Embracing the Perfect Storm:
A Theory of Optimal Catastrophe Risk Financing

Carolyn W. Chang,[a] Jack S.K. Chang,[b] Min-Teh Yu[c]

Abstract

Unprecedented increases in extreme loss events have raised important concerns about the inadequacy of catastrophe risk financing. However, the Federal Reserve's ultra-low interest rate policy following the 2007-2009 financial crisis has transformed the catastrophe space into a new normal. Demand shocks from an influx of third-party capital have induced capacity expansion, risk capital redistribution, and premium reduction; and catalyzed a convergence of the traditional reinsurance and the securitized catastrophe bond markets. We develop a novel theory of catastrophe risk financing that optimally allocates these two inherently different products, in relation to their characteristics as well as catastrophe arrival intensity and severity, toward providing *ex ante* full risk intermediation and financing. Numerical results demonstrate the stylized post-crisis convergence process and parametric sensitivity.

JEL classification: G20; G22; G28
Keywords: Fed's QE; Optimum catastrophe risk financing; Optimum allocation; Traditional reinsurance; Catastrophe bonds.

[a] Department of Finance, California State University, Fullerton, *cchang@fullerton.edu*
[b] Department of Finance & Law, California State University, Los Angeles, *jskchangg@gmail.com*
[c] Institute of Finance, National Chiao Tung University, Taiwan, *mtyu@nctu.edu.tw*

* We would like to thank seminar participants at the 2014 Actuarial Research Conference, the 2014 Western Risk and Insurance Conference, Temple University, the National University of Singapore, and the ICW Group of Insurance Companies, and especially Hua Chen, David Cummins, Jin Duan, Yisong S. Tian, and Mary Weiss for valuable comments. We would also like to thank Owen Jeng and Don Mango at Guy Carpenter, as well as Kimberly Anthony and Shawn Adams at the ICW Group for industry insights. All remaining errors are ours.

Embracing the Perfect Storm: A Theory of Optimal Catastrophe Risk Financing

1. Introduction

Over the last two decades, an unprecedented increase in both natural and man-made catastrophic events[1] has occurred. Recent climate change studies have also provided empirical evidence that global warming has more than doubled the likelihood of extreme weather events with more devastating cyclones, floods, and clustered tornadoes.[2] In the U.S. alone, hurricane-induced economic losses have increased exponentially with estimated annual total losses averaging $10.1 billion in 1990-1995, and jumped to $35.8 billion in 2001-05.[3] In 2012, combined losses from Super Storm Sandy and the year-long Midwest and Plains drought totaled over $100 billion, amounted to about 0.6% of the 2012 U.S. GDP.[4]

Traditional tailor-made reinsurance has been the de facto mechanism for transferring catastrophe (cat hereafter) risks, which comprise the largest segment of the global reinsurance market. In general, the non-life insurance industry, inclusive of both governmental agencies and private insurers, collects approximately $2 trillion in premiums per year from policy holders around the world; it then uses about $200 billion per year of this income to buy its own insurance, the so-called reinsurance. In other words, about 10% of the insurance premium that consumers pay for homes, cars, and businesses ideally passes into a global reinsurance pool that serves as a full-functioning risk-sharing mechanism for extreme losses. These risks may include a hurricane in eastern U.S., a tsunami in Japan, or flooding in Europe.

In reality, however, cat risks have been notoriously known to be under-covered. On the supply side, Froot (2001) and Froot and O'Connell (2008) find that reinsurers consistently charge premium/expected loss multiples at 5-8 times, even though cat losses are generally uncorrelated with returns on other major asset classes,[5] such that one would expect cat risk premiums to approximate expected losses after adjusting for idiosyncrasies. Moreover, these multiples tend to jump even higher in the aftermath of a major cat event due to reinsurers' "reloading" of balance sheets. The authors attribute these high multiples to reinsurers' monopolistic power and high costs of capital resulting from

---

[1] Examples of such events include the 9/11 terrorist attacks; Hurricanes Katrina, Rita, and Wilma in 2005; the Tohoku earthquake/tsunami in 2011; Super Storm Sandy in 2012; Typhoon Haiyan in 2013; flooding in the UK and South America in 2014; and the Ebola pandemic in West Africa in 2014.

[2] Elsner, Kossin and Jagger (2008), Webster, Holland, Curry and Chang (2005), Emanuel (2005), and Schiermeier (2011) provide empirical evidence.

[3] National Science Board, "Hurricane warning: the critical need for a national hurricane research initiative", January 12, 2007.

[4] See Aerts, Botzen, Emanuel, Lin, de Moel and Michel-Kerjan (2014) for a recent study about the loss impact of Hurricane Sandy through sea surge.

[5] The extremely rare case when a major cat event causes a market collapse is an exception, such as the systemic risk induced by the Tohoku earthquake/tsunami in 2011.

inefficient corporate forms.[6] Reinsurers' capital pool is also often seriously impaired after a cat event, with the usual means of rebuilding capacity by new company formation through initial public offerings, seasonal equity offerings, and capital increases being insufficient.

On the demand side, Froot (2001) notes that, contrary to the intuition that reinsurance purchases should prioritize the more severe events; mid- and small-sized events are more prevalent. In addition, insurers intermediate only a small fraction of cat exposures and tend to retain rather than share cat risks, with increases in retention after a large event. Ibragimov, Jaffee and Walden (2008) further suggest that private insurers may choose not to offer catastrophe insurance due to a non-diversification trap when tail risk is heavy and the insurers have limited liability. As a result of this supply/demand interplay, reinsurers tend to insure the most severe event the least, insurers prefer to purchase low reinsurance loss layers that are subject to a high probability of penetration versus full protection against ruin, and the total number of reinsurance transactions often falls sharply after a major event.

Amidst the recent upsurge of critical cat events with the subsequent decline in reinsurance capacity and rise in pricing, insurance-linked securities (ILS)[7] have emerged as a functional capital market alternative to the reinsurance norm. The cat bond, or 'Act of God' bond as it was first called, has become known as one of the most successful ILS (Cummins and Weiss (2009)).[8] Since its debut in the 1990s as a major risk-transfer innovation in the insurance sector, cat bonds have incentivized both insurers and reinsurers to participate as sellers, with the former looking for a cheaper and default-free substitute to hedge underwriting exposures and the latter aiming to expand underwriting capacity and enhance reinsurance value. At the same time, cat bonds incentivize demand from fund managers for higher returns than those of equally-rated corporate bonds, and near-zero beta to achieve further diversification beyond investing in other asset classes.

The cat bond market initially experienced slow growth due to market entry barriers, illiquidity, investors' unfamiliarity, and loss parameter uncertainty (Cummins and Weiss (2009)), resulting in high issuance costs with a bond spread of about 6.5 times the expected loss (Cummins, Lalonde and Phillips 2004). The market was only $1-2 billion of issuances per year during the 1998-2001 period, but jumped to over $2 billion per year following the 9/11 attack in 2001, which created an adverse effect on available reinsurance capacity across perils. Issuances jumped again to about $4 billion on an annual basis in 2006 following Hurricanes Katrina, Rita and Wilma and then grew further to record-breaking

[6] Traditional reinsurers tend to invest in illiquid and information-intensive financial activities, and so they charge premiums based on correlations with their own pre-existing portfolios and U.S. nationwide cat risks, rather than with any market portfolio, resulting in significantly higher cost of capital.

[7] ILS refer to instruments, such as cat bonds, cat swaps, cat options, sidecars, collateralized quota shares, and industry loss warrants, that are designed to pass life and non-life insurance risks on to the financial markets. Among them, cat bonds, borne out of a desire to broaden reinsurance capacity in the aftermath of Hurricane Andrew in 1992, have been viewed as the most successful ILS (Cummins and Weiss (2009)).

[8] Munich Re 2013 Market Review reports that cat bonds are the leading source of ILS capacity at 43%.

annual issuances of more than $7 billion in 2007.[9] During the same period, bond spread/expected loss multiples steadily declined to an average of 2.9 (Lane and Mahul 2008). As markets for structured products collapsed with too "little skin in the game" (Domiroglu (2012)) in the heydays of the 2007-2009 subprime crisis, the cat bond market also dried up due to a flawed market risk transfer mechanism and limited appetite from investors.[10]

Interestingly enough, the 2007-2009 subprime crisis turned out to be a blessing in disguise for the cat bond market. The Federal Reserve's ultra-low interest rate policy, with three phases of quantitative easing (QE) and operation twist,[11] led fund managers to go "risk-on" with their capital and aggressively seek higher spreads within their risk profiles across all available offerings in catastrophe and other structured product spaces (e.g. CDOs). This demand shock, coupled with the softness of the reinsurance underwriting cycle, has transformed the cat space into a "new normal". This transformed space is characterized by a virtuous cycle starting with fund managers seeking higher bond returns as a new and lucrative alternative investment class,[12] which leads to more bond issuances[13] and improved risk-transfer mechanisms, increased investors' sophistication in the bond structure backed by state-of-the-art catastrophe modeling, greater competition between cat bonds and reinsurance contracts, and significant downward pressure on reinsurance pricing toward cat bond pricing, and eventually results in a significant lower spread multiple of about 2.0 in 2014.[14] Today, the cat space comprises about 16% of

[9] See Figure 1 for the growth history of cat bonds over 2003-2013.

