

Reinsurance or CAT Bond? How to Optimally Combine Both *

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June 3, 2015

*We thank the Montréal Exchange, the Institute de Finance Mathématique de Montréal (IFM2), the Royal Bank of Canada, the Fonds Conrad Leblanc, and the Social Sciences and Humanities Research Council of Canada for their financial support. We also thank Yoann Racine for excellent research assistance. All errors and omissions are the sole responsibility of the authors.

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Abstract

Härdle and Cabrera (2010) show empirically that a mix of reinsurance and CAT bond provides a catastrophe insurer with coverage at a lower cost and lower exposure at default than reinsurance alone. To underpin theoretically this empirical finding, we develop a model to optimally combine reinsurance and CAT bonds. We use a contingent claims framework to show that it is optimal to cover small losses using reinsurance and hedge higher losses by issuing a CAT bond. Our results demonstrate that this strategy significantly lowers the insurer's cost of protection, expands his underwriting capacity and yields higher shareholder values.

Keywords: Catastrophe bonds, Reinsurance, Risk Management, Contingent claims analysis

1 Introduction

Increases in the frequency and severity of natural catastrophes in recent years have propelled the use of alternative risk transfer (ART) instruments for managing catastrophic risks. As a matter of fact, the total volume of new insurance-linked securities issued during the second quarter of 2014 reached a historical record of \$4.634 billion.¹ The catastrophe bond (CAT bond), the most important type of ART instrument in terms of volume, constitutes an interesting complement to traditional reinsurance for a catastrophe insurance firm seeking to expand its risk-bearing capacity. CAT bonds traded in capital markets exhibit different characteristics to traditional reinsurance offering some advantages over it. For instance, CAT bonds are usually fully collateralized, and their credit risk is therefore small, especially in the case of the new hybrids launched since the subprime crisis, which are protected against the credit risk of the total return swap counterparty.² In contrast, the reinsurer is inherently not without default risk, which can substantially reduce the insuring/hedging effectiveness of reinsurance. In fact, the reinsurance firm usually has pre-existing risk exposure so that its ability to take on additional catastrophe risks might be limited. It is documented empirically that reinsurers charge very high premiums for protection against high layers of losses that have low occurrence probabilities (see Froot (2001), Figure 4, p. 540). Meanwhile, reinsurance presents advantages over ART instruments. For instance, traditional reinsurance offers potentially mutual benefits in business relationships, whereas a CAT bond sponsor receives no benefit from long-term business relationships with investors. Moreover, the reinsurer can monitor the situation much more effectively than CAT bond investors can, meaning that the reinsurer should be less exposed to moral hazard,³ which could reduce the cost of protection. Later in this paper, we show that the differences between reinsurance and CAT bonds are such that these instruments actually complement each other.

¹See article "Q2 2014 Catastrophe Bond & ILS Market Report" at <http://www.artemis.bm>.

²See article "Catastrophe Bonds Evolve to Address Credit Risk Issues" by Tower Watson (2010) for a discussion on CAT bonds' credit risk in the context of a total return swap.

³This is based on results produced by Doherty and Smetters (2005), who show that the cost of moral hazard is significantly less when the insurer and the reinsurer are affiliates. See also Bond and Crocker (1997) for a model on the effectiveness of monitoring in reducing moral hazard in insurance. Further, Cummins and Song (2008) show that a reinsurance firm should charge a lower loading to an insurer if a long-term repeat-business relationship exists between them (due to a reduction in the asymmetric information component).

Previous literature analyzes interactions between ART instruments and reinsurance. Croson and Kunreuther (2000) argue that combining traditional reinsurance with the (then new) ART instruments should expand the underwriting capacity of (re)insurers and lower the price of protection from its current level. Doherty and Richter (2002) use a mean-variance framework to show the benefits of combining indemnity-based gap reinsurance with an index hedge. Nell and Richter (2004), by way of an expected utility approach, underscore the substitution effects between traditional reinsurance and parametric CAT bonds for large losses. Their rationale is based on the reinsurance from market imperfections stemming from reinsurers' higher risk aversion for large losses in reinsurance contracting. Lee and Yu (2007) use a contingent claims framework to study how the value of a reinsurer's contract can be increased by issuing CAT bonds. Cummins and Song (2008) use a mean-variance framework to show that traditional reinsurance and derivative hedging display a substitution effect when the insurer assets and liabilities do not exhibit natural hedging characteristics.

Cummins and Trainar (2009) argue that reinsurance should be preferred than securitization for relatively small and uncorrelated risks, but that securitization becomes more advantageous as the magnitude of potential losses and the correlation of risks begin to increase. Cummins and Weiss (2009) explain how the convergence of financial and (re)insurance markets has been driven by the increase in the frequency and severity of catastrophe risk, and argue that securitization expands (re)insurers' risk-bearing capacity. Finken and Laux (2009) argue that private information about the insurers' risk affects competition in the reinsurance market. Because information-insensitive CAT bonds with parametric triggers are not subject to adverse selection, the availability of CAT bonds with sufficiently low basis risk reduces cross-subsidization of high-risk insurers by low-risk insurers and the incumbent reinsurer's rents. Barrieu and Loubergé (2009) argue that downside risk aversion and ambiguity aversion limit the success of CAT bonds, and propose hybrid CAT bonds to provide protection against market risks. Härdle and Cabrera (2010) examine an empirical example of a hedging strategy that combines a reinsurance contract and a parametric CAT bond, and find that this strategy is optimal in the sense that it provides coverage for a lower cost and lower exposure at default than reinsurance itself. Lakdawalla and Zanjani (2012)

show that CAT bonds have important uses when buyers and reinsurers cannot contract over the division of assets in the event of insolvency. They then use numerical simulations to illustrate how CAT bonds improve efficiency in markets with correlated risks, or with uneven exposure of the insured parties to reinsurer default.

Gatzert and Kellner (2013) use a contingent claims framework to examine how gap reinsurance is useful for reducing the basis risk created by an index-linked instrument. They show that gap reinsurance can increase the hedge effectiveness by reducing basis risk, increasing shareholder value and lowering insurers' shortfall risk. Gibson et al. (2014) argue that the choice between traditional reinsurance and CAT bonds depends crucially on the information acquisition cost structure and on the degree of redundancy in the information produced. Hagendorff et al. (2014) observe empirically that insurance firms that issue CAT bonds tend to have less risky underwriting portfolios with less exposure to catastrophe risks and overall less need to hedge catastrophe risk. They also find firms issuing CAT bonds to experience a reduction in their default risk, and to increase their catastrophe exposure, following a CAT bond issuance.

As pointed out by Cummins and Trainar (2009), securitization is likely to play an important role in facilitating (re)insurers to shift optimal combinations of risk to the capital markets, especially catastrophic risks. However, theoretical models that describe how traditional reinsurance and ART instruments can be combined in an optimal way are scarce. Nell and Richter (2004) show that reinsurance coverage will be substituted by index-linked coverage for large losses, but their results are ambiguous for small losses. In this paper, taking this analysis further, our objective is to demonstrate the optimal combination of reinsurance and CAT bonds by a catastrophe insurer so as to minimize hedging costs and maximize shareholder value. In Nell and Richter (2004)'s model, the explanation for the insurer's demand for hedging is its risk aversion. However, as they point out, firms are often considered as risk neutral. In this context, it is well-known that the corporate demand for hedging can be motivated by the desire to reduce the expected costs of financial distress (see e.g., Mayers and Smith (1983)), a key feature that we explicitly model in our paper. Also, the model of Nell and Richter (2004) uses a CAT bond with a parametric trigger. However, since 73% of risk capital issued during the second quarter of 2014 used an indemnity

trigger,⁴, we employ this type of trigger in our model. Finally, Nell and Richter (2004) neglect both the risk of moral hazard and the reinsurer's risk of default. We find that these risk factors play a very important part in determining the optimal mix of reinsurance and CAT bond. Note that we focus on excess-of-loss hedging strategies, but our approach could be extended to other types of contracts.

As mentioned earlier, Härdle and Cabrera (2010) study an empirical example of a mix of a reinsurance contract and a CAT bond that leads to coverage at a lower cost than reinsurance on a stand alone basis. Following this line of inquiry, we first investigate how a traditional reinsurance contract can be optimally combined with a CAT bond to obtain the desired coverage at the lowest cost under a constraint on the credit risk of the hedge. Our results show that it is optimal to hedge small (and frequent) losses using reinsurance and hedge higher (and rarer) losses by issuing a CAT bond, which complements the findings of Nell and Richter (2004). We show that this is because moral hazard risk induces the CAT bond's investors to require higher loadings when the attachment point of the contract is low, whereas the financial distress costs drive the reinsurer to demand higher loadings when the attachment point is high. In addition, we confirm the analysis of Cummins and Trainar (2009), who argue that traditional reinsurance is very effective when risks are relatively low and weakly correlated, but the effectiveness of this reinsurance market breaks down as the magnitude of potential losses and the correlation of risks increase.

We also investigate how an optimal mix of reinsurance and CAT bond allows an insurer to increase his shareholder value. To this end, we use a contingent claims framework to determine the optimal risk exposure and the optimal hedging strategy for an insurer that seeks to maximize his shareholder wealth. The impact of shareholder value maximization on risk management has been studied in a few articles. For instance, Krvavych and Sherris (2006) investigate the demand for change-loss reinsurance when the insurer's objective is to maximize shareholder value under a solvency constraint and in the presence of frictional costs. They find that, if reinsurance is not taken into account to reduce the required minimal level of risk capital, then reinsurance can increase shareholder value only if frictional costs are present. By studying the optimal capital

⁴See article "Q2 2014 Catastrophe Bond & ILS Market Report" at <http://www.artemis.bn>.

structure of an insurer that seeks to maximize shareholder value also under a solvency constraint and in the presence of frictional costs, Yow and Sherris (2008) show that, if the insured parties are risk-averse, there is a trade-off between improving the firm’s solvency condition and incurring costs of economic capital. As indicated earlier, for the case of an index-linked instrument, Gatzert and Kellner (2013) show that gap reinsurance can increase hedging effectiveness by reducing basis risk, increasing shareholder value and, at the same time, lowering shortfall risk. Meanwhile, we study how an optimal mix of reinsurance and CAT bond enables an insurer to deliver higher shareholder value in the presence of a frictional cost caused by the policyholders sensitivity to the insurer’s default risk. We find that an optimal mix of CAT bond and reinsurance allows an insurer to significantly increase its shareholder value. We show that this is because the cost of hedging is lower under this strategy, which allows the insurer to purchase protection that increases its capacity to sell more insurance policies profitably. In addition, we show that our results are consistent with the findings of Hagendorff et al. (2014), who empirically observe that insurance firms that issue CAT bonds tend to have less exposure to catastrophe risks.

The article continues as follows. Section 2 presents the model framework, the formulation of optimization problems and the numerical methods. Section 3 then presents and discusses the numerical analysis results. Section 4 summarizes our findings and concludes.

