us consider utility function $\hat{u}$. Under-insurance is optimal if rejoicing function h is more concave than $\hat{h}$ in the sense that

$$-\frac{h''(x)}{1+h'(x)} \geq -\frac{\hat{h}''(x)}{1+\hat{h}'(x)}.$$

From the above analysis, we confirm our intuitive conclusion, *i.e.* when risk effect dominates rejoicing effect, over insurance is optimal, and vice versa. Here, the relative importance is measured by their degree of concavity.

Finally, we will examine the effect of compensation in the no-loss state on mixed insurance demand. Let us consider the above situation, *i.e.* the full insurance is optimal. We denote the compensation $\hat{C}$ that satisfies (13). It is easily calculated as follows:

$$\mathrm{sgn}\left\{\frac{\partial\left(\frac{u'(W_L)}{u'(W_{NL})}\right)}{\partial C}\right\} = -(1-\pi)u''(W_L)u'(W_{NL}) - \pi u''(W_{NL})u'(W_L) \geq 0,$$

[6] Unlike utility function, rejoicing function is not preserved under affine transformations.

$$\operatorname{sgn}\left\{\frac{\partial\left(\frac{1+h'(0)}{1+h'(L)}\right)}{\partial C}\right\}=\frac{(1-\pi)h''(L)u'(W_L)}{\left(1+h'(L)\right)^2}\leq 0.$$

From the above, we have the following inequality for the compensation C such that $\hat{C}\geq(<)C$ by setting u as $\hat{u}$, and h as $\hat{h}$

$$\frac{\hat{u}'(W_L)}{\hat{u}'(W_{NL})}\geq(<)\frac{1+\hat{h}'(0)}{1+\hat{h}'(L)}$$

$$\Leftrightarrow \hat{u}'(W_L)\left(1+\hat{h}'(L)\right)-\hat{u}'(W_{NL})\left(1+\hat{h}'(0)\right)\geq(<)0$$

By this argument, we have the following result:

Result 7

Let us consider that the compensation $\hat{C}$ satisfies (13). Over-insurance (under-insurance) is optimal if the compensation is large (small) in the sense that $\hat{C}\geq(<)C$.

By the reasoning of the analysis, the compensation increases the mixed insurance demand from the viewpoint of the risk effect and decreases one from the rejoicing effect. When the compensation is enough large (small), the former dominates (is dominated by) the latter, so that an agent has a motive that she holds (does not hold) the mixed insurance compared to the full insurance level.

6. Concluding Remarks

This paper examines how regret and rejoicing on mixed insurance choice and demand. Both an agent may prefer to hold mixed insurance to term insurance under both the regret formation and the rejoicing formation. This observation provides a theoretical ground why people purchase mixed insurance. We determine conditions on how amounts of mixed insurance are bought using the rejoicing formation. The relative strength of rejoicing effects plays a key role to determine mixed insurance demand. When rejoicing function is enough concave, under-insurance is optimal.

There are two directions of future research: one is about the formation. Braun and Muermann (2004) opened up the possibility that the regret and rejoicing formation is a useful and promising approach to analyze insurance problems. This formation can explain why mixed insurance is traded. It is worth pursuing insurance problems using this formation. Behavioral approach including the regret and rejoicing formation, may be powerful to understand them.

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Figure 1