[10] Several cat bonds issued by All State defaulted, because of the Lehman Brother debacle.

[11] Dobb, Lund, Koller and Shwayder (2013) argue that the Fed's low interest rate policies have produced distributional effects through impacts on interest rate income and expenses of different sectors of the economy. Lower rates have reduced interest rate expenses for borrowers, while at the same time diminishing the interest income of savers. There has been a net transfer of wealth from insurers, pension funds, households, and the rest of world to the U.S. federal government, non-financial corporations, and banks. In particular, life insurers and pension funds that rely heavily on fixed-rate policies have been facing a squeeze.

[12] Alternative investment strategies that were once the domain of institutional and high net worth investors are going main stream in response to fund companies' demand for diversification and/or downside protection in the wake of the 2008 market collapse. Cat risk has become an attractive asset class for alternative investments, because of its lucrative spreads and near-zero beta. Since the financial crisis, the insurance-linked fund industry has broadened to include other catastrophe products such as collateralized reinsurance, which has ballooned to more than $40 billion of assets under management and has established itself as a mainstream asset class. Hedge funds tend to opt for higher returns in the opportunity set, whereas pension funds and similar investors opt for a lower, more stable return profile.

[13] A recent Wall Street Journal article dated 4/23/2014, entitled "Investors embrace 'cat bonds'", reports that cat bond issuance is now exploding, with 2013 first quarter issuances more than doubling from the previous year to $1.2 billion and second-quarter issuances hitting an all-time high above $3.5 billion. At the same time, the average quarter yield on cat bonds has sunk to its lowest level in nine years, down to 5.22% from 9.61% in 2012. Aon's 2013 annual report further shows that large investor inflows drove declining spreads, which in turn sparked increasing sponsor interest; and as of the year ending June 30, 2013, total cat bonds issued were at an all-time high of $17.5 billion, surpassing the previous record of $16.2 billion on June 30, 2008.

[14] This information is taken from Lane Financial's Quarterly Market Performance Report Q2 2014.

ILS, up from 2-3% in the late 1990s.[15] More than ever before, cedents are choosing to incorporate ILS into their overall risk transfer programs.[16] As a result, the role of ILS has evolved from that of a threatened reinsurance substitute to one being a complementary product, underpinning the ultimate convergence of the two markets.[17]

Figure 1 about Here

Against the backdrop of this convergence, our research contributes to the extant literature in several ways. First, to the best of our knowledge, our study is the first to develop a theory of optimal allocation between cat bonds and cat reinsurance with the aim of promoting full financing of insured risks so as to increase social welfare. Reinsurance by itself has provided at best suboptimal financing of cat risks, while the emergence of cat bonds and other ILS has served to expand (re)insurers' underwriting capacity. Thus, an optimum allocation of the two would push the efficient frontier further to the left towards achieving optimum catastrophe risk financing. However, this pursuit of optimality is not as straightforward as de facto asset allocation in portfolio analysis because of the inherent differences between the two contracts, with standardized cat bonds being fully-collateralized and default-free, and customized cat reinsurance being only partially-collateralized and thus default-risky.[18] The difficulty also stems from the fact that a reinsurance arrangement blends art and science as both the reinsurer and insurer are well-informed, can sell cat bonds, and are free to negotiate pricing and coverage in the spirit of open competition. As a result, there is an unavoidable pseudo game-theoretic supply and demand interaction between the two parties to determine the eventual optimum risk intermediation (e.g. He and Krishnamurthy, 2013). Due to these difficulties, the (re)insurance industry has been approaching allocation decisions in an ad-hoc manner.[19]

[15] According to a 2014 report by McKinsey & Company, entitled "Could third-party capital transform the reinsurance markets?", 16% of the approximately $300 billion in catastrophe reinsurance capacity worldwide is provided by third-party capital, up from 2-3% in the late 1990s.

[16] The Deutsche Bank 2013 Annual Report states that cat bonds rank as the fifth-best performer among all global financial assets since September 2008 with a total return of more than 52% in terms of the Swiss Re Global Cat Bond Index.

[17] Today, the ILS marketplace features a solid, expanding core of experienced dedicated fund investors. Rating agencies and modeling firms have also played critical roles in enhancing the confidence of market participants by working on and providing analysis of transactions. Issuers too have become far more comfortable with using ILS strategies, and the use of ILS as hedging instruments has evolved from being somewhat experimental to an integral part of the risk management tool-kit, with benefits ranging from managing earnings volatility to capital management and the ability to monetize illiquid assets.

[18] Section 3.1 contains a more detailed comparison of cat bonds and cat insurance.

[19] For example, the Mexico Fund (FONDEN) arranged a $450 million reinsurance contract against earthquake risk in 2006, sponsored by the Mexican government. In this arrangement, the reinsurer EFR (European Financial Reinsurance) and a subsidiary of Swiss Re decided to retain $290 million of reinsurance exposure by issuing a

Our optimal allocation methodology is rooted in the structure of a cat bond, as illustrated in Figure 2,[20] issued by a (re)insurer-sponsored Special Purpose Vehicle (SPV). The function of the SPV, acting as an independent reinsurance company, is to simultaneously sell reinsurance to the sponsor and issue cat bonds to investors as a "pure play" in catastrophe risk. In this sense, the SPV offers reinsurance protection to the reinsurer, known as re-reinsurance. As the reinsurer's objective by issuing cat bonds through the SPV is to reduce default risk through the re-reinsurance protection to enhance the value of the reinsurance offer to the insurer, its optimum allocation for each coverage layer (defined from attachment point to detachment point)[21,22] can be achieved by maximizing the NPV of the cash flows from a short reinsurance-long re-reinsurance mix with respect to a cat bond's structural parameters. The layer with the largest NPV across all of the coverage layers then locates a reinsurer's sell-side global optimum allocation. On the other hand, since the insurer issues cat bonds to supplement its reinsurance purchase as a cheaper hedging alternative, its long reinsurance-long re-reinsurance allocation mix is optimized, for each reinsurance layer offered by the reinsurer, by minimizing the total hedging cost of the allocation with respect to the cat bonds' structural parameters and the insurer's external hedging needs. The layer with the smallest total hedging cost across all coverage layers offered by the reinsurer then locates the insurer's buy-side global optimum allocation. As the insurer's global optimum allocation generally does not coincide with that of the reinsurer's, negotiations will follow to reach the final allocation.

Figure 2 about Here

Our second contribution to the literature is to derive the optimum catastrophe risk financing solutions through a sequential dual optimization framework starting with the reinsurers' optimum allocation. We first apply Merton's (1974, 1977) structural model to endogenize the reinsurer's default in the presence of interest rate and credit risks. In a no-arbitrage martingale framework, we then

---

$160 million cat bond - in other words, an ad-hoc allocation of 64% in traditional reinsurance and 36% in cat bonds. In August 2011, the California Earthquake Authority (CEA) successfully launched a ground-breaking sale of $150 million earthquake cat bonds, with a total of $650 million cat bonds now outstanding as of Q2 2013 - an allocation in its funding program of about 65% in traditional reinsurance and 23% in cat bonds, with the remaining 12% going to other variations.

[20] See Section 3.2 for more details about the allocation methodology nested in the cat bond structure.

[21] Layering is a common form of insurance coverage. Layered programs involve a series of insurers' coverage at various levels formed on an excess of loss basis, ultimately providing an insured with a high total limit of coverage.

[22] Figure 3 in Section 3 demonstrates a reinsurer's short default-risky reinsurance–long default-free re-reinsurance position with the insurer and the SPV as the two counterparties.

respectively value the default-risky reinsurance in relation to the evolution of the reinsurer's balance sheet and catastrophe arrival uncertainties, and the default-free cat bond, with and without basis risk, in relation to catastrophe arrival uncertainties. Next, based on the NPV maximization approach discussed above, we derive the reinsurer's sell-side global optimum allocation and a schedule of reinsurance pricing across layers. Given the sell-side pricing schedule, we then derive the insurer's buy-side global optimum allocation, based on its layering need and by minimizing its total hedging cost, to form its funding program (see Figure 3 for a detailed sample depiction of a typical insurer's funding program). For example, in one scenario as shown in our convergence simulation[23] where the reinsurance market is hard but the cat bond market is soft, the reinsurer's sell-side optimum allocation is (2.10374, 90, 10, 90, 37) - i.e. to sell reinsurance coverage over loss layer (90, 10) and simultaneously issue a cat bond over loss layer (90, 37) to reach a maximum NPV of 2.10374. But the insurer's buy-side optimum allocation is (1.73904, 60, 30, 80, 30) - i.e. purchase reinsurance coverage over loss layer (60, 30) and simultaneously issue a cat bond over loss layer (80, 30) to reduce the total hedging cost to 1.73904. As expected, in this scenario the reinsurer prefers to sell more coverage so as to maximize NPV, while the insurers prefers to buy less coverage with a substitution from issuing the cheaper cat bonds to minimize total hedging cost.