2 Model framework

Let us consider a *catastrophe insurance* firm that seeks to maximize its shareholder value. The firm can sell catastrophe coverage and can also buy hedging in the form of a traditional reinsurance contract and/or a CAT bond. The insurer’s demand for hedging is driven by the desire to avoid financial distress costs and by the sensitivity of the insurance premium loading with respect to insolvency risk. Buying coverage is costly, but it allows the insurer to increase its risk-bearing capacity. In our model, the advantage of using reinsurance comes from the presumed existence of a beneficial business relationship between the insurer and the reinsurer, which reduces the moral hazard cost. However, the reinsurer is vulnerable to default credit. Moreover, the premium loading charged by the reinsurer increases if the new contract exacerbates its expected cost of

financial distress. On the other hand, the CAT bond is assumed to have no credit risk, but its investors require a substantial premium to compensate them for their exposure to moral hazard risk, which is present since CAT's investors have few means of monitoring the CAT sponsor. In the following, we present the theoretical details of our simple model.

2.1 Modeling the equity and the asset side

We consider a single-period model in which the insurer receives insurance premiums at time 0 and has to indemnify a stochastic underwriting loss L_T at time T . In addition, as in Krvavych and Sherris (2006), we model financial distress costs by means of the insurer's deadweight losses⁵ incurred if the terminal net asset value is below the financial distress barrier, which is assumed to be equal to an exogenously predetermined proportion k_L of the initial value of the insurer's assets. We also assume limited liability, so that the terminal payoff to the shareholders is bounded below by zero even if the terminal asset value A_T is below the value of the insurance liability L_T . If the terminal net asset value $A_T - L_T$ is above the financial distress barrier K , then the firm is considered to be financially healthy and shareholders receive a liquidating dividend equal to $A_T - L_T$. However, if financial distress happens (i.e., $A_T - L_T \leq K$), then the insurer takes deadweight losses $1 - \omega_L$ as a proportion of the asset values, so that the terminal value of equity is given by $\max\{\omega_L A_T - L_T, 0\}$. Therefore, the equity value at time T is given by

$$S_T = (A_T - L_T) 1_{A_T - L_T > K} + \max\{\omega_L A_T - L_T, 0\} 1_{A_T - L_T \leq K}, \quad K = k_L A_0, \quad (1)$$

where A_0 is the initial value of assets, and $\omega_L \in [0, 1]$ and $k_L \in [0, 1]$ are exogenous constants. The assets include the insurer's initial capital X , plus the premium π_P received at $t = 0$ from the policyholders. The insurer can also buy two hedging instruments: a reinsurance contract that costs π_{RE} at $t = 0$ and reimburses a cash flow RE_T at $t = T$, and a CAT bond that costs π_{CB} at $t = 0$ and pays a cash flow CB_T at $t = T$. We also assume that the only other investment

⁵As explained in Jarrow and Purnanandam (2007), financial distress costs can be both direct (e.g., legal costs) and indirect (e.g., loss of reputation). Some empirical studies estimate these costs to be around 10-20% of the market value of assets for financial companies (see, for example, Andrade and Kaplan (1998)), and they are likely to be higher for insurance companies due to policyholders' aversion to insolvency risk.

available to the insurer returns the risk-free rate r .⁶ Since the hedging instruments are considered as assets for the purpose of financial accounting (see Doherty (2000)), the terminal asset value is given by

$$A_T = (X + \pi_P - \pi_{CB} - \pi_{RE}) e^{rT} + CB_T + RE_T. \quad (2)$$

Next we present the insurance policies pricing model.

2.2 Modeling the pricing of insurance policies

Since it is a limited liability for the shareholders, the insurance premium π_P , the amount paid by the policyholders for catastrophic insurance to the insurer, is given by the following expressions

$$\pi_P = E^{\mathbb{Q}}[P_T] e^{-rT} (1 + \delta_P), \quad P_T = \min \{L_T, A_T\}, \quad (3)$$

where \mathbb{Q} is the risk-neutral measure,⁷ P_T is the amount paid at time T to the policyholders⁸ and δ_P is the loading the policyholders are willing to pay for insurance. As in Gatzert and Kellner (2013), we assume that the insurance company is not in a monopolistic position and cannot fix δ_P arbitrarily. On the contrary, we assume that the loading is endogenous and reflects the policyholders' risk aversion. Specifically, we assume that δ_P decreases with an increase in the insurer's objective default probability PD. This is supported by the experimental results presented by Zimmer et al. (2009) and Zimmer et al. (2012), who find that the policyholders' willingness to pay higher loadings decreases with the insurer's default probability. Following Gatzert and Kellner (2013), we assume that the loading is given by

$$\delta_P = \max \left\{ (1 - q \cdot \text{PD}) \delta_P^{\max}, -1 \right\}, \quad \text{PD} \equiv \mathbb{P}(A_T < L_T), \quad (4)$$

where \mathbb{P} is the real-world measure, δ_P^{\max} is an exogenous constant denoting the maximum loading the policyholders are willing to pay if the insurer is default free, and q represents the policyholders'

⁶An interesting extension of our model could be to allow the insurer to invest in risky assets as well.

⁷Following Yow and Sherris (2008) and Gatzert and Kellner (2013), we assume the market to be arbitrage free, such that the risk-neutral measure exists without necessarily being unique for the case considered here.

⁸We assume that the policyholders are not affected by financial distress. We will see later that relaxing this assumption does not change our results since it is optimal for the insurer to have a near-zero probability of financial distress.

risk aversion to insolvency risk, which is assumed to be constant over time. Note that δ_P is bounded below by -1 to ensure that $\pi_P \geq 0$. Since we will show later that financial distress costs and policyholders' risk aversion make it optimal for the insurer to have a near-zero default probability, solvency constraints are not necessary for our model. This feature aligns with experimental results by Zimmer et al. (2012), who find that policyholders' risk aversion induces insurers to exhibit no default risk. In fact, following our experiments, we find that imposing different types of solvency constraints (as in Bernard and Tian (2009), for instance) does not change our results.

2.3 Modeling the pricing of hedging instruments

The price paid by the insurer for the CAT bond, π_{CB} , is given by ⁹

$$\pi_{CB} = E^{\mathbb{Q}} [CB_T] e^{-rT} (1 + \delta_{CB}), \quad (5)$$

where δ_{CB} is the loading demanded by investors to compensate them for the inherent moral hazard risk stemming from the indemnity trigger feature. Moral hazard affects CAT bond pricing, as supported by empirical results from Galeotti et al. (2013), who find that investors demand higher premiums when an indemnity trigger is used.¹⁰ Following Gatzert and Kellner (2013), the loading is assumed to be proportional to the ratio of the expected payment from the CAT bond to the expected loss of the insurer (both under the real-world measure \mathbb{P}) ¹¹

$$\delta_{CB} = \max \left\{ \theta_{CB} \left(\frac{E^{\mathbb{P}} [CB_T]}{E^{\mathbb{P}} [L_T]} \right), \delta_{CB}^{\min} \right\}, \quad (6)$$

where $\theta_{CB} \in \mathbb{R}$ is a constant capturing the investors' exposure to moral hazard and the parameter δ_{CB}^{\min} is a constant representing the minimum loading that investors would require when dealing

⁹For details on the risk-neutral valuation of CAT bonds, see Vaugirard (2003), Jarrow (2010), Nowak and Romaniuk (2013), and Ma and Ma (2013).

¹⁰Likewise, Lee and Yu (2002) show by simulation that moral hazard reduces the value of CAT bonds substantially. Further, Hagedorff et al. (2014) document empirically that insurers increase their catastrophe exposure following a CAT bond issue, which is a type of moral hazard.

¹¹In Gatzert and Kellner (2013), the convexity of the cost function can be controlled by an additional parameter denoted by k . This parameter is set to 1 in our work, thereby we assume a linear cost function.

with an insurer bearing no moral hazard risk. We assume the CAT bond has neither counterparty risk nor basis risk. Still, as mentioned previously, the fact that CAT bonds are fully collateralized makes them significantly less exposed to default risk than traditional reinsurance, especially in the case of the new hybrids marketed since the subprime crisis that are protected against credit risk of the total return swap counterparty.

On the contrary, credit risk is inherent in the reinsurance contract. Let \mathcal{A} and π_{RE} denote the reinsurer's assets prior to entering into the new contract and the price paid by the insurer for the new contract. The initial value of the reinsurer's total assets is given by $\mathcal{A} + \pi_{\text{RE}}$ and is assumed invested at the risk-free rate. If \mathcal{L}_T denotes the reinsurer's liability that is senior to the new contract, then the terminal value of the reinsurance contract to the insurer is given by

$$\text{RE}_T = \min \left\{ \text{RE}_T^f, \max \left\{ (\mathcal{A} + \pi_{\text{RE}}) e^{rT} - \mathcal{L}_T, 0 \right\} \right\}, \quad (7)$$

where RE_T^f is the cash flow that the insurer receives in full faith and credit, i.e., the payoff if the reinsurer had no default risk. We assume that the reinsurer takes financial distress costs into account in his pricing of the reinsurance contract, as follows:

$$\pi_{\text{RE}} = \max \left\{ \text{RE}_0 (1 + \delta_{\text{RE}}^{\text{MH}}), \text{RE}_0^* \right\}, \quad \delta_{\text{RE}}^{\text{MH}} = \max \left\{ \theta_{\text{RE}} \left(\frac{E^{\mathbb{P}} [\text{RE}_T]}{E^{\mathbb{P}} [\mathcal{L}_T]} \right), \delta_{\text{RE}}^{\text{min}} \right\}, \quad (8)$$

where $\delta_{\text{RE}}^{\text{min}}$ is the competitive loading, $\theta_{\text{RE}} \in \mathbb{R}$ represents the reinsurer's exposure to moral hazard, RE_0 represents the actuarially fair value of the contract, and RE_0^* represents the price of the reinsurance contract that would leave the reinsurer's equity unchanged. We thus have

$$\text{RE}_0 = E^{\mathbb{Q}} [\text{RE}_T] e^{-rT}, \quad \text{RE}_0^* = \arg \left\{ \mathcal{S}_0^{\text{RE}}(\pi_{\text{RE}}) = \mathcal{S}_0 \right\}, \quad (9)$$

where \mathcal{S}_0 denotes the reinsurer's equity prior to entering into the new contract and $\mathcal{S}_0^{\text{RE}}(\pi_{\text{RE}})$ denotes the reinsurer's equity after the sale of the new contract at price π_{RE} . The reinsurer's equity is modeled in the same fashion as in Equation (1), with terminal assets given by $(\mathcal{A} + \pi_{\text{RE}})e^{rT}$, terminal liabilities given by $\mathcal{L}_T + \text{RE}_T^f$, and financial distress parameters denoted by $\omega_{\mathcal{L}}$ and $k_{\mathcal{L}}$.

Equation (8) means that the reinsurance firm would like to offer the loading δ_{RE}^{MH} , which includes a moral hazard premium but not financial distress costs. However, the reinsurance firm will charge a higher loading if this price decreases shareholder wealth once distress costs are taken into account. The price offered by the reinsurer is thus bounded below by the value that would leave its equity unchanged.¹² Note that our reinsurance pricing model is similar in spirit to the one used by Froot and O’Connell (2008), which posits that the reinsurance firm wishes to protect future investment opportunities, and additional risk exposure could inflict losses forcing it to raise costly external funds. In our model, financial distress costs are meant to capture this kind of loss.¹³ In fact, as shown later, as financial distress costs are prevalent, the reinsurer demands higher loadings as protection against high layers of losses that have low occurrence probabilities, consistent with empirical observations (see Froot (2001), Figure 4, p. 540).