Figure 3 about Here

Our third contribution is to demonstrate how our optimum allocation models simulate the post-crisis convergence process through three stages: the initial stage when the cat bond market softens due to surging demand from fund managers but the reinsurance market is still hard; the intermediate stage when the reinsurance market begins to soften due to competitive pressure from the cat bond market; and the final stable stage when both markets have softened to converge onto the new normal with fair pricing. We also conduct an extensive sensitivity analysis to study the parametric impact[24] on the allocations. We find that first, as interest rate risk increases, reinsurers will issue more cat bonds to hedge higher default risk, and the greater the reinsurer's leverage is, the more the allocation will be. Second, when basis risk is present, because the cat bond's payoffs are linked to a cat loss index rather than the reinsurer's actual cat loss, insurers will rely less on the use of cat bonds due to lower hedging effectiveness, while reinsurers will increase the size of the cat bond issuance but with a lower trigger

[23] See Section 6.1, Tables 2.2 and 3.
[24] These include a reinsurer's markups, layering, capital/debt positions, and interest rate/credit risk exposures; a cat bond's spread, trigger point and size, and basis risk; and the catastrophe arrival frequency and loss volatility, among others.

level in order to enhance hedging effectiveness. Finally, increases in both catastrophe arrival intensity and loss volatility raise the reinsurer's NPV as reinsurance becomes more valuable, leading to a higher total hedging cost for the insurer. These results suggest that as catastrophe arrival intensifies and the loss becomes more unpredictable, reinsurers should increasingly slice off lower layer exposures as the expected losses on these layers accentuate, by issuing cat bonds with lower trigger levels. For insurers, hedging becomes more expensive for both reinsurance and cat bonds, but optimum allocation does not change significantly. We further observe that the loss volatility effect is more acute than the loss arrival intensity effect, suggesting that (re)insurers should be more concerned with the severity of a catastrophe than with the frequency of the arrival.

Looking forward a natural question arises: will the convergence continue, stop, or even recede, after policy reversal from the Federal Reserve. On this issue, we are cautiously optimistic that the convergence will develop further toward *ex ante* full catastrophe risk financing, though the development may not be a monotonic one and may be intertwined with the concurrent development of an *ex post* government public catastrophe loss program to address the unprecedented growth in post-disaster relief.[25] Our optimism stems from the maturity of market participants who have familiarized themselves with ILS during the post-crisis convergence process, and that cat bond funds have become an increasingly popular alternative investment class. The prospect of continued growth of third-party capital participation in the catastrophe space is conceivable, considering the sheer size of global managed assets of more than $100 trillion, compared with a total property catastrophe reinsurance market of $200 billion in premiums. Several major investors could take a large position in the market to show their long-term commitment with balance sheets that have the depth to absorb market volatility. General demand for new issuances has mainly been driven by dedicated cat funds, which have seen continuous cash inflows from end investors as a result of the low yield environment in corporate debt markets, allowing sponsors to reduce risk spreads and increase issuances. Pension funds have also allocated substantial capital to this sector over the last several years, providing collateral capital and enabling the development of a new source of catastrophe risk financing - collateralized reinsurance. Mutual funds as a whole are a future source for increased direct participation in the ILS sector through a greater development of reinsurance-based mutual funds.

How fast this trend will continue depends on the development of several structural and macroeconomic factors: 1) whether third-party capital products can mimic traditional insurance, 2) whether third-party capital remains more efficient providers of capital than reinsurers, and 3) whether property catastrophe markets continue to attractive investors. Nevertheless, although increased capacity from third-party capital has benefited insurers, it does challenge traditional reinsurers to innovate and to

[25] Jaffee and Russel (2013) use the tools of welfare economics to analyze the appropriate mix of *ex ante* private and *ex post* government responses to catastrophe events, and the potential role for mandatory insurance.

remain competitive in this customarily lucrative line of business. However, due to the relational nature of the reinsurance business, the reinsurer will remain a pivotal player in the cat market.

In the rest of the paper, we discuss related research in Section 2. Section 3 sets up the framework of our optimum allocation methodology based upon the structure of a cat bond. Section 4 models the reinsurer's balance sheet to endogenize default, values catastrophe reinsurance and bonds to implement the reinsurer's optimum allocation, and derives the resulting sell-side optimum layer and reinsurance pricing schedule. Section 5 determines the insurer's optimum allocation vis-à-vis reinsurance pricing and cat bond issuing cost across all reinsurance coverage offered by the reinsurer. Section 6 conducts a simulation to demonstrate how the Fed's policy impacts optimum allocation in the catastrophe space, leading to convergence of the reinsurance and capital markets. We also study the impacts of the input parameters on allocation via sensitivity analysis. Finally, Section 7 concludes the paper.

2. Related Research

There is a large strand of finance and insurance literature on cat risk management in relation to reinsurance and cat bonds. For the convergence of cat reinsurance and cat bond markets, Croson and Kunreuher (2000) first discuss cat bond management and the issues that may emerge in their combination with reinsurance. Froot (2008) points out that in a well-functioning insurance market, one would expect a large degree of sharing, especially for large losses, but there are persistent inefficiencies in the cat reinsurance market with unusually high premiums and limited supply. Ibragimov, Jaffee and Walden (2008) suggest that private insurers may choose not to offer cat insurance due to a non-diversification trap when tail risk is heavy and the insurers have limited liability, which limits the demand of cat reinsurance. Lee and Yu (2007) examine how a reinsurance company can increase the value of a reinsurance contract by reducing its default risk with the issuance of cat bonds. Barrieu and Louberge (2009) point out that downside risk-aversion and ambiguity aversion have limited the success of cat bonds and propose replacing simple cat bonds with hybrid cat bonds so as to provide protection against market risk, e.g. a stock market crash. Finken and Laux (2009) argue that parametric or index cat bonds provide low-risk insurers with an alternative to reinsurance contracts, leading to less cross-subsidization in the reinsurance market. Cummins and Weiss (2009) give a comprehensive review of the slow convergence of (re)insurance and financial markets driven by the increase in the frequency and severity of catastrophe risk, inefficiency created by (re)insurance underwriting cycles, advances in IT technologies, and the emerging success of ILS issuances in capital markets, arguing that securitization expands out (re)insurers' risk-bearing capacity. Hardle and Lopez Cabrera (2010) apply May 2006 Mexican earthquake cat bond data to calibrate a pure parametric cat bond and argue under a homogenous Poisson setting that the implied trigger intensity rate can measure the relative costs of reinsurance and cat bond allocations. This stream of research has demonstrated the importance of nesting reinsurance and cat bonds in providing protection against catastrophes. Nevertheless, as Cummins and Trainar (2009) have suggested, models about how they can be optimally mixed have been lacking, because of the inherently different trading mechanisms, risk structures and contract designs of these two products.

For the issue of pricing, Chang, Cheung and Krinsky (1989) value reinsurance in an option theoretical framework as an alternative to the actuary norm of expected loss plus risk loading. Lee and Yu (2002) model the cat bond spread in a contingency-pricing no-arbitrage martingale framework as an alternative to the actuary norm, incorporating issuer default, moral hazard, and basis risk. Froot (2001) and Froot and O'Connell (2008) demonstrate the existence of unusually high reinsurance spreads vis-à-vis expected loss using a unique reinsurance issuance dataset and attribute them, in a clinical examination setting, to capital market imperfections, reinsurers' market power, and high marginal cost of capital. Zanjani (2002) shows in a multi-line pricing and capital allocation framework that the marginal capital requirement for catastrophe reinsurance is about five times the premium with a price impact of about 30%. Cummins, Lalonde and Phillips (2004) note empirically that cat bonds' basis risk is not significant for large insurers with a diversified book, and the bond spread is about 6.5 times the expected loss in earlier years. Lane and Mahul (2008) employ a quarterly secondary-market cat bond dataset to analyze cat bond spreads using the actuary approach and ascribe the cat bond spread of about 2-4 times the expected loss to the wider capital market cycles, the risk profile of the transaction, and the peril contained. Dieckmann (2010) finds that cat bond spreads are equal to 3-4 times the expected losses after controlling for bond-specific characteristics and attributes the spread to a habit process that a catastrophe shock could bring investors to their subsistence level, thus demanding higher yield spreads vis-à-vis equally-rated corporate bonds. Cummins and Weiss (2009) suggest that market imperfections, e.g. thinness in cat bond trading, fund managers' unfamiliarity with cat bonds, and parameter uncertainty might have also been contributors to the wide cat bond spreads observed.