For our purposes, we assume moral hazard risk to be significantly smaller for the reinsurer than for the CAT bond investors (or $\theta_{RE} < \theta_{CB}$), the reasons being that the reinsurer’s ability to monitor the insurer is much greater than capital market investors’ ability to carry out surveillance on the CAT bond sponsor, and that a long-term business relationship is assumed to exist between the insurer and the reinsurer. As stated earlier, this assumption is based on results by Doherty and Smetters (2005), who show that the cost of moral hazard is significantly lower when the insurer and the reinsurer are affiliates. In other words, the reinsurance pricing is mainly driven by financial distress costs, whereas the CAT bond pricing is impacted by moral hazard. Furthermore, the CAT bond is assumed to have no default risk, whereas the reinsurance is credit vulnerable. In the context of our model, this feature summarizes the differences between these two hedging instruments.¹⁴

¹²We could also assume that the reinsurer requires the new insurance contract issuer to increase its equity by some amount, but we find in our experiments that this does not change our general findings.

¹³Note also that the results derived from our model and shown later have properties that agree with the three-factor model presented by Froot (2007). That is, the price offered by the reinsurer depends on the asymmetry of the loss distribution and the correlation with its pre-existing portfolio.

¹⁴We reckon that other factors might affect their pricing. For instance, Zhu (2011) demonstrates that ambiguity aversion could explain why CAT bond investors require higher loadings when the probability of the first loss is very small (i.e., $< 2\%$). We do not model this effect since the probability of loss is always higher than 10% for the simulations considered in this study. Also, Perrakis and Bolorforoosh (2013) show that CAT bond investors should demand higher loadings if the insurer’s loss is significantly correlated with traditional financial instruments. In our model, we implicitly assume that this correlation is very small. See also the empirical models presented in

2.4 Modeling the liability side

The insurer's loss L_T and the reinsurer's loss \mathcal{L}_T are assumed to follow a geometric Brownian motion, and their real-world dynamics are given by

$$\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_{L_t}^{\mathbb{P}}, \quad \frac{d\mathcal{L}_t}{\mathcal{L}_t} = \mu_{\mathcal{L}} dt + \sigma_{\mathcal{L}} dW_{\mathcal{L}_t}^{\mathbb{P}}, \quad (10)$$

with drifts $\mu_L, \mu_{\mathcal{L}}$; volatilities $\sigma_L, \sigma_{\mathcal{L}}$; and $W_{L_t}^{\mathbb{P}}, W_{\mathcal{L}_t}^{\mathbb{P}}$ denoting correlated \mathbb{P} -Brownian motions.

Under the risk-neutral measure \mathbb{Q} , these stochastic processes are given by

$$\frac{dL_t}{L_t} = r dt + \sigma_L dW_{L_t}^{\mathbb{Q}}, \quad \frac{d\mathcal{L}_t}{\mathcal{L}_t} = r dt + \sigma_{\mathcal{L}} dW_{\mathcal{L}_t}^{\mathbb{Q}}, \quad \text{corr} \left(dW_{L_t}^{\mathbb{Q}}, dW_{\mathcal{L}_t}^{\mathbb{Q}} \right) = \rho, \quad (11)$$

where r is the risk-free rate and where the correlation coefficient ρ is assumed to be constant to allow for the effect of different degrees of dependence between the insurer's and the reinsurer's losses. The solution to these equations is well-known (see, e.g., Björk (2009))

$$L_T = L_0 e^{(r - \sigma_L^2/2)T + \sigma_L W_{L_T}^{\mathbb{Q}}}, \quad \mathcal{L}_T = \mathcal{L}_0 e^{(r - \sigma_{\mathcal{L}}^2/2)T + \sigma_{\mathcal{L}} W_{\mathcal{L}_T}^{\mathbb{Q}}}, \quad \text{corr} \left(W_{L_T}^{\mathbb{Q}}, W_{\mathcal{L}_T}^{\mathbb{Q}} \right) = \rho, \quad (12)$$

where $W_{L_T}^{\mathbb{Q}}$ and $W_{\mathcal{L}_T}^{\mathbb{Q}}$ are standard \mathbb{Q} -Brownian motions, and L_0 and \mathcal{L}_0 are the initial conditions of the processes. The resulting lognormal distribution (of the geometric Brownian motion) is in alignment with empirical observations for the Property Claims Services (PCS) index in the United States (see Burnecki et al. (2000)) and this process is frequently used to represent loss estimates between $t = 0$ and $t = T$ (see Wu and Chung (2010) and Braun (2011) among others). Alternatively, we also use Merton (1976)'s jump diffusion model, as in Gatzert and Schmeiser (2008) and Lo et al. (2013). This approach explicitly models catastrophic events, and the resulting loss distributions have heavier tails. In a robustness check, we find that using this type of model does not change our general findings.¹⁵

Bodoff and Gan (2012) and Galeotti et al. (2013). We do not use such empirical models since the purpose of this work is to combine reinsurance and CAT bonds while the results in these articles have been obtained by focussing on CAT bonds only.

¹⁵These results are available from the authors upon request.

In the subsequent sections, we analyze the hedging strategies, then set-up the optimization problems, and finally present and discuss the results obtained by using numerical methods.

2.5 Analyzing the hedging strategies

In this paper, we only consider excess-of-loss (XL) hedging strategies,¹⁶ so that the analyzed strategies are all identical in terms of payoff function and differ only in terms of loading and credit risk. We analyze four strategies which stem from the following reinsurance contract and CAT bond¹⁷

$$\text{RE}_T^f = \min \left\{ \max \left\{ L_T - H_{\text{RE}}, 0 \right\}, M_{\text{RE}} \right\}, \quad \text{CB}_T = \min \left\{ \max \left\{ L_T - H_{\text{CB}}, 0 \right\}, M_{\text{CB}} \right\}, \quad (13)$$

where H_{RE} and H_{CB} denote the attachment for the reinsurance contract and the CAT bond respectively, and where M_{RE} and M_{CB} denote the layer limit for the reinsurance contract and the CAT bond respectively. The four strategies differ from one another with respect to how the reinsurance contract and the CAT bond are combined. These strategies are displayed in Table 1 and are easily interpreted. Strategy RE uses reinsurance but no CAT bond, whereas Strategy CB uses a CAT bond but no reinsurance. Strategy RECB uses a reinsurance contract with attachment H_{RE} and limit M_{RE} , and also a CAT bond with attachment $H_{\text{RE}} + M_{\text{RE}}$ and limit M_{CB} . The resulting payoff of this strategy is the following XL function

$$\text{RE}_T^f + \text{CB}_T = \min \left\{ \max \left\{ L_T - H_{\text{RE}}, 0 \right\}, M_{\text{RE}} + M_{\text{CB}} \right\}. \quad (14)$$

Under Strategy RECB, therefore, lower losses are hedged with reinsurance and higher losses are hedged with a CAT bond. Strategy CBRE does exactly the opposite and the resulting payoff function is

$$\text{RE}_T^f + \text{CB}_T = \min \left\{ \max \left\{ L_T - H_{\text{CB}}, 0 \right\}, M_{\text{RE}} + M_{\text{CB}} \right\}. \quad (15)$$

¹⁶Other strategies, as spelled for instance in Gatzert and Kellner (2013) may be studied but doing so would not change the main results of this paper.

¹⁷Recall that RE_T^f denotes the payoff of the reinsurance contract without default risk. The actual payoff with default risk is given by Equation (7). Note also that both the reinsurance contract and the CAT bond are triggered by the insurer's actual loss L_T , implying that basis risk is assumed to be negligible as indicated before.

We see that all strategies follow a payoff function identical to that of a single XL contract (see Figure 1 for an example). Therefore, since they all span the same set of payoff functions, it is legitimate to compare these four hedging strategies. This is an important point, since comparing hedging strategies with different payoffs does not pin down, i.e., isolate, the beneficial effect of the hedge’s cheaper price and the reduced credit risk, which is the aim of this article.

2.6 Minimization of hedging cost

How can the insurer optimally combine reinsurance with a CAT bond to minimize its hedging cost?¹⁸ Let us suppose that the insurer seeks to obtain an XL hedging contract with attachment H and layer limit M .¹⁹ Under Strategy RE (reinsurance) or Strategy CB (CAT bond), the insurer is constrained to use a single hedging contract. Intuitively, both of these strategies will lead likely to the insurer having to pay a significant loading for the prices. For instance, using reinsurance alone might apply too much pressure on the reinsurer, which could result in a contract with a significant probability of default, and a higher price to compensate the reinsurer for bearing financial distress costs. On the other hand, only using a CAT bond might result in the insurer having to pay a high price loading to compensate investors for their exposure to moral hazard. To handle this problem, the insurer could instead use Strategy CBRE, which hedges low losses through the issuing of a CAT bond and covers higher losses with reinsurance. The objective of the insurer is then to use this strategy to replicate the desired XL contract (i.e., with attachment H and limit M) at the lowest price possible. We also assume that the insurer imposes a constraint on the hedge’s probability of default HPD, which is given by the probability that the reinsurance instrument defaults,

$$\text{HPD} = \mathbb{P}\left(\text{RE}_T < \text{RE}_T^f\right), \quad (16)$$

as a result of the assumption that the CAT bond component of the hedging strategy is without default risk. For our simulation exercises, we suppose that the insurer requires the hedge’s default

¹⁸Seeking answers to this question is relevant since it will help us to understand how the insurer can maximize shareholder value by combining a reinsurance contract with a CAT bond, which is the question addressed in the next section of this paper.

¹⁹Although the choice of the parameters H and M is exogenous, it is not made arbitrarily in the sense that the values are drawn from the results of the problem of maximizing shareholder value. In other words, we will later show that there is, in fact, in our model, a demand from the insurer for such XL hedging contracts.

probability HPD to be below a predetermined level p , which is assumed to be low (e.g., 0.2%). The insurer then has to determine the optimal amount of reinsurance M_{RE} that solves the following problem ²⁰

$$\begin{aligned} \underset{M_{\text{RE}}}{\text{Minimize}} \quad & \Pi \equiv \pi_{\text{RE}} + \pi_{\text{CB}}, \quad \text{with } H_{\text{CB}} = H, \quad M_{\text{CB}} = M - M_{\text{RE}}, \quad H_{\text{RE}} = H_{\text{CB}} + M_{\text{CB}} \\ \text{subject to} \quad & M_{\text{RE}} \in [0, M], \quad \text{HPD} \leq p. \end{aligned} \tag{17}$$

Alternatively, the insurer could use Strategy RECB, which covers small losses with reinsurance and covers high losses through the issuing of a CAT bond. The corresponding optimization problem is then

$$\begin{aligned} \underset{M_{\text{RE}}}{\text{Minimize}} \quad & \Pi \equiv \pi_{\text{RE}} + \pi_{\text{CB}}, \quad \text{with } H_{\text{CB}} = H_{\text{RE}} + M_{\text{RE}}, \quad M_{\text{CB}} = M - M_{\text{RE}}, \quad H_{\text{RE}} = H \\ \text{subject to} \quad & M_{\text{RE}} \in [0, M], \quad \text{HPD} \leq p. \end{aligned} \tag{18}$$

For p sufficiently small, it is interesting to note that minimizing the hedging cost is equivalent to minimizing the *effective loading* δ_{EFF} paid by the insurer, which can be written as²¹

$$\delta_{\text{EFF}} \equiv \frac{\pi_{\text{RE}} + \pi_{\text{CB}}}{\text{RE}_0 + \text{CB}_0} - 1 = \frac{\text{RE}_0 \cdot \delta_{\text{RE}} + \text{CB}_0 \cdot \delta_{\text{CB}}}{\text{RE}_0 + \text{CB}_0}, \tag{19}$$

where $\pi_{\text{RE}} + \pi_{\text{CB}}$ is the total cost of the hedge, $\text{RE}_0 = E^{\mathbb{Q}}[\text{RE}_T]e^{-rT}$ is the actuarially fair value of the reinsurance component, $\text{CB}_0 = E^{\mathbb{Q}}[\text{CB}_T]e^{-rT}$ is the actuarially fair value of the CAT bond component, δ_{RE} is the loading paid to the reinsurer and δ_{CB} is the loading paid to the CAT bond's investors. Equation (19) shows that the effective loading is a weighted average of the loading paid to the reinsurer and that paid to the CAT bond's investors. The goal of the insurer is to minimize this weighted average, and we will see later that Strategy RECB is the optimal one.