In reinsurers' capital allocation decision, the most common allocation utilized is the Euler allocation rule (e.g. Denault (2001)), but Mango (2003) argues that capital must not be physically allocated a priori based on a risk measure. Instead, all contracts share the same capital stock from which capital can be called upon when the contract is running an operational deficit. Bodoff (2009) proposes a compromised approach in which one must allocate capital separately on each layer, but perform the capital allocation across all layers based on a set of simulated portfolio-wide loss scenarios. In reinsurers' optimum layering decision for a given book of business, Boyer and Nyce (2013) develop a two-factor cost model based on a new theory of optimum risk sharing to minimize the cost and total premium associated with cat events.

For insurers' optimum layering in their primary policy offers, Fu and Klury (2010) propose a layering methodology based on maximizing the risk-adjusted underwriting profit within a classical mean-variance framework. It improves upon prior literature by considering both catastrophic and non-catastrophic losses and by applying a lower partial moment risk measure.

3. The Allocation

3.1 Reinsurance vs. Cat Bonds

Traditional reinsurance is default-risky with only partial collateralization and opaque contract terms, especially for those reinsurers that are less than well-capitalized and highly-rated, but can be customized to cover virtually any peril(s), region(s), or exposure(s). On the other hand, cat bonds are default-free with full-collateralization and are traded on capital markets, offering transparency, liquidity, and diversification to investors. Cat bonds, however, must be homogeneously termed to be investor-friendly, i.e. offer easy analysis for investors in capital markets in relation to their own portfolios in modeling, documentations, etc., and thus are generally more difficult for underwriters to place. In addition, securitization can be very costly and time consuming in the traditional Rule 144A+ offering process. Recently however Tokio Solution Management Ltd. and Guy Carpenter Securities have teamed up to create and launch a private cat bond platform called the Tokio Tensai™ Platform, as a cost-effective alternative to the traditional offering process.[26] Finally, traditional reinsurance has a one-year policy term with the possibility of renewal, but cat bonds are typically multi-year, allowing purchasers to lock in a single annualized premium for several years. There are also differences in accounting, regulatory, and rating agency treatments.

3.2 Cat Bond Structure and Optimum Allocation Methodology

Our proposed allocation methodology is rooted in the structure of a cat bond as illustrated in Figure 2. Cat bonds are issued by a Special Purpose Vehicle (SPV) set up by a sponsor, normally a (re)insurer, such that it is not an affiliate of the sponsor, but rather an independent reinsurance company with the sole purpose of simultaneously issuing cat bonds to investors and selling reinsurance to the sponsor as a "pure play" in catastrophic risk. Reinsurers (supply side

[26] The traditional process entails the need to obtain a rating for bond issues; the solicitation of counsel on legal, accounting, and regulatory matters; and the time spent on back office support in risk modeling, documentation, and disclosure. The Tokio Tensai Platform aims to allow clients the same kind of flexibility, but with a significantly reduced cost that a normal cat bond issuance allows to customize their coverage using a range of triggers and reinsurance structures while still offering the speed to market. Therefore, the securitization cost has become significantly lower since.

of traditional reinsurance) issue cat bonds against their traditional reinsurance obligations in order to lay off exposures and enhance reinsurance values, insurers (demand side of traditional reinsurance) issue cat bonds to supplement their reinsurance purchases so as to lower hedging costs, while fund managers (demand side of cat bonds) purchase cat bonds for higher returns and diversification. Cat bonds are designed as an embedded bond with alternative triggers and event structures that do not necessarily synchronize with the sponsor's actual (re)insurance losses.

The SPV has three counterparties: the sponsor, the investor/fund manager, and the Interest Rate Swap (IRS hereafter) counterparty. Potential insurance payments by the SPV to the sponsor, in this case a reinsurance company in Figure 2, are pre-funded by the proceeds of the bond issue that are invested in a collateral account for a fixed return. Investors receive periodic interest payments reset to LIBOR plus a fixed spread as the "premium". To immunize the interest rate exposure, the SPV typically enters into an IRS arrangement with a counterparty to exchange the fixed return from the collateral account for floating payments of LIBOR minus a "spread" to match the payment schedule to the investor. Any shortfall, i.e. the premium plus the spread, is then captured by the re-reinsurance payment made to the SPV by the sponsor as a "pure play" in catastrophic risk, with the net payoff to all three counterparties being zero. In other words, by issuing cat bonds, (re)insurers actually enter into a long position in re-reinsurance with the SPV free from default and interest rate risks under a parity relationship in which the re-reinsurance premium paid is tied to the sum of the bond premium and the IRS spread.

Optimum allocation shall proceed in a sequential dual optimization process starting with the reinsurers' optimum allocation in Section 4 below. Given the cat bond structure, we see that the reinsurance company has cash flows from two counterparties – the insurer and the SPV – with a short default-risky reinsurance–long default-free re-reinsurance position. Thus, in order to solve for the reinsurer's optimum allocation, we first maximize, for each insurance layer, the NPV of the position's cash flows with respect to the cat bond structural parameters to the point where the marginal increase of the reinsurance value is exactly offset by the marginal increase of the cat bond issuing cost. These results enable the reinsurer to find its sell-side reinsurance pricing schedule across all layers and furthermore its sell-side global optimum allocation layer as the layer with the highest NPV.

By taking the above reinsurer's optimum pricing schedule as given and assuming that the aggregate loss process of the catastrophe and the total external hedging needs of the insurer are exogenous,[27] the insurer then determines its optimum allocation by minimizing the total hedging cost out of its purchases of reinsurance and issuance of cat bonds, with respect to the cat bond structural parameters across all reinsurance layers offered, i.e. to locate its buy-side global optimum allocation layer.

4. The Optimum-Allocation Model for Reinsurers

In this section we derive the reinsurer's optimal allocation results. The reinsurer allocates between sale of default-risky reinsurance and issuance of default-free cat bonds, with the aim to maximize the NPV of the allocation. We start with modeling the default-risky reinsurer by employing Merton's (1974, 1977) structural approach to model the balance sheet of the reinsurer in a continuous-time no-arbitrage martingale framework with the incorporation of the stylized dynamics of the fundamental variables. The Merton approach has the advantage of linking the valuation of financial claims to the firm's assets and capital structure, thus endogenizing default. Unlike the more recent applications of the approach to incorporate jumps and to integrate with the capital asset pricing model (CAPM) in order to analyze corporate bond spreads (e.g. Collin-Dufresne, Goldstein and Yang (2012) and Coval, Jurek and Stafford (2009)), our structural model of analyzing catastrophe spreads builds upon the works of Cummins (1988), Duan, Moreau, and Sealey (1995), Duan and Yu (2005), and Lee and Yu (2007) to properly allow for the asset, liability, interest rate, and cat loss dynamics.

We consider a reinsurer that sells a cat event-linked excess-of-loss (XOL) reinsurance policy over a loss layer to an insurer in the reinsurance market and at the same time sponsors and sets up a SPV to issue tailor-made cat bonds with varying size, tenor, trigger, and event structures in the capital markets as a "pure play". The reinsurer buys a re-reinsurance policy from the SPV with specific terms to lay off exposure and extend underwriting capacity. In this reinsurance mix the reinsurer has a short default-risky reinsurance position as a call option spread and a long default-free re-reinsurance position as another call option spread, resulting in a net option spread. Maximization in the option mix occurs at the point where the marginal benefit of any additional cat bond issuance in enhancing the reinsurance premium and capacity is exactly offset by the marginal cost of such additional cat bond issuance. Since the reinsurer's asset-liability structure and catastrophe loss specification are important factors in determining the contract values, this section begins with specifying the reinsurer's asset, liability, and catastrophe loss dynamics.

4.1 Asset, Interest Rate, Liability, and Catastrophe Loss Dynamics

[27] Here, we assume the aggregate loss process is exogenously provided by a catastrophe modeling firm (e.g. AIR Worldwide for hurricanes and windstorms and Risk Management Solutions (RMS) for earthquakes), and the total external hedging need is exogenously provided by the insurer's optimum layering decision.

As it is common for reinsurance companies to hold a large proportion of fixed-income assets in their portfolios, we model the asset dynamics taking into account explicitly the impact of stochastic interest:

$$\frac{dV_t}{V_t} = \mu_V dt + \phi_V dr_t + \sigma_V dW_{V,t}, \tag{1}$$

where V_t is the asset value of the reinsurance company at time t; r_t is the instantaneous interest rate at time t; $W_{V,t}$ is the Wiener process that denotes the credit risk on the assets of the reinsurance company; σ_V is the volatility of the credit risk; and ϕ_V is the instantaneous interest rate elasticity of the assets of the reinsurance company. The term credit risk refers to all risks orthogonal to the interest rate risk.

The instantaneous interest rate is assumed to follow the squared-root process of Cox, Ingersoll, and Ross (1985):

$$\mathrm{d}r_t = \kappa(m - r_t)dt + v\sqrt{r_t}\, dZ_t, \tag{2}$$

where r_t denotes the instantaneous interest rate at time t; κ is the mean-reverting force measurement; m is the long-run mean of the interest rate; v is the volatility parameter for the interest rate; and Z_t is a Wiener process independent of $W_{V,t}$.

Combining (1) and (2), the asset dynamics of a reinsurance company can be described as follows:

$$\frac{dV_t}{V_t} = (\mu_V + \phi_V \kappa m - \phi_V \kappa r_t)dt + \phi_V v\sqrt{r_t}\, dZ_t + \sigma_V dW_{V,t}. \tag{3}$$

Next, the dynamics for the interest process under the risk-neutralized pricing measure, denoted by Q, can be written as:

$$dr_t = \kappa^*(m^* - r_t)dt + v\sqrt{r_t}\, dZ_t^*, \tag{4}$$

where $\kappa^* = \kappa + \lambda_r$, $m^* = \dfrac{\kappa m}{\kappa + \lambda_r}$, $dZ_t^* = dZ_t + \dfrac{\lambda_r\sqrt{r_t}}{v}dt$, λ_r is the market price of interest rate risk and is constant under the assumption of Cox et al. (1985), and Z_t^* is a Wiener process under Q. Thus, the asset dynamics can be risk neutralized to:

$$\frac{dV_t}{V_t} = r_t dt + \phi_V v\sqrt{r_t}\, dZ_t^* + \sigma_V dW^*_{V,t}, \tag{5}$$

where $W_{V,t}^*$ is a Wiener process under Q and is independent of Z^*.

In addition to the liability of providing reinsurance coverage for catastrophes, the reinsurer also faces a liability that comes from providing reinsurance coverage for other lines. Since this liability represents the present value of future claims related to the non-catastrophic policies, its value, denoted as L_t, can be modeled as follows:

$$dL_t = (r_t + \mu_L)L_t dt + \phi_L L_t dr_t + \sigma_L L_t dW_{L,t}, \quad (6)$$

where ϕ_L is the instantaneous interest rate elasticity of the reinsurer's non-catastrophic liability. The continuous diffusion process reflects the effects of interest rate changes and other day-to-day small shocks. Term μ_L denotes the risk premium for the small shock. Since the day-to-day small shocks, denoted as $W_{L,t}$, pertain to idiosyncratic shocks to the capital market, we assume a zero risk premium for this risk. The interest rate shock can be risk-neutralized using the result in equation (4) so as to substitute out Z_t with Z^* as:

$$dL_t = r_t L_t dt + \phi_L v\sqrt{r_t}\, L_t\, dZ_t^* + \sigma_L L_t dW_{L,t}, \quad (7)$$

We next model the aggregate catastrophe loss as a compound Poisson process as in Bowers, Gerber, Hickman, Jones, and Nesbitt (1986). The accumulative catastrophe loss at time *t*, denoted as C_t, is described as follows:

$$C_t = \sum_{j=1}^{N(t)} c_j. \quad (8a)$$

Although the reinsurance trigger is usually one of indemnity, the cat bond trigger type can be either indemnity or an industry index. In order to estimate the impact of the basis risk on the cat bond valuation, we also specify the dynamics of a composite industry index of catastrophe losses, denoted by $C_{index,t}$, as follows:

$$C_{index,t} = \sum_{j=1}^{N(t)} c_{index,j}, \quad (8b)$$

where the process $\{N(t)\}t \geq 0$ is the loss number process assumed to be driven by a Poisson process with intensity λ. Term c_j ($c_{index,j}$) denotes the amount of loss caused by the *jth* catastrophe covered by the reinsurance contract during the specific period, where $j = 1, 2, ..., N(t)$, and is assumed to be mutually independent, identical, and lognormally-distributed, and independent of the loss number process, and the logarithmic means and standard deviations are denoted as μ_c and σ_c, respectively. The correlation coefficients of the logarithms of c_j and $c_{index,j}$ for all $j = 1, 2, ..., N(t)$ are assumed to be equal to ρ_c.

In industry practice, this loss process is endogenized by a catastrophe modeler through initial calibration as part of the IPO package of an initial cat bond offer or through subsequent re-calibration, e.g. AIR Worldwide for hurricanes and windstorms and RMS for earthquakes. In this paper we assume the loss process as exogenous. We also make the common assumption that catastrophe derivatives are zero-beta assets, and thus both the loss number process $\{N(t)\}$ and the amount of losses(C_t) have a zero risk premium.[28] The loss process, Equation (8), thus retains its original distributional characteristics after changing from the physical probability measure to the risk-neutralized pricing measure.

[28] Gürtler, Hibbeln and Winkelvos (2012) show that this premium can be significant when a mega-cat strikes.

4.2 Cat bond and Cat Reinsurance Valuation, and Optimum Allocation

We first value a cat bond with and without basis risk. Since the SPV simultaneously issues cat bonds to investors, enters into an IRS with an investment bank, and sells re-reinsurance to the sponsor for a pure play on catastrophic risk, free from default and interest rate exposures, a parity relation must hold in which the reinsurance premium paid to the SPV exactly offsets the cat bond premium and the IRS negative spread paid by the SPV. Consequently, we do not need to model the balance sheet of the SPV. In other words, at the equilibrium, the cost of the SPV reinsurance must be equal to the cost of issuing cat bonds, taking into consideration the IRS negative spread.

At maturity, the payoffs of the cat bond, denoted as $PO_{Cat,T}$, can be specified as follows:

$$PO_{Cat,T} = \begin{cases} F_{Cat} & \text{if } C_T < K, \\ F_{Cat} - \delta_T & \text{if } C_T \geq K, \end{cases} \tag{9}$$

where C_T can be either the accumulated amount of actual cat losses or the value of the loss index $C_{index,T}$ specified in the bond contract; F_{Cat} is the full face value of the cat bond that its holders will receive when C_T does not reach the predetermined trigger level K, where $A \leq K \leq M$; and δ_T represents the total amount that will be forgiven by cat bondholders when the trigger level has been pulled, in which case bondholders will receive only $F_{Cat} - \delta_T$. We assume that:

$$\delta_T = \begin{cases} C_T - K & \text{if } K \leq C_T < K + F_{Cat}, \\ F_{Cat} & \text{if } C_T \geq K + F_{Cat}, \end{cases} \tag{10}$$

where the payoff to the reinsurer is a linear function of cat losses when the losses exceed the trigger level (K), but is bounded by the full face value of the bond. In the special case where $\delta_T = F_{Cat}$, the payoff to the bondholder constitutes a standard binary call. Basis risk occurs if the underlying loss of the cat bond is set to be a composite industry loss index rather than the actual loss of the reinsurer, in which case the payoffs of the reinsurance contract remain the same except that the contingent savings from the cat bond become $\delta(C_{index,T})$. In this case, since the debt forgiven of the cat bond does not correlate perfectly with the actual loss, the realized losses and savings may not match and may therefore affect the insolvency of the reinsurer and the value of the reinsurance contract in a way that differs from that without basis risk.

Since the payoff of a regular default-free bond, $PO_{B,T}$, is:

$$PO_{B,T} = F_{Cat}, \tag{11}$$

the additional variable cost of issuing a cat bond, Δ_0, in addition to the fixed cost to set up a cat bond program, can be computed using the no-arbitrage martingale pricing approach[29] under the risk-neutralized pricing measure Q as follows:

$$\Delta_0 = E_0^Q[e^{-\int_0^T r_s\, ds} \times PO_{B,T}] \ - \ E_0^Q[e^{-\int_0^T r_s\, ds} \times PO_{Cat,T}] \ = \ E_0^Q[e^{-\int_0^T r_s\, ds}] \times \Delta_T, \tag{12}$$

where $\Delta_T = E_0^Q\ [\delta_T]$ and Δ_0 essentially represents the value of the catastrophe option embedded in a cat bond contract.