²⁰We assume that the insurer always has access to the reinsurance and CAT bond markets, regardless of the desired hedging contract, i.e., there exists a supply of protection for any values of H_{RE} , M_{RE} , H_{CB} and M_{CB} .

²¹We use the relations $\pi_{\text{RE}} = \text{RE}_0(1 + \delta_{\text{RE}})$ and $\pi_{\text{CB}} = \text{CB}_0(1 + \delta_{\text{CB}})$ to obtain the right hand side of Equation (19).

2.7 Maximization of shareholder value

By how much can the insurer increase equity using an optimal combination of reinsurance and CAT bond? This problem is obviously related to the minimization of the hedging cost. In fact, the linkage between these problems is evident if we decompose the value of the insurer's equity into

$$S_0 = X + \delta_P \cdot P_0 - \delta_{CB} \cdot CB_0 - \delta_{RE} \cdot RE_0 - CDF, \quad (20)$$

where X is an exogenous constant denoting the initial equity capital available to the insurer,²² $P_0 = E^{\mathbb{Q}}[P_T]e^{-rT}$ is the actuarially fair value of the insurance policies and CDF denotes the expected cost of financial distress.²³ The insurer goal is to simultaneously determine the optimal risk exposure L_0 and the optimal hedging strategy. Here, we assume that the insurer is free to choose the initial value L_0 of the loss process. In practice, it might be true that the insurer has little control on the demand for insurance, which means that it could be more realistic to impose bounds on the value of L_0 . Nevertheless, we let L_0 be an unconstrained parameter since our goal is to study how the underwriting *capacity* of the insurer is enhanced by an optimal mix of reinsurance and CAT bond. Note that the insurer is assumed to have no pre-existing risk exposure,²⁴ in that the total value of insurance liabilities at time 0 is L_0 .

Equation (20) allows us to understand how the values of the controls (or decision variables) affect the equity value S_0 . First, an increase in the risk exposure L_0 increases equity by increasing P_0 . However, an excessively high value of L_0 also decreases equity by increasing the distress costs CDF and decreasing the loading δ_P policyholders are willing to pay. The underwriting capacity of the insurer is thus limited. However, the insurer can buy hedging instruments, which can increase equity by increasing both δ_P and P_0 and also by decreasing CDF. On the other hand, hedging also decreases equity by $-\delta_{CB} \cdot CB_0 - \delta_{RE} \cdot RE_0$, justifying the insurer motive to find an optimal

²²As in Garven and Loubergé (1996) and Gatzert and Kellner (2013), we assume that it is too costly for the insurer to adjust its initial capital, hence considering X to be a constant.

²³By definition, this quantity is the difference between the equity value S_0 priced as if financial distress were devoid and the actual (true) equity value when the prevalent financial distress is taken into account.

²⁴We assume the volatility of loss σ_L to be an exogenous constant that is independent of the risk exposure L_0 . We thus neglect potential diversification effects and consider the insurer underwriting portfolio to contain only one representative type of catastrophic risk.

mix of reinsurance and CAT bond that minimizes the effective loading δ_{EFF} for the strategy.

The formulation of the corresponding optimization problem depends on the hedging strategy used by the insurer. For Strategy RE (reinsurance only), the optimization problem takes the form

$$\underset{L_0, H_{\text{RE}}, M_{\text{RE}}}{\text{Maximize}} \quad S_0 = E^{\mathbb{Q}}[S_T]e^{-rT}, \quad \text{subject to } L_0 \geq 0, H_{\text{RE}} \geq 0, M_{\text{RE}} \geq 0. \quad (21)$$

For Strategy CB (CAT bond only), the optimization problem takes the form

$$\underset{L_0, H_{\text{CB}}, M_{\text{CB}}}{\text{Maximize}} \quad S_0 = E^{\mathbb{Q}}[S_T]e^{-rT}, \quad \text{subject to } L_0 \geq 0, H_{\text{CB}} \geq 0, M_{\text{CB}} \geq 0. \quad (22)$$

For Strategy RECB (optimal mix of reinsurance and CAT bond), the optimization problem is

$$\begin{aligned} & \underset{L_0, H_{\text{RE}}, M_{\text{RE}}, M_{\text{CB}}}{\text{Maximize}} \quad S_0 = E^{\mathbb{Q}}[S_T]e^{-rT}, \quad \text{with } H_{\text{CB}} = H_{\text{RE}} + M_{\text{RE}} \\ & \text{subject to} \quad L_0 \geq 0, H_{\text{RE}} \geq 0, M_{\text{RE}} \geq 0, M_{\text{CB}} \geq 0. \end{aligned} \quad (23)$$

Since the results from the problem of minimizing hedging costs will show that Strategy CBRE is not optimal relative to Strategy RECB, we do not consider Strategy CBRE when studying the maximization of shareholder value.

2.8 Numerical methods

We resort to numerical methods to solve the optimization problems presented above since they do not have analytical solutions. For example, Equation (3) for the insurance premium can be written as

$$\pi_P = E^{\mathbb{Q}} \left[\min \left\{ L_T, (X + \pi_P - \pi_{\text{CB}} - \pi_{\text{RE}}) e^{rT} + \text{CB}_T + \text{RE}_T \right\} \right] e^{-rT} (1 + \delta_P). \quad (24)$$

The premium π_P appears on both sides of the equation in such a way that it cannot be separated, implying that it does not have an analytical form. We use the Monte Carlo approach to estimate the objective functions of the optimization problems. We also use antithetic variates and quadratic resampling (see Huynh et al. (2008)) to improve statistical accuracy. The optimization problems are solved using MATLAB's pattern search algorithm. This is a direct search method, introduced by Hooke and Jeeves (1961), which performs a search through more than one basin of attraction.

This method does not require the gradients of the problem and is suitable even when the objective function has discontinuities. We also use a multistart approach to reduce the probability of finding a local minimum. For instance, for the optimization problem given by Equation (23), we start by generating a relatively large (e.g., 300000) random sample for the value of the input vector $(L_0, H_{RE}, M_{RE}, M_{CB})$. The objective function is then calculated for each guess, and a small number (say 20) of the best guesses are each used as the starting point of the pattern search algorithm. The best found minimum is then taken as the solution. For our optimization purposes, the parameters H_{RE}, M_{RE}, M_{CB} are scaled by L_0 , since this parameter determines their order of magnitude. In other words, each parameter is expressed as a multiple xL_0 and the optimization bounds are on x . This approach also greatly facilitates the definition of the search space for the optimization algorithm.

3 Numerical analysis

In this section we study how a catastrophe insurer can optimally combine a traditional reinsurance contract with a CAT bond to minimize the hedging cost and maximize shareholder value. As indicated earlier, we compare four hedging strategies (see Table 1) with XL payoffs that differ only in terms of pricing and credit risk. One strategy uses reinsurance alone (RE), one relies only on a CAT bond (CB), one chooses a CAT bond to cover smaller losses in conjunction with a reinsurance contract to cover larger losses (CBRE), and one employs a reinsurance contract to cover smaller losses and a CAT bond to protect against larger losses (RECB). The aim of this analysis is to determine which strategy allows the insurer to obtain the desired coverage at the lowest cost, and whether this allows the insurer to significantly increase shareholder value. We also perform sensitivity analyses to identify the main factors contributing to the effectiveness of an optimal mix of reinsurance and CAT bond.

3.1 Parameters

The baseline input parameters are presented in Table 2, where the attachment H and the layer limit M of the desired XL contract values are those obtained from the shareholder value-maximizing

exercise.²⁵ The values for the financial distress barriers are drawn from those used by Krvavych and Sherris (2006). The deadweight cost of financial distress is taken from some empirical studies (see, e.g., Andrade and Kaplan (1998)), which show that this cost can be 20% of the market value of assets for financial companies. Due to policyholders' aversion to insolvency risk, this cost is likely to be higher for insurance companies. The minimum CAT bond loading and the minimum reinsurance loading are set equal to each other, since, according to Cummins and Weiss (2009), CAT bonds are nowadays competitive with respect to traditional reinsurance. Most other parameters are either inspired by Gatzert and Kellner (2013) or chosen for illustrative purposes. We also conduct robustness tests on all input parameters to validate the generality of our findings. As stated earlier, the numerical results are obtained by Monte Carlo simulations with a total of 300,000 sample paths and the statistical precision is further improved by the use of antithetic variates and quadratic resampling (see, e.g., Huynh et al. (2008)).

3.2 Results for the minimization of hedging costs

In this section, we investigate how a reinsurance contract and a CAT bond can be combined optimally by an insurer so as to minimize the price of a desired excess-of-loss (XL) coverage.

Intuitively, the optimal proportion of reinsurance for the insurer to purchase depend on the correlation between the insurer's and the reinsurer's losses. Table 3 exhibits the results for the four hedging strategies we study, (see Table 1), for different values of that correlation ρ . For all strategies, we report the results for the reinsurance layer limit M_{RE} which is related to the CAT bond layer limit M_{CB} by the relation $M_{RE} + M_{CB} = M$, where M is the layer limit of the desired XL contract to be synthesized. By definition, we always have $M_{RE} = M$ and $M_{CB} = 0$ for Strategy RE (reinsurance), whereas we have $M_{RE} = 0$ and $M_{CB} = M$ for Strategy CB (CAT bond). For Strategies CBRE and RECB, the optimal values of M_{RE} are respectively derived by solving the optimization problems described by Equations (17) and (18). Table 3 also presents the corresponding proportion of reinsurance $\frac{RE_0}{RE_0 + CB_0}$, where RE_0 is the actuarially fair value of the reinsurance contract and CB_0 is the actuarially fair value of the CAT bond. The hedge total cost

²⁵These results are presented below in Subsection 3.3 entitled "Results from the maximization of shareholder value".

Π , the hedge's effective loading δ_{EFF} (see Equation (19)), and the hedge's objective probability of default HPD (see Equation (16)) are also presented in Table 3.