Since the issuance of cat bonds as a hedge reduces the default probability of the traditional reinsurance contract, the reinsurer's future payoff to the insurer can be specified as:

$$PO_{R,T} = \begin{cases} M - A & \text{if } C_T \geq M \text{ and } V_T + \delta_T \geq L_T + M - A, \\ C_T - A & \text{if } M > C_T \geqq K \text{ and } V_T + \delta_T \geq L_T + C_T - A, \\ \left(\frac{V_T + \delta_T}{L_T + M - A}\right)(M - A) & \text{if } C_T \geq M \text{ and } V_T + \delta_T < L_T + M - A, \\ \left(\frac{V_T + \delta_T}{L_T + C_T - A}\right)(C_T - A) & \text{if } M > C_T \geqq K \text{ and } V_T + \delta_T < L_T + C_T - A, \\ C_T - A & \text{if } K > C_T \geq A \text{ and } V_T \geq L_T + C_T - A, \\ \left(\frac{V_T}{L_T + C_T - A}\right)(C_T - A) & \text{if } K > C_T \geq A \text{ and } V_T < L_T + C_T - A, \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

where V_T is the value of the reinsurer's assets at *T*; L_T is the value of the reinsurer' liabilities at *T*; C_T is the catastrophe loss covered by the reinsurance contract; δ_T stands for the amount forgiven from cat bonds for each corresponding scenario; and *M* and *A* are respectively the cap and attachment points arranged in the reinsurance contract with the assumption that $M \geq K \geq A$. For example, when C_T is larger than the reinsurance cap *M* and the reinsurer's total assets inclusive of payment from the SPV (δ_T) are larger than total liability inclusive of the reinsurance obligation *M-A*, the payoff is *M-A*. When the reinsurer's total assets inclusive of payment from the SPV (δ_T) are smaller than total liability inclusive of the reinsurance obligation *M-A*, the reinsurer will default, and the payoff to the insurer now is only $\left(\frac{V_T + \delta_T}{L_T + M - A}\right)(M - A)$.

The present value of the future payoff determines the value of the reinsurance contract and can be computed as a martingale under the risk-neutralized pricing measure Q as:

$$PV_{R,0} = E_0^Q\left[e^{-\int_0^T r_s\, ds} \times PO_{R,T}\right], \tag{13a}$$

[29] The alternative is to apply the actuarial pricing approach, which is practical but however inconsistent with the finance theory.

where E_0^Q denotes the expectation taken on the issuing date under the risk-neutralized pricing measure Q. In Appendix 1, we demonstrate how to compute $PV_{R,0}$ via simulation. It is expected, as Δ_0 increases with a larger cat bond issue, $PV_{R,0}$ increases to reflect a more valuable reinsurance contract, as the issuance of cat bonds as a hedge can help reduce default probability and expand underwriting capacity.

We next describe the net payoff of the allocation inclusive of the SPV proceed - which is a combination of a short position of a default-risky traditional reinsurance as a call option spread and a long position of a default-free re-reinsurance from the SPV, denoted by $PO_{N,T}$ - as the payoff of a net call spread:

$$PO_{N,T} = \begin{cases} M - A - \mathrm{A}\delta_T & \text{if } C_T \geq M \text{ and } V_T + \delta_T \geq L_T + M - A, \\ C_T - A\,\delta_T & \text{if } M > C_T \geqq K \text{ and } V_T + \delta_T \geq L_T + C_T - A, \\ \left(\frac{V_T + \delta_T}{L_T + M - A}\right)(M - A) - \delta_T & \text{if } C_T \geq M \text{ and } V_T + \delta_T < L_T + M - A, \\ (\frac{V_T + \delta_T}{L_T + C_T - A})(C_T - A) - \delta_T & \text{if } M > C_T \geqq K \text{ and } V_T + \delta_T < L_T + C_T - A, \\ C_T - A & \text{if } K > C_T \geq A \text{ and } V_T \geq L_T + C_T - A, \\ (\frac{V_T}{L_T + C_T - A})(C_T - A) & \text{if } K > C_T \geq A \text{ and } V_T < L_T + C_T - A, \\ 0 & \text{otherwise.} \end{cases} \quad (13b)$$

$PV_{N,0}$, the PV of $PO_{N,T}$, can be computed as a martingale under the risk-neutralized pricing measure Q as:

$$PV_{N,0} = E_0^Q\left[e^{-\int_0^T r_s\,ds} \times PO_{N,T}\right], \quad (13c)$$

where it can be quickly checked that $PV_{N,0} = PV_{R,0} - \Delta_0$.

To set up the allocation's NPV, we let the sell-side reinsurance pricing be $(1+u)PV_{R,0}$, where *u* denotes the reinsurance pricing markup, higher in a hard market but lower in a soft market. According to Froot (2001) and Froot and O'Connell (2008), capital market imperfections, reinsurers' market power, and a high marginal cost of capital had resulted in unusually high markups with spreads at around 5-8 times the expected loss. Don Mango at Guy Carpenter notes that the traditional reinsurers' targets are 15%+ ROE, and Zanjani (2002) states that the marginal capital requirement for catastrophe reinsurance is about five times the premium with a price impact of about 30%. We let the cat bond pricing be $(1+d)\Delta_0$, where *d* denotes the cat bond markup. According to Cummins, Lalonde and Phillips (2004), cat bonds had been notorious for having high spreads in earlier years at around 6.5 time of the expected loss.

As discussed before, however, an influx of capital market participation in the new normal has resulted in the convergence of reinsurance and cat bond pricing with significantly lower markups now.

According to a recent report by GC Securities,[30] the growth in convergence capital since the Fed initiated its ultra-low interest rate policy in 2009 has resulted in some ILS - e.g. cat bonds, collateralized reinsurance[31], side cars, and industry loss warrants - offering the most competitive terms to reinsurance buyers. A strong appetite for U.S. hurricane cat bonds, for example, has tightened spreads in the secondary market by an average of 45% on a weighted notional basis since 2012 (down to 5.22% from 9.61% in 2012 according to the WSJ report discussed in footnote 13). Despite the significant recent decrease in ILS pricing, investor demand continues to be robust. Indeed, projections by Guy Carpenter indicate that the cat bond market alone could reach $23 billion by the end of 2016. This force of capital market participation is completely disrupting the reinsurance market right now. Reinsurance pricing is being driven down towards the fixed income required returns of pension funds, ~ 400bp spreads, which are nothing like the traditional 15%+ ROE targets of reinsurers. There is some evidence during the 2013 renewal season that competition was spilling over into other business segments as traditional reinsurers' capital was redeployed, convergence participants started to explore other modeled risk classes, and a new wave of hedge fund-sponsored reinsurers gained traction. Absent significant catastrophe losses in 2013, market expectations are for these trends to continue throughout the year and into the January 1, 2014 renewal. [32]

As we have formulated, the NPV of the reinsurer's investment in the allocation is $(1+u)PV_{R,0}$ $-PV_{N,0}-[f+\ (1+d)\Delta_0]$, where $(1+u)PV_{R,0}$ denotes the reinsurance price, $PV_{N,0}$ denotes the PV of net future payoffs to the insurer, and $(1+d)\Delta_0$ the variable cost of issuing a cat bond. This formulation can be quickly simplified to $uPV_{R,0}-d\Delta_0$. As the reinsurer issues more cat bonds, the NPV will increase to reflect a more valuable reinsurance contract, but decrease to reflect a more costly bond issue. To reach local optimality, we maximize the NPV of the allocation for a given reinsurance coverage layer with respect to the cat bond's structure parameters *F* and *K* as:

$$Max_{\ F,K}\ \left(uPV_{R,0}-\ d\Delta_0\right), \qquad (13d)$$

where optimization is achieved at the point where the PV of the marginal increase in the reinsurance profits is exactly offset by the PV of the marginal increase in the cat bond issuing cost. We then identify the global optimum allocation layer as the one that has the highest NPV across all coverage

[30] http://www.gccapitalideas.com/2013/10/02/convergence-capitals-impact-on-the-reinsurance-market/

[31] Nephila Capital and other dedicated funds have teamed up with pension funds to issue collateralized reinsurance. These hybrids are backed by pension fund allocations as specialized fixed income investments, with returns only ~300bp above the risk-free rate, provide zero counterparty risk (collateralized account in T-Bills), and sell at near cat-bond price levels, while getting reinsurance accounting treatment rather than investment. Investors consider it a fixed income investment, similar to a cat bond, but without any security issuance costs or disclosure requirements.

[32] For recent articles on these trends check http://www.artemis.bm/.

layer results. The $(1+u)PV_{R,0}$ value achieved at the maximum in equation (13d) represents the reinsurance premium the reinsurer charges the insurer, i.e. the sell-side reference price.

5. The Optimum Allocation Model for Insurers

The insurer allocates between purchase of reinsurance and issuance of cat bonds, with the aim to minimize the total hedging cost. The payoffs of a cat bond at maturity, denoted as $PO_{I,t}$, can be specified as follows:

$$PO_{I,T} = \begin{cases} F_I & \text{if } C_T < K_I\,, \\ F_I - \delta_{IT} & \text{if } C_T \geq K_I\,, \end{cases} \tag{14a}$$

where C_T can be either the accumulated amount of actual cat losses or the index loss specified in the bond contract; F_I is the full face value of the cat bond that its holders will receive when C_T does not reach the predetermined trigger level K_I, where $A \leq K_I \leq M$; and δ_T represents the total amount that will be forgiven by cat bondholders when the trigger level has been pulled, in which case bondholders will receive only $F_I - \delta_{IT}$. We assume that:

$$\delta_{IT} = \begin{cases} C_T - K_I & \text{if } K_I \leq C_T < K_I + F_I, \\ F_I & \text{if } C_T \geq K_I + F_I, \end{cases} \tag{14b}$$

where the payoff to the reinsurer is a linear function of cat losses when the losses exceed the trigger level K_I, where $A \leq K_I \leq M$.

The total cost of the allocation is composed of the cost of buying the (*M*,*A*) coverage from the reinsurer with pricing $(1+u)PV_{R,0}$, and the cost of issuing a (F_I, K_I) cat bond in the capital markets with pricing $(1+d)\Delta_{I0}$, i.e. $(1+u)PV_{R,0} + (1+d)\Delta_{I0}$. We employ a simplified external total hedging capital constraint $M - A + F_I = R$ here for computational convenience, which can be easily extended. Optimization is achieved across all (M, A) layers of coverage offered by the reinsurer at the point where the total hedging cost is minimized:

$$Min_{\ M,A} \left((1+u)PV_{R,0} + (1+d)\Delta_{I0}\right), \tag{14c}$$

s.t. $M - A + F_I = R$.

To implement the minimization, the reinsurance pricing schedule, i.e.$(1+u)PV_{R,0}$, must first be provided by the reinsurer for all layers of coverage, whereas the insurer has the leverage to choose the particular layer of coverage that minimizes its total hedging cost. The resulting chosen layer of coverage, that has the smallest total hedging cost, represents the insurer's buy-side global optimum, as opposed to the reinsurer's sell-side global optimum. Capital market participation in the collateralized cat bond market in recent years has offered the insurer leverage to dictate reinsurance coverage by replacing the purchase of traditional reinsurance with the issuance of cheaper cat bonds to minimize total hedging cost. This competitive pressure causes reinsurance re-pricing to lower the reinsurance markup in order to sell more reinsurance, as demonstrated in Section 6.1.

6. Simulation and Sensitivity Analysis

We employ Monte Carlo simulation to demonstrate how the optimization models simulate the post-crisis convergence process and how the input parameters effect optimum allocation.

6.1 Simulation Results

We simulate the post-crisis convergence process in three stages: the initial stage when the cat bond market softens due to surging demand from fund managers with a small markup of 5%, but the reinsurance market is still hard with a markup of 40%; the intermediate stage when the reinsurance market softens due to competitive pressure from the cat bond market with a lower markup of 10%; and the final stage when the two markets have fully converged with zero markups. We consider a base case in which all of the parameter values in the asset, liability, interest rate, and catastrophe loss dynamics are consistent with previous literature.[33] These parameter values are summarized in Table 1. The simulations are run on a monthly basis with 20,000 paths.

Table 2.1 reports the reinsurer's optimum allocation result in the initial stage for a specific loss coverage layer (M, A) = (70, 10) as Optimum (NPV_{max}, F_R, K_R) = (2.10095, 33, 37). In other words, the optimum allocation for a reinsurance coverage layer (70, 10) is to issue a cat bond with face value 33 and trigger 37 over layer (70, 37), which maximizes the allocation's NPV to 2.10095, under which the marginal benefit of any additional cat bond issuance in enhancing the reinsurance premium and capacity is exactly offset by the marginal cost of such an issuance. This result suggests that the reinsurer should slice off the (70, 37) layer's exposure from its reinsurance offering of layer (70, 10).

Next, in Table 2.2 and Figure 4 we produce optimization results for all coverage layers with M ranging from 60 to 90 and A ranging from 10 to 30. We can then identify the reinsurer's sell-side global optimum allocation that has the largest NPV across all layers to be (90, 10), with a maximum NPV of 2.10374 and an optimum cat bond issuance of (52, 37). In other words, the best deal for the reinsurer is to sell the layer with the highest cap and the lowest attachment point, i.e. layer (90, 10), while hedging off a subset of this layer, layer (89, 37), by issuing a cat bond with face value 52 and trigger 37. The corresponding reinsurance premiums across all layers of coverage, denoted as $(1+u)PV_{R,0}$, are summarized in Table 2.3 as input to the insurer's optimization scheme.

In Table 3 we identify the insurer's global optimum allocations in each of the three simulation stages, i.e. the buy-side optimum. For simplicity, we choose $K_I = A$ and set the total required external

[33] See Cummins (1988), Lee and Yu (2002) and Duan and Simonato (1999).

hedging capital at R=80. In each stage, we first determine, for each of the coverage layer, the optimum allocation that minimizes the total hedging cost ($(1+u)PV_{R,0}$ + $(1+d)\Delta_0$), and then identify across all layering results the global optimum allocation as the one with the smallest total hedging cost. The results in Panel I show that in the initial stage when the reinsurance market is hard but the cat bond market is soft with markup combination (0.4, 0.05), the optimum allocation for the insurer (THC_{min}, M, A, F_I, K_I) is (1.73904, 60, 30, 50, 30), where THC_{min} denotes the minimum total hedging cost - that is, the insurer's buy-side optimum allocation is to purchase reinsurance coverage layer (M,A) = (60, 30) and issue a cat bond (50, 30) with a (reinsurance, cat bond) allocation of (30, 50), i.e. 37% in reinsurance and 63% in cat bonds out of a total risk limit of 80. For this allocation the reinsurance's NPV per Table 2.2 is only 0.27942. In contrast, the reinsurer's sell-side optimum allocation as reported in Table 2.2 is (NPV_{max}, M, A, F_R, K_R) = (2.10374, 90, 10, 52, 37), with an NPV of 2.10374 and an allocation of (80, 52) to provide the insurer with all of its hedging need of 80.

The above finding indicates that when the cat bond market is soft while the reinsurance market is still hard, the insurer prefers to purchase much less reinsurance coverage with cat bond substitution, in which case the reinsurer's NPV will dwindle from 2.10373 to 0.27942. This competitive pressure from the cat bond market will cause the reinsurer to re-price its offer, causing its markup to go down.

Panel II in Table 3 demonstrates that in the intermediate stage when the reinsurance markup decreased to 0.1 from 0.4, coverage (60,30) is still the insurer's best choice with a total hedging cost dropping down to 1.52935 from 1.73904 in the initial stage, but its edge over other choices is now significantly smaller. As the reinsurance market further softens, THC_{min} continuously moves down and eventually when u=d=0 in the final stable stage, we observe, in Panel III of Table 3, that the insurer's optimum allocation reverts back to the purchase of a larger reinsurance limit with (THC_{min}, M, A, F_I, K_I) = (1.42324, 80, 30, 30, 30). In other words, as the two markets converge, the insurer's optimum allocation now is to purchase significantly more reinsurance coverage than in previous stages, with an optimum coverage layer of (80,30) vis-à-vis (60,30) in previous stages, or 63% in reinsurance and 37% in cat bonds vis-à-vis 37% in reinsurance and 63% in cat bonds in previous stages. Furthermore, the reinsurer's NPV per Table 2.2 is now 0.28788, a small improvement over the reinsurer's optimum NPV of 0.27942 in the initial stage, but still far below its global optimum NPV of 2.10374.

To wit, the insurer's leverage in the new normal to dictate reinsurance coverage by issuing cat bonds helps drive down reinsurance pricing significantly toward cat bond pricing, underpinning the current rapid growth of the cat bond market, and leads to "convergence" of the two markets toward fair-pricing equilibrium. The influence of products and pricing available from convergence market participants and the competition from hedge fund-sponsored reinsurers pose important questions for the traditional reinsurance market. Some reinsurers have already responded to the challenge by developing strategies to exploit the demand for the asset class from investors by hiring capital market executives and establishing operating divisions of their own to attract and manage capital from these investors. The recent wave of reinsurer-sponsored ILS asset management products and platforms differs from previous activities and is in direct response to the threat from the convergence market, while previous moves into this market were demand-driven in response to capacity needs and market hardening, following major catastrophe events. This redeployment of capacities allows reinsurers the opportunity to securitize the most capital-intensive parts of the business while providing valuable cost-efficient capacity to their clients by leveraging their access to the business and depth of underwriting, risk management, and claims management expertise.

6.2 Sensitivity Analysis Results

In this section we conduct sensitive analysis to study the parametric impacts on the allocations, including the interest rate risk, the leverage, the markup, the basis risk and the catastrophe risk effects.

6.1 The Interest Rate Risk, Leverage, and Markup Effects

We extend the base value scenario as reported in Table 1 to consider three interest rate elasticity scenarios: (ϕ_V, ϕ_L) = (0, 0), (-3×(*L/V*), -3), and (-5, -3), three leverage scenarios: *V/L* = (1,1, 1.3, 1.5), with decreasing leverage, and three markup scenarios: (*u*,*d*) = (0.4,0.05), (0.1, 0.05) and (0,0). Under the case of (ϕ_V, ϕ_L) = (0, 0), both assets and liabilities are interest rate insensitive, whereas the scenario of (-3×(*L/V*), -3) has a zero size-adjusted interest rate elasticity gap and can constitute an interest risk management scenario that strives to eliminate the interest risk confronting the reinsurer by adjusting the asset-liability mix. The case of (-5, -3) represents the interest rate risk scenarios with a positive duration gap on the asset-liability structure.