First of all, Table 3 shows that Strategy RECB results in lower hedge prices Π , and yields effective loadings δ_{EFF} that are about twice as small as those for other strategies. These results indicate that hedging small losses with reinsurance and hedging higher losses by issuing a CAT bond (Strategy RECB) is optimal in the sense that it provides the desired coverage for a lower cost. Strategy CBRE actually leads to higher prices than Strategy RE, which is due to the constraint $\text{HPD} \leq 0.2\%$ on the hedge's probability of default. For Strategy RE, the default probability is in fact always above 0.2%, which is the upper bound imposed in Strategies RECB and CBRE. The results in Table 3 also show that the optimal proportion of reinsurance decreases with an increasing value of the correlation ρ between the insurer's and the reinsurer's losses. In fact, under Strategy RE (reinsurance), the hedge's default probability, its loading and its cost all increase with an increasing value of ρ . We find that this is because a higher correlation ρ exacerbates the reinsurer's risk of insolvency and expected cost of financial distress. We see from Equation (8) that the reinsurer will require a larger loading in such circumstances. When CAT bonds are available, we conclude from these results that correlation can not only limit the efficiency of reinsurance but also reduce the demand for it.

Another parameter that intuitively plays a crucial role is the reinsurer's risk exposure \mathcal{L}_0 . Table 4 illustrates the results for all of the studied hedging strategies under different values of the reinsurer's risk exposure \mathcal{L}_0 . Again we notice that using reinsurance for lower layers of losses and the CAT bond for higher layers of losses (Strategy RECB) gives rise to both lower hedging costs Π and effective loadings δ_{EFF} . The optimal proportion of reinsurance decreases with an increasing value of the reinsurer's risk exposure \mathcal{L}_0 . When we use reinsurance only (Strategy RE), the hedge's default probability, its loading and its cost all increase with an increasing value of \mathcal{L}_0 . This is explained by the fact that a reinsurer with greater risk exposure has less capacity to take on additional catastrophe risk without increasing its expected cost of financial distress, and will therefore charge a higher loading. In other words, a reinsurer with high pre-existing risk exposure \mathcal{L}_0 is less creditworthy and pricier in loading. We conclude that the limited capacity of reinsurers

may limit the usefulness of this market, and may also reduce the demand for reinsurance when CAT bonds are available.

We next investigate how the insurer's risk exposure L_0 affects the demand for reinsurance when CAT bonds are available. The lefthand graph of Figure 2 illustrates how the optimal proportion of reinsurance under Strategy RECB decreases with an increasing risk size L_0 , for three different correlations ρ . When the exposure L_0 is relatively small and the correlation ρ is low, the reinsurance component constitutes around 80% of the hedge. However, for larger risks and higher correlations, the optimal proportion of reinsurance drops to around 45%. Therefore, reinsurance is an important part of the hedge when the insurer's risks are relatively small and weakly correlated to the reinsurer's risks. We also study how the attachment point H of the desired XL coverage impacts the optimal proportion of reinsurance under Strategy RECB. The righthand graph of Figure 2 illustrates how the optimal proportion of reinsurance decreases with an increasing attachment point H , for three values of correlation. When the attachment point is relatively low and the correlation ρ is weak, we see that the reinsurance component constitutes about 75% of the hedging strategy. However, the optimal proportion of reinsurance quickly decreases with an increasing value of the attachment point, which indicates that it is optimal to use CAT bonds when the desired attachment point is high. This also explains why hedging smaller losses with reinsurance and higher losses with a CAT bond (Strategy RECB) is optimal. These findings are further explored with the help of Figure 3, which illustrates how the premium loading δ_{EFF} changes with the attachment point H under Strategy RE (reinsurance only) and Strategy CB (CAT bond only). We see that the CAT bond's loading is a decreasing function of the attachment H . This can be explained by the fact that, in our framework, the risk of moral hazard is higher when the expected payoff of the CAT bond to the insurer is high compared to the insurer's expected loss (see Equation (6)). In our model, this implies that the loading demanded by the CAT bond's investors is larger when the attachment H is low. On the contrary, the reinsurer's loading is increasing in the attachment point. This is because the reinsurer's expected cost of financial distress is higher for such contracts.

Summing up, our results demonstrate that a combination of a CAT bond with a traditional

reinsurance contract can be optimal in the sense that it provides the desired coverage for a significantly lower cost, which is in accord with the empirical findings of Härdle and Cabrera (2010). We show that the optimal strategy is to use the CAT bond to hedge higher layers of losses and reinsurance to cover lower layers of losses. We show that this strategy is optimal mainly because in the presence of financial distress costs, the reinsurer charges higher loadings for protection against higher layers of losses, which is in line with empirical observations (see Froot (2001), Figure 4, p. 540). Our findings complement the results of Nell and Richter (2004), who find substitution effects between reinsurance and a parametric CAT bond for large losses only. Furthermore, we confirm the intuition discussed in Cummins and Trainar (2009), that traditional reinsurance is very useful when risks are relatively small and weakly correlated, but that the efficiency of this market breaks down as the magnitude of potential losses and the correlation of risks increase.

3.3 Results for the maximization of shareholder value

In the previous section, we showed that the insurer's cost of protection can be reduced significantly through the combination of a reinsurance contract with a CAT bond. In this section, we study how an optimal mix of reinsurance and CAT bond allows the insurer to yield a higher shareholder value.

Table 5 exhibits results for the three strategies under consideration for different values of the insurer's loss volatility σ_L . For all strategies, the optimal values of the risk exposure L_0 , the reinsurance attachment H_{RE} , the reinsurance layer limit M_{RE} , the CAT bond attachment H_{CB} and the CAT bond layer limit M_{CB} are presented. For Strategy RE (reinsurance only), we obtain these parameters by solving the optimization problem in Equation (21). For Strategy CB (CAT bond only) and Strategy RECB (reinsurance and CAT bond), these parameters are derived by solving the problems in Equations (22) and (23), respectively. Table 5 also presents the maximum shareholder value $S_0 = E^Q[S_T]e^{-rT}$ (see Equation (1)), the insurer's default probability PD (see Equation (4)) and the other outputs that are required to decompose the shareholder value into (see Equation (20))

$$S_0 = X + \delta_P \cdot P_0 - \delta_{\text{EFF}} \cdot (\text{RE}_0 + \text{CB}_0) - \text{CDF}, \quad (25)$$

where $X = \$100M$ denotes the insurer's initial equity capital (see Table 2). Here, δ_P is the insurance loading (see Equation (4)), δ_{EFF} is the hedge effective loading (see Equation (19)), $P_0 = E^{\mathbb{Q}}[P_T]e^{-rT}$ is the actuarially fair value of insurance policies (see Equation (3)), CDF is the insurer's expected cost of financial distress²⁶ and the sum $\text{RE}_0 + \text{CB}_0$ is the actuarially fair value of the hedge, with $\text{RE}_0 = E^{\mathbb{Q}}[\text{RE}_T]e^{-rT}$ and $\text{CB}_0 = E^{\mathbb{Q}}[\text{CB}_T]e^{-rT}$ (see Equation (13)).

The results from Table 5 show that an optimal mix of reinsurance and CAT bond (Strategy RECB) enables the insurer to gain shareholder values S_0 that are about 2-3% higher than obtained using the other strategies. This is mainly because the optimal risk exposure L_0 is significantly larger under this strategy, which leads to a higher fair value of insurance P_0 . Moreover, the policyholders' loading δ_P is always close to its upper bound (10%), which is due to the fact that the insurer's probability of default PD is, most of the time, very close to zero. In this setting, it is optimal for the insurer to be nearly default free, which explains why the actuarially fair value of insurance policies P_0 is always so close to the value of the insurer's risk exposure L_0 . In addition, we note that expected financial distress costs CDF are always very small ($\sim \$0.05M$). The insurer also uses more hedging under Strategy RECB, buttressed by the hedge's fair value $\text{RE}_0 + \text{CB}_0$. In fact, as shown in Table 5, the ratio of the hedge's fair value $\text{RE}_0 + \text{CB}_0$ to the optimal risk exposure L_0 is always higher under Strategy RECB, which means that the insurer covers a larger fraction of its losses when this strategy is used.²⁷ Despite this, we see that the hedge's effective loading δ_{EFF} is always smaller for this strategy, which means that the ratio of the price of the hedge to its actuarially fair value is closer to one (see Equation (19)). In summary, our results indicate that an optimal mix of reinsurance and CAT bond enables the insurer to obtain a higher equity value due to the fact that the cost of hedging is lower under this strategy, which allows the insurer to purchase more protection so as to increase its underwriting capacity (i.e., the optimal

²⁶The expected cost of financial distress CDF is defined as the reduction in the equity value S_0 that is caused by the state of financial distress. We estimate CDF as the difference between the equity value in the absence of financial distress (i.e., with $\omega_L = 1$) and the actual (true) equity value when financial distress is present.

²⁷For instance, for a volatility $\sigma_L = 0.4$, the ratio $\frac{\text{RE}_0 + \text{CB}_0}{L_0}$ is 16.22% under Strategy CB, 20.43% under Strategy RE, and 31.93% under Strategy RECB. The insurer hedges a larger fraction of its risk exposure when Strategy RECB is used, because that hedging is cheaper under this strategy.

value of L_0). These findings are further investigated by way of Figure 4, where the lefthand graph plots the maximum shareholder value S_0 against the insurer's loss volatility σ_L , and the righthand graph plots the optimal insurer's risk exposure L_0 against σ_L . For all strategies, both shareholder value and risk exposure decrease with increases in loss volatility, because insurance policies are then riskier and more expensive to hedge. More importantly, it can be seen in Figure 4 that Strategy RECB dominates the other strategies in terms of shareholder value S_0 and insurance capacity L_0 , which further confirms the results displayed in Table 5. Therefore, we conclude that both low-volatility and high-volatility insurers would benefit from combining reinsurance and CAT bonds.

The results presented in the previous section on minimizing the hedging cost show that the reinsurer's risk exposure \mathcal{L}_0 is a key driver in determining the usefulness of traditional reinsurance. We now investigate this aspect from a shareholders' value maximization perspective. Table 6 displays the results from the maximization of shareholder value for all hedging strategies under consideration, for different values of the reinsurer's risk exposure \mathcal{L}_0 . These results further confirm that an optimal mix of reinsurance and CAT bond yields higher shareholder value, for the same reasons pointed out in the previous paragraph. That is, the lower cost of hedging under this strategy enables the insurer to increase its optimal risk exposure L_0 while remaining free of default risk. In this case, we can also clearly see that the optimal reinsurance layer limit M_{RE} decreases with an increasing value of the reinsurer's risk exposure \mathcal{L}_0 , meaning that less reinsurance is used when the reinsurer has high pre-existing risk exposure. Recall that the same finding was obtained in the previous section, in which we tackled the problem of minimizing the hedging cost. Again, the reason is that a reinsurer with a greater pre-existing risk exposure has less capacity to take on additional catastrophe risk without increasing its expected cost of financial distress, and will therefore charge a larger premium. Another reason for this is that, a reinsurer with a high risk exposure \mathcal{L}_0 is more credit risky. It can be seen from Table 6 that, if the reinsurer's risk exposure is $\mathcal{L}_0 = \$500M$, then the optimal reinsurance layer limit is $M_{RE} = 0$ under Strategy RECB, meaning that no reinsurance is purchased by the insurer. In this particular case, the insurer's optimal risk exposure L_0 and optimal hedging parameters H_{RE} , M_{RE} , H_{CB} ,

M_{CB} are then identical to those obtained under Strategy CB (CAT bond only), which means that Strategy RECB converges to Strategy CB as the value of \mathcal{L}_0 increases. These findings are further investigated by means of Figure 5, where the lefthand graph plots maximum shareholder value S_0 against reinsurer's risk exposure \mathcal{L}_0 , and the righthand graph shows the optimal insurer's risk exposure L_0 against \mathcal{L}_0 . If the reinsurer's risk exposure is sufficiently small ($\mathcal{L}_0 \sim \$200M$), then Strategy RE (reinsurance only) and Strategy RECB converge to the same values of S_0 and L_0 , because it is then optimal to rely entirely on reinsurance. However, as \mathcal{L}_0 increases, the maximum shareholder value and optimal risk exposure remain higher under Strategy RECB, and eventually converge to the solution obtained under Strategy CB. In other words, it is optimal to use 100% reinsurance if the reinsurer's risk exposure \mathcal{L}_0 is sufficiently small, whereas it is optimal to resort to 100% CAT bonds if \mathcal{L}_0 is sufficiently large. These results show that CAT bonds complement reinsurance very well, especially when the reinsurance industry has a limited capacity.