Table 4 reports the interest rate risk, leverage, and markup effects on the minimum total hedging cost for insurers (THC_{min}) and the maximum NPV for reinsurers (NPV_{max}) across combinations of (*M*, *A*). We first observe that the optimum allocations for the zero gap and insensitivity scenarios are very similar, indicating that active interest risk management can achieve the same effect as sticking exclusively to interest rate insensitive assets and liabilities. Next, the presence of interest rate risk slightly decreases the reinsurer's NPV and increases the allocation to cat bonds, and the larger the leverage is, the stronger the effects. This is because the reinsurance contract becomes less valuable with larger default risk, in which case the cat bond's hedging effect becomes more desirable. For insurers, the presence of interest rate risk decreases the total hedging cost, as now the reinsurance becomes cheaper, and the larger the leverage is, the stronger the effect. We also observe that larger leverage correlates with more allocation to cheaper reinsurance, while the cost of issuing cat bonds is insensitive to interest rate risk and reinsurer default.

For the markup effect, we observe that when the markups are large at (0.4, 0.05), the larger the leverage is, the more the reinsurers will issue cat bonds to slice off more exposure in order to increase the allocation's NPV, while the insurers will issue less cat bonds to lower total hedging cost as reinsurance becomes cheaper. When the markups are smaller or zero, however, the reinsurers will not issue cat bonds because of the cost incurred.

Finally, in all cases, the reinsurers prefer to sell more coverage while the insurers prefer less with substitution from issuing cat bonds. This again shows that the competitive capital market force drives down reinsurance pricing.

6.2.2 The Basis Risk Effect

When the payoffs to cat bonds are linked to a cat loss index rather than the reinsurer's actual cat loss, basis risk is present. Table 5 reports the basis risk effect by extending the base value scenario in Table 1 to consider three basis risk level: ρ_c = (0.8, 0.5, 0.3), three leverage scenarios: *V/L* = (1,1, 1.3, 1.5), with decreasing leverage, and three markup scenarios: (*u*,*d*) = (0.4,0.05), (0.1, 0.05) and (0,0). A larger basis risk in a cat bond issue, resulted from a smaller correlation coefficient between the actual loss and the index loss, induces a lower hedging effectiveness but a higher hedging cost. Table 5 reports that, in this scenario, insurers should rely less on cat bonds but more on reinsurance for hedging in order to reduce total hedging cost. For the reinsurers, the NPV is lower with a larger basis risk, while the impact on allocation is small with the cat bond trigger level moving down but issue size moving up in order to enhance hedging effectiveness.

6.2.3 The Catastrophe Risk Effect

Table 6 presents the cat risk effect by extending the base value scenario in Table 1 to consider three levels of catastrophe arrival intensity: λ = (0.5, 1, 2), and three levels of loss volatility: σ_c = (0.5, 1, 2). It shows that increases in both catastrophe arrival intensity and loss volatility raise the reinsurer's NPV as reinsurance becomes more valuable, which leads to higher total hedging cost for the insurer. As catastrophe arrival intensifies and the loss becomes more unpredictable, reinsurers should increasingly slice off lower layer exposures as the expected losses on these layers accentuate, by issuing cat bonds with lower trigger levels. For insurers, hedging becomes more expensive with both reinsurance and cat bonds, but optimum allocation does not change.

We further observe that the loss volatility effect is more acute than the loss arrival intensity effect. For example, when V/L is 1.1 and the markup is (0.4,0.05), doubling the arrival intensity from 0.5 to 1 increases the reinsurer's NPV from 1.9457 to 3.2283, but doubling the loss volatility from 0.5 to 1 increases the reinsurer's NPV from 1.9457 to 5.1078. The same is true for the insurer's total hedging cost but to a lesser degree. These findings suggest that, consistent with the findings of Chang, Chang and Wen (2014) in hedging hurricane risk using hurricane futures contracts, (re)insurers should be more concerned with the severity of a catastrophe than with the frequency of the arrival.

7. Concluding Remarks

Unprecedented increases in extreme loss events over the past decades have raised important concerns about inadequate catastrophe risk financing. The Federal Reserve's ultra-low interest rate policy following the 2007-2009 subprime crises, however, has transformed the catastrophe space into a new normal with an influx of third-party capital, inducing capacity expansion, risk capital redistribution, and premium reduction; and catalyzed a convergence of the traditional reinsurance and the securitized catastrophe bond markets. This research addresses a critical challenge resulting from the transition – how to allocate these inherently different products optimally to promote *ex ante* full risk intermediation and sharing. We present a novel theory of optimal catastrophe risk financing at the interface of the two markets in a dynamic setting in which insurers are price takers of the pricing schedule provided by NPV maximizing reinsurers, but have the leverage to dictate risk layers by substituting reinsurance with cat bonds. The resulting models (i) provide optimization methodologies and computational methods for optimum catastrophe risk financing; (ii) illustrate the stylized post-crisis convergence process; and (iii) demonstrate, through sensitivity analysis, the impacts of interest rate risk, leverage, markups, basis risk, and the frequency and severity of catastrophe events on the optimal allocation.

註解 **[DC1]:** is there a reason why yo

註解 **[DC2]:** u added 'the'? maybe I don't understand what's better protocol as outsider but to me without 'the' is better

註解 **[DC3]:** why the apostrophe? Maybe yo can tell me

We find that as the cat bond yields move down due to increasing capital market participation, insurers would prefer to purchase less reinsurance coverage with cat bond substitution, while reinsurers would prefer the insurer to purchase more reinsurance coverage without cat bond substitution. Competitive pressure from the cat bond market would lead to a resurgence of cat bonds in the catastrophe space, exerting significant downward pressure on reinsurance pricing. Moreover, as the reinsurance market softens, the insurer then becomes more agreeable to purchase more reinsurance coverage to reach convergence.

We also discover through sensitivity analysis that first, as interest rate risk increases, reinsurers will issue more cat bonds to hedge higher default risk, and the higher the reinsurer's leverage is, the more the allocation. Second, when basis risk is present, insurers will rely less on the use of cat bonds due to lower hedging effectiveness because the cat bond's payoffs are linked to a cat loss index rather than the reinsurer's actual cat loss, while reinsurers will increase the size of cat bond issuance, but at a lower trigger level to enhance hedging effectiveness. Finally, increases in both catastrophe arrival intensity and loss volatility raise the reinsurer's NPV as reinsurance becomes more valuable, leading to a higher total hedging cost for the insurer. As catastrophe arrival becomes more frequent and the loss becomes more unpredictable, reinsurers should increasingly slice off lower layer exposures as the expected losses on these layers accentuate, by issuing more cat bonds with lower trigger levels. For insurers, hedging becomes more expensive for both reinsurance and cat bonds, but optimum allocation does not change. We further observe that the loss volatility effect is more acute than the loss arrival intensity effect, suggesting that (re)insurers should be more concerned with the severity of a catastrophe than with the frequency of the arrival.

We are cautiously optimistic that the convergence will develop further even after policy reversal from the Federal Reserve. Our optimism stems from the maturity of market participants who have familiarized themselves with ILS during the post-crisis convergence process, and that cat bond funds have become an increasingly popular alternative investment class. Continued growth of third-party capital participation in the catastrophe space is conceivable, considering the sheer size of global managed assets of more than $100 trillion, compared with a total property catastrophe reinsurance market of $200 billion in premiums. Several major investors could take a large position in the market to show their long-term commitment with balance sheets that have the depth to absorb market volatility. General demand for new issuances has mainly been driven by dedicated cat funds, which have seen continuous cash inflows from end investors as a result of the low yield environment in corporate debt markets, allowing sponsors to reduce risk spreads and increase issuances. Pension funds have also allocated substantial capital to this sector over the last several years, providing collateral capital and enabling the development of a new source of catastrophe risk financing - collateralized reinsurance. Mutual funds in general provide a future source for increased direct participation in the ILS sector through the development of reinsurance-based mutual funds.

However, it is worth noting that although increased capacity from third-party capital has benefited insurers, it does challenge traditional reinsurers to innovate in order to remain competitive in this traditionally lucrative line of business. Nevertheless, due to the relational nature of its business, the reinsurer will remain a pivotal player in the cat market.

Our optimum allocation results can be expanded to incorporate collateral type, more trigger types, tranching, payment flow, reinstatements, collateral release and commutation, and premium adjustment in the cat reinsurance and cat bond structures, and one can study not only optimum allocation but also optimum contract design. The optimum allocation methodology can also conceivably be applied to other capital market products in the catastrophe space such as collateralized reinsurance, side cars, and ILWs