Finally, we study how the insurer's risk exposure L_0 is related to the optimal proportion of CAT bond required. The goal here is to check whether our model can explain an empirical observation documented by Hagendorff et al. (2014), that CAT bonds tend to be issued by insurance firms with low risk exposure. We again derive the optimal hedge parameters H_{RE} , M_{RE} , H_{CB} , M_{CB} by solving the optimization problem in Equation (23), but in this case with an additional constraint that $L_0 = L_0^{\text{exo}}$. In other words, the insurer's risk exposure is now considered exogenous and cannot be changed by the insurer. Figure 6 illustrates the optimal proportion of CAT bond $\frac{CB_0}{RE_0+CB_0}$ and the hedge's relative fair value $\frac{RE_0+CB_0}{L_0^{\text{exo}}}$ against the insurance demand L_0^{exo} available to the insurer. There is less need to hedge when L_0^{exo} is small, which results in very small values ($\sim 5\%$) of the hedge's relative fair value. We also note that, as observed by Hagendorff et al. (2014), a larger proportion of CAT bond is then utilized in the hedge and this proportion decreases with an increasing L_0^{exo} . This is because the insurer needs only to hedge higher layers of losses when the value of L_0^{exo} is small. As we have shown, such reinsurance contracts tend to be very expensive, making it optimal to issue a CAT bond.

To sum up, our results reveal that an optimal mix of CAT bond and reinsurance allows the insurer to deliver higher shareholder value. We showed earlier that this is because hedging costs are then

cheaper. The results in this section show that this strategy allows the insurer to purchase more protection so as to increase its capacity to sell a larger amount of risk exposure in the insurance market. We have shown that CAT bonds thus complement reinsurance very well, especially when the reinsurance industry has a limited capacity. Our results also show that both low-volatility and high-volatility insurers would benefit from mixing reinsurance and CAT bonds. Further, our results are in line with Hagendorff et al. (2014)'s empirical finding that CAT bonds tend to be issued by insurance firms with less catastrophe risk exposure and less need to hedge. The rationale we provide for this is that insurers with a low risk exposure need only to hedge higher layers of losses, which tend to be very expensive to cover using traditional reinsurance. We underline that we conducted further analyses to validate the general findings obtained in the above sections and found that they were robust to changes of the input parameters.

4 Conclusion

This paper examines how a catastrophe insurer can optimally combine a traditional reinsurance contract with a CAT bond so as to minimize its hedging cost and maximize its shareholder value. To this end, we compare four hedging strategies, all of which have an excess-of-loss payoff function. The first strategy uses reinsurance only, the second uses a CAT bond only, the third employs a CAT bond to cover small losses and a reinsurance contract to cover larger losses and the fourth strategy resorts to a reinsurance contract to cover small losses and a CAT bond to cover larger losses. Our results indicate that the fourth strategy is optimal with respect to the minimization of the hedging cost and also with respect to the maximization of shareholder value.

Our results demonstrate that this combination of a CAT bond with a traditional reinsurance contract provides the desired coverage for a lower cost. We show that this strategy is optimal due to the fact that, with financial distress costs present, the reinsurer charges higher loadings to cover higher layers of losses. Moreover, in our framework, the risk from moral hazard is reduced when the attachment of the contract is high, and CAT bond investors thus demand smaller loadings for such contracts. It is therefore optimal to hedge smaller losses with reinsurance and higher losses through the issuing of a CAT bond. Our results also indicate that traditional reinsurance is very

useful when risks are relatively small and weakly correlated. However, we find that the efficiency of the reinsurance market breaks down as the magnitude of potential losses and the correlation of risks increase, and that CAT bonds should be favored in such circumstances.

We also establish in this paper that an optimal mix of reinsurance and CAT bond allows the insurer to deliver a higher shareholder value. This is because hedging is cheaper under this strategy, which allows the insurer to purchase more protection so as to increase its risk-bearing capacity. In addition, we show that the optimal proportion of reinsurance to purchase depends crucially on the pre-existing risk exposure of the reinsurer. For low-risk reinsurers it can be optimal to use only reinsurance, whereas for high-risk reinsurer it can be optimal to use only a CAT bond. Reinsurance and CAT bonds thus complement each other very well, especially when the reinsurance industry has a limited capacity. Moreover, we show that a mix of reinsurance and CAT bond remains optimal for different values of the insurer's loss volatility. In other words, both low-volatility and high-volatility insurers would benefit from combining reinsurance and CAT bonds. Finally, we also find that insurance firms with less catastrophe risk exposure should issue a CAT bond rather than purchase reinsurance, assuming that the fixed cost of issuing a CAT bond is not prohibitive for such firms. We explain that this is because such firms need only to hedge higher layers of losses, which are very expensive to cover with reinsurance.

Unlike previous works that study how to combine reinsurance and alternative risk transfer (ART) instruments (see, e.g., Nell and Richter (2004)), we explicitly model the credit risk of the reinsurer and the effect of financial distress on the pricing of reinsurance contracts. We find that modeling these features is crucial in determining the optimal proportion of reinsurance to purchase, since they allow us to determine the usefulness of reinsurance, and include the effects of correlation, the catastrophe losses exceedance probability, and risk exposure on the pricing of reinsurance contracts.

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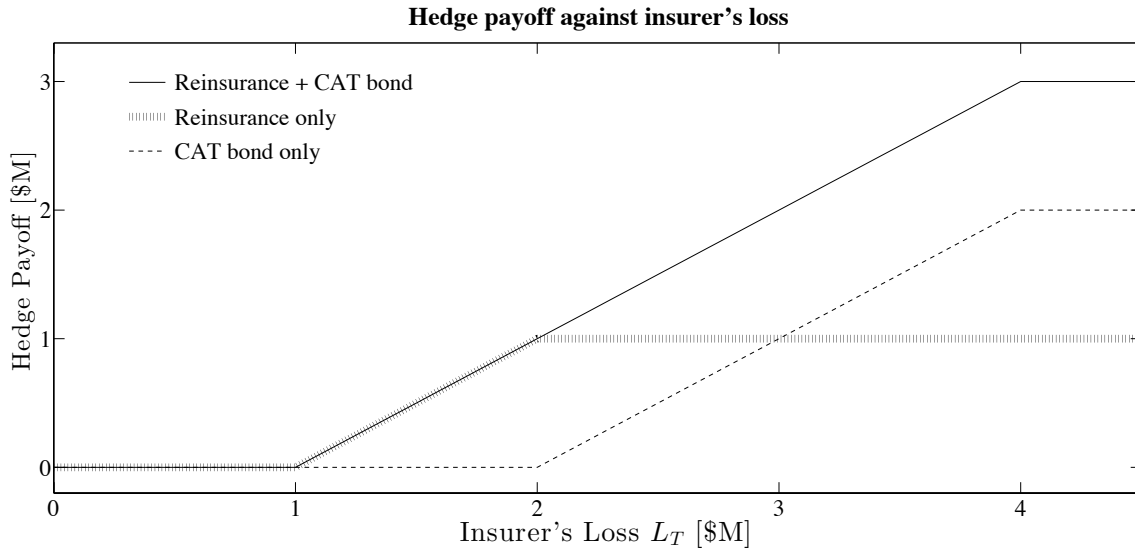
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Table 1: Analyzed excess-of-loss hedging strategies

Strategy	Description	Relevant parameters	Payoff
RE	Reinsurance only	$H_{RE} \geq 0, M_{RE} \geq 0$	RE_T
CB	CAT bond only	$H_{CB} \geq 0, M_{CB} \geq 0$	CB_T
CBRE	CAT bond for low losses & Reinsurance for high losses	$H_{CB} \geq 0, M_{CB} \geq 0, M_{RE} \geq 0,$ $H_{RE} = H_{CB} + M_{CB}$	$RE_T + CB_T$
RECB	Reinsurance for low losses & CAT bond for high losses	$H_{RE} \geq 0, M_{RE} \geq 0, M_{CB} \geq 0,$ $H_{CB} = H_{RE} + M_{RE}$	$RE_T + CB_T$

The reinsurance contract's attachment point and layer limit are denoted by H_{RE} and M_{RE} , whereas the CAT bond's attachment point and layer limit are denoted by H_{CB} and M_{CB} respectively. The payoff functions are contained in Equation (13), and the impact of the reinsurer's default risk is described by Equation (7).

Figure 1: Decomposition of the hedge payoff function for Strategy RECB as the sum of a reinsurance contract (with attachment $H_{RE} = \$1M$ and limit $M_{RE} = \$1M$) and a CAT bond (with attachment $H_{CB} = \$2M$ and limit $M_{CB} = \$2M$). The sum has an attachment of $\$1M$ and a limit of $\$3M$.



This figure illustrates how Strategy RECB works. We see that an XL hedging contract with an attachment point of $\$1M$ and a layer limit of $\$3M$ is obtained by combining a reinsurance contract with attachment $H_{RE} = \$1M$ and layer limit $M_{RE} = \$1M$ with a CAT bond with attachment $H_{CB} = \$2M$ and layer limit $M_{CB} = \$2M$.

Table 2: Baseline input parameters

Maturity of all instruments	T	1 year
Risk-free rate	r	0.02
Insurer's initial equity capital	X	\$100M
Reinsurer's initial asset value	\mathcal{A}	\$1000M
CAT investors' exposure to moral hazard	θ_{CB}	0.7
Reinsurer's exposure to moral hazard	θ_{RE}	0.1
Policyholders' risk aversion	q	5
Maximum insurance loading	δ_P^{\max}	0.1
Minimum CAT bond loading	δ_{CB}^{\min}	0.05
Minimum reinsurance loading	δ_{RE}^{\min}	0.05
Insurer's financial distress	ω_L, k_L	0.7, 0.35
Reinsurer's financial distress	$\omega_{\mathcal{L}}, k_{\mathcal{L}}$	0.7, 0.35
Correlation between losses	ρ	0.4
Insurer's and reinsurer's volatility of losses	$\sigma_L, \sigma_{\mathcal{L}}$	0.5, 0.35
Insurer's and reinsurer's drift of losses	$\mu_L, \mu_{\mathcal{L}}$	0.025, 0.025
Reinsurer's initial value of losses	\mathcal{L}_0	\$350M
Insurer's initial value of losses	L_0	\$120M
Desired XL hedging contract	H, M, p	$0.7 L_0, 5 L_0, 0.2\%$

The parameters L_0, H, M, p are exogenous for the problem of minimizing the hedging cost, but are determined endogenously for the problem of maximizing shareholder value.

Table 3: Results for the minimization of the hedging cost for all hedging strategies presented in Table 1 for different values of correlation ρ between the losses of the insurer and the reinsurer

	Reinsurance layer limit $M_{RE} [L_0]$	Proportion of reinsurance $\frac{RE_0}{RE_0+CB_0}$	Hedge's total price $\Pi [\$M]$	Hedge's loading $\delta_{EFF} [\%]$	Hedge's default probability HPD [%]
$\rho = 0.4$					
RE	5.000	1.000	49.654	13.52	0.284
CBRE	4.572	0.431	50.795	15.89	0.200
RECB	0.658	0.727	47.545	8.16	0.113
$\rho = 0.5$					
RE	5.000	1.000	50.244	15.08	0.342
CBRE	4.122	0.172	53.370	21.75	0.200
RECB	0.596	0.691	47.910	8.96	0.101
$\rho = 0.6$					
RE	5.000	1.000	50.676	16.53	0.422
CBRE	3.773	0.084	54.216	23.74	0.199
RECB	0.527	0.644	48.216	9.67	0.119
CB	0.000	0.000	55.40	25.85	0.000

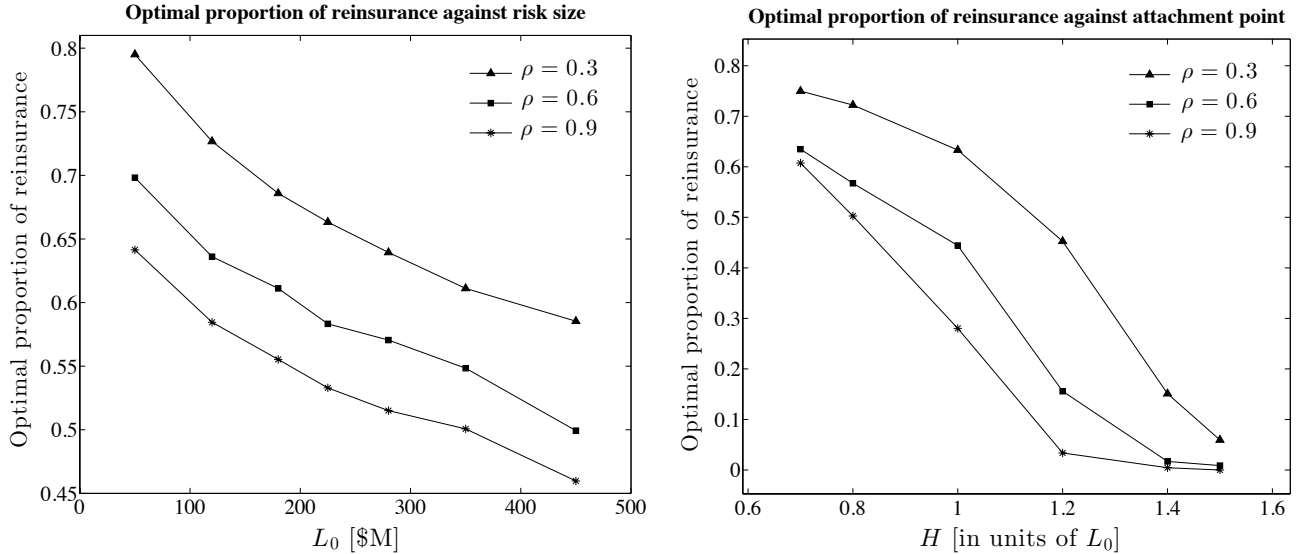
For Strategies CBRE and RECB, the optimal values of M_{RE} are respectively derived by solving the optimization problems described by Equations (17) and (18). The values of M_{RE} are presented in units of L_0 for ease of interpretation. Note that the results for Strategy CB are presented only once since they do not depend on ρ .

Table 4: Results for the minimization of the hedging cost for all hedging strategies presented in Table 1 for different values of the reinsurer’s risk exposure \mathcal{L}_0

	Reinsurance layer limit $M_{RE} [L_0]$	Proportion of reinsurance $\frac{RE_0}{RE_0+CB_0}$	Hedge’s total price $\Pi [\$M]$	Hedge’s loading $\delta_{EFF} [\%]$	Hedge’s default probability HPD [%]
$\mathcal{L}_0 = \$300M$					
RE	5.000	1.000	47.81	8.81	0.098
CBRE	4.994	0.981	47.78	8.80	0.099
RECB	0.832	0.823	46.23	5.03	0.027
$\mathcal{L}_0 = \$325M$					
RE	5.000	1.000	48.84	11.34	0.162
CBRE	4.885	0.813	48.60	10.78	0.155
RECB	0.697	0.747	46.80	6.38	0.054
$\mathcal{L}_0 = \$370M$					
RE	5.000	1.000	50.04	14.65	0.402
CBRE	4.071	0.156	53.52	22.02	0.199
RECB	0.566	0.673	48.06	9.35	0.171
CB	0.000	0.000	55.40	25.85	0.000

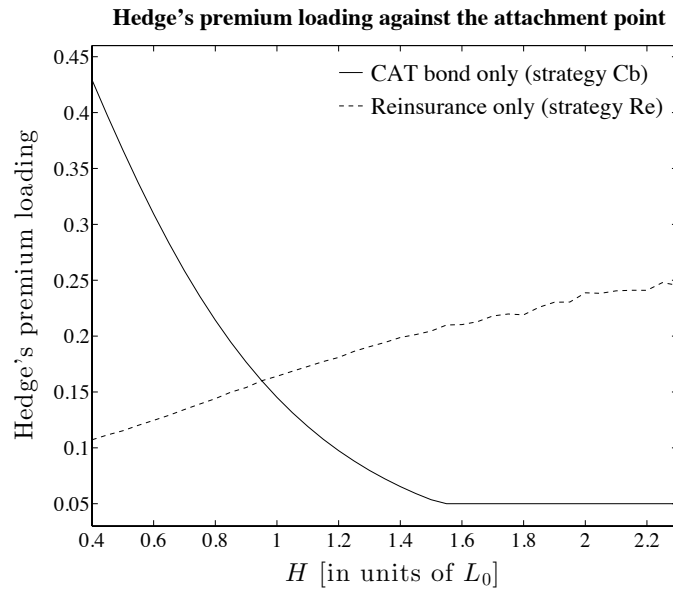
For Strategies CBRE and RECB, the optimal values of M_{RE} are derived by solving the optimization problems described by Equations (17) and (18) respectively. The values of M_{RE} are presented in units of L_0 for ease of interpretation. Note also that the results for Strategy CB are presented only once since they do not depend on \mathcal{L}_0 .

Figure 2: Optimal proportion of reinsurance $\frac{RE_0}{RE_0+CB_0}$ under Strategy RECB as a function of risk size L_0 (left) and attachment point H (right), for three different correlations ρ



The optimal proportion of reinsurance $\frac{RE_0}{RE_0+CB_0}$ is obtained by minimizing the insurer’s hedging cost, i.e., by solving the problem described in Equation (18). L_0 is the insurer’s risk exposure, H is the attachment point of the desired XL contract, and ρ is the correlation between the insurer’s and the reinsurer’s losses. The other parameters are presented in Table 2.

Figure 3: Loading of the CAT bond and of reinsurance against the attachment point



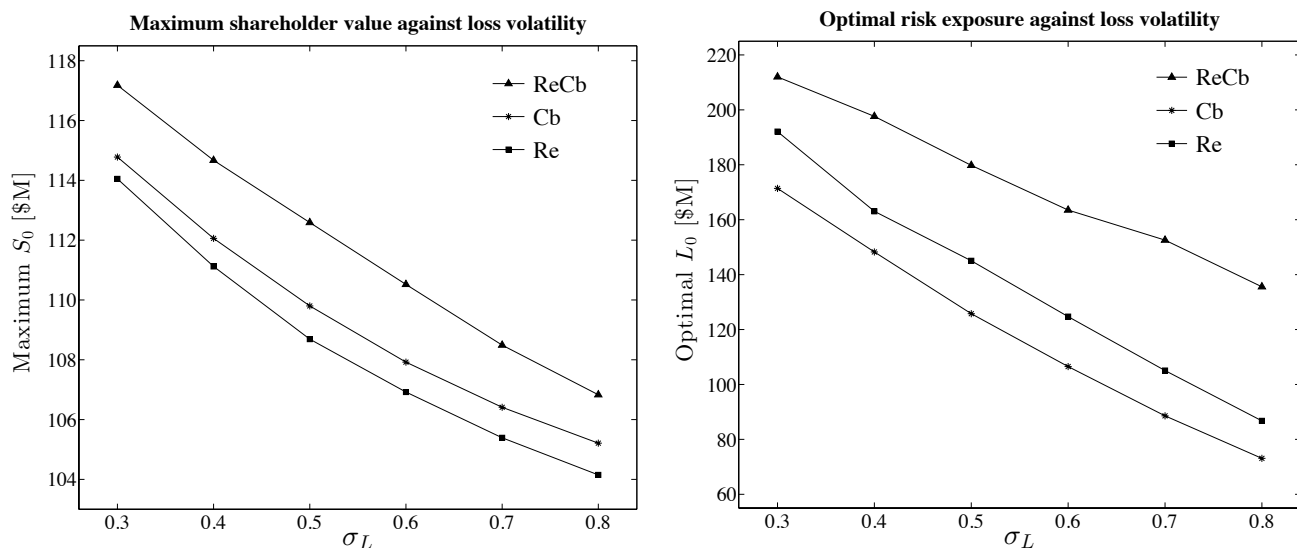
For Strategy CB (CAT bond only), the premium loading is obtained directly from Equation (5). For Strategy RE (reinsurance only), the premium loading is given by $\pi_{RE}/RE_0 - 1$, where RE_0 is the actuarially fair value of the reinsurance contract and π_{RE} is its price as given by Equation (8).

Table 5: Results for the maximization of shareholder value for all hedging strategies under consideration, for different values of the insurer's loss volatility σ_L

	Optimal risk exposure and hedging parameters				Insurance load	Hedge load	Insurance fair value	Hedge fair value	Financial distress cost	Maximum equity	Default probability	
	L_0 [\$M]	H_{RE} [L_0]	M_{RE} [L_0]	H_{CB} [L_0]								M_{CB} [L_0]
$\sigma_L = 0.4$												
CB	148.24	-	-	1.011	5*	10.00	11.51	148.24	24.05	0.000	112.06	0.00
RE	163.06	0.918	5*	-	-	9.95	15.14	162.97	33.32	0.071	111.12	0.10
RECB	197.63	0.733	0.431	1.164	5*	10.00	7.99	197.63	63.10	0.053	114.67	0.00
$\sigma_L = 0.5$												
CB	125.76	-	-	1.075	5*	9.99	12.51	125.74	22.23	0.009	109.80	0.01
RE	145.05	0.937	5*	-	-	9.92	16.77	144.87	33.45	0.076	108.69	0.16
RECB	179.79	0.748	0.537	1.285	5*	10.00	8.89	179.78	60.18	0.048	112.59	0.01
$\sigma_L = 0.6$												
CB	106.51	-	-	1.155	5*	9.99	13.43	106.49	20.25	0.004	107.92	0.01
RE	124.70	0.998	5*	-	-	9.89	17.76	124.41	29.93	0.075	106.92	0.23
RECB	163.50	0.760	0.587	1.347	5*	9.99	9.93	163.48	58.23	0.044	110.52	0.01
$\sigma_L = 0.7$												
CB	88.56	-	-	1.275	5*	9.97	13.84	88.47	17.37	0.016	106.41	0.05
RE	105.08	1.093	5*	-	-	9.85	19.12	104.62	25.45	0.074	105.38	0.29
RECB	152.57	0.747	0.726	1.474	5*	9.95	11.11	152.39	59.62	0.042	108.49	0.10

For Strategy RE (reinsurance only), the optimal risk exposure and hedging parameters are derived by solving the optimization problem in Equation (21). For Strategy CB (CAT bond only) and Strategy RECB (reinsurance and CAT bond), these parameters are derived by solving the optimization problems given by Equations (22) and (23), respectively. Numbers marked by * do not have a precise optimal value and need only be sufficiently large. Note also that Strategy CBRE is not investigated since the results on minimizing hedging costs showed that such a combination of reinsurance and CAT bond is not optimal. The results presented in this table show that an optimal mix of reinsurance and CAT bond (Strategy RECB) allows the insurer to attain higher shareholder value S_0 .

Figure 4: (left) Maximum shareholder value S_0 and (right) optimal risk exposure L_0 against loss volatility σ_L for the different hedging strategies under consideration



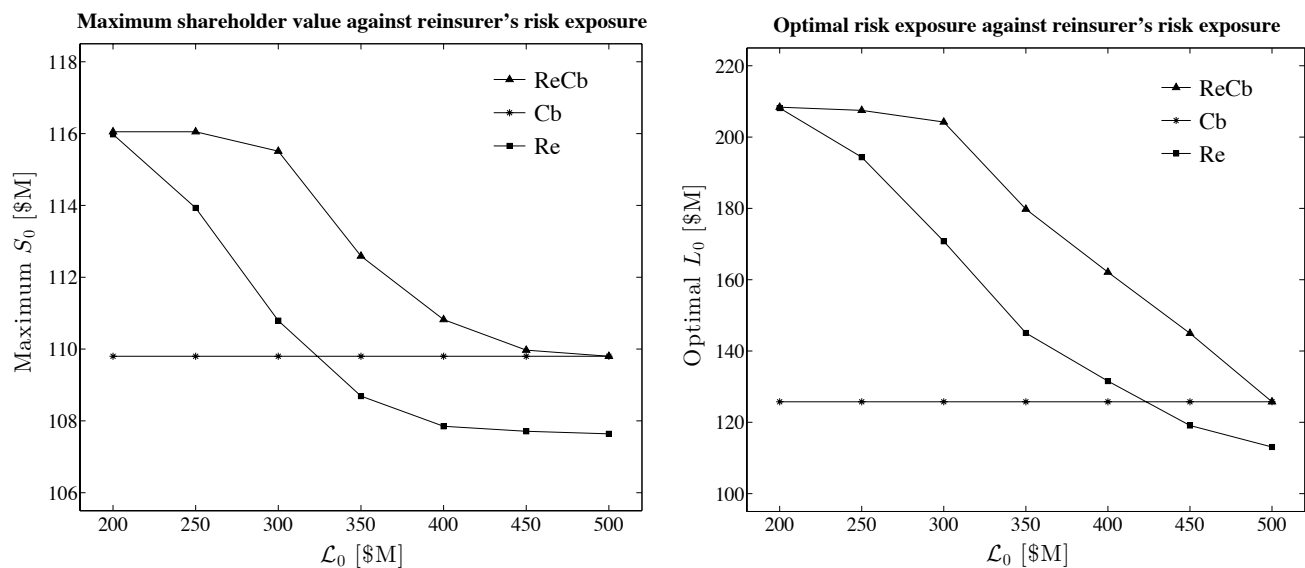
For Strategy RE (reinsurance), the optimal parameters are derived by solving the optimization problem given by Equation (21). For Strategy CB (CAT bond) and Strategy RECB (reinsurance and CAT bond), these parameters are derived by solving the optimization problems in Equations (22) and (23), respectively. We can see that Strategy RECB leads to higher shareholder value and risk exposure.

Table 6: Results for the maximization of shareholder value for all hedging strategies under consideration for different values of the reinsurer's risk exposure \mathcal{L}_0

	Optimal risk exposure and hedging parameters					Insurance load	Hedge load δ_{EFF} [%]	Insurance fair value P_0 [\$M]	Hedge fair value $\text{RE}_0 + \text{CB}_0$ [\$M]	Financial distress cost	Maximum equity S_0 [\$M]	Default probability
	L_0 [\$M]	H_{RE} [L_0]	M_{RE} [L_0]	H_{CB} [L_0]	M_{CB} [L_0]							
$\mathcal{L}_0 = \$300M$												
RE	170.78	0.790	5*	-	-	9.95	11.66	170.64	52.66	0.052	110.79	0.10
RECB	204.22	0.604	0.768	1.372	5*	10.00	5.50	204.20	89.12	0.014	115.51	0.01
$\mathcal{L}_0 = \$350M$												
RE	145.05	0.937	5*	-	-	9.92	16.77	144.87	33.45	0.076	108.69	0.16
RECB	179.79	0.748	0.537	1.285	5*	10.00	8.89	179.78	60.18	0.048	112.59	0.01
$\mathcal{L}_0 = \$400M$												
RE	131.56	1.024	5*	-	-	9.85	19.66	131.26	25.15	0.144	107.85	0.30
RECB	162.11	0.846	0.365	1.211	5*	9.99	11.61	162.09	44.95	0.160	110.82	0.01
$\mathcal{L}_0 = \$450M$												
RE	119.12	1.115	5*	-	-	9.75	19.65	118.72	18.49	0.240	107.71	0.49
RECB	144.95	0.950	0.243	1.194	5*	10.00	12.67	144.94	32.65	0.390	109.97	0.00
$\mathcal{L}_0 = \$500M$												
RE	113.07	1.171	5*	-	-	9.62	18.54	112.52	15.07	0.410	107.64	0.75
RECB	125.76	1.075	0.000	1.075	5*	9.99	12.51	125.74	22.23	0.009	109.80	0.01
CB	125.76	-	-	1.075	5*	9.99	12.51	125.74	22.23	0.009	109.80	0.01

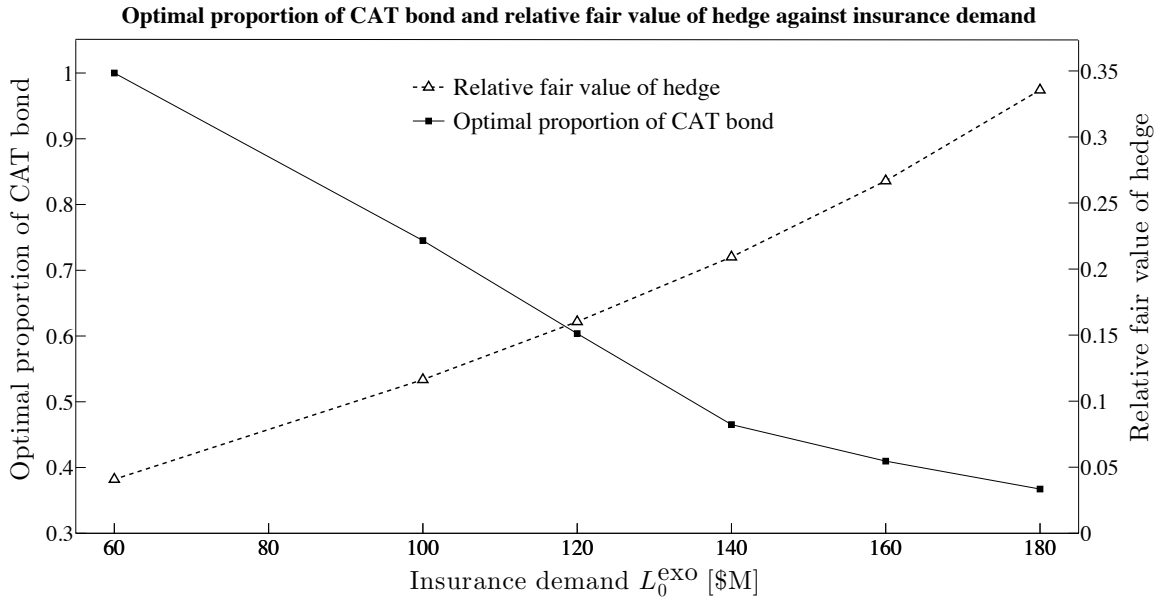
For Strategy RE (reinsurance only), the optimal risk exposure and hedging parameters are derived by solving the optimization problem in Equation (21). For Strategy CB (CAT bond only) and Strategy RECB (reinsurance and CAT bond), these parameters are derived by solving the optimization problems in Equations (22) and (23), respectively. Numbers marked by * do not have a precise optimal value and need only be sufficiently large. Also, note that the results for Strategy CB are presented only once since they do not depend on \mathcal{L}_0 . Finally, note that Strategy CBRE is not investigated since the results on minimizing hedging costs show that such a combination of reinsurance and CAT bond is not optimal. The results presented in this table show that an optimal mix of reinsurance and CAT bond (Strategy RECB) allows the insurer to attain higher shareholder value S_0 .

Figure 5: (left) Maximum shareholder value S_0 and (right) optimal risk exposure L_0 against reinsurer's risk exposure \mathcal{L}_0 for the different hedging strategies under consideration



For Strategy RE (reinsurance), the optimal parameters are derived by solving the optimization problem shown in Equation (21). For Strategy CB (CAT bond) and Strategy RECB (reinsurance and CAT bond), these parameters are derived by solving the optimization problems in Equations (22) and (23), respectively. We can see that the values under Strategy RECB converge to those under Strategy RE when \mathcal{L}_0 is sufficiently small, but to those under Strategy CB when \mathcal{L}_0 is sufficiently large.

Figure 6: (left axis) Optimal proportion of CAT bond $\frac{CB_0}{RE_0+CB_0}$ and (right axis) relative fair value of hedge $\frac{RE_0+CB_0}{L_0^{\text{exo}}}$ against insurance demand L_0^{exo} available to the insurer



This figure depicts how the insurer’s risk exposure is related to the optimal proportion of CAT bond. The optimal hedge parameters H_{RE} , M_{RE} , H_{CB} , M_{CB} are derived by solving the optimization problem in Equation (23) with an additional constraint that $L_0 = L_0^{\text{exo}}$. We can see that there is less need to hedge when L_0^{exo} is small, which results in very small values of the hedge’s relative fair value. We can also see that a larger proportion of CAT bond is then required in the hedging strategy, and that this proportion decreases with an increasing value of L_0^{exo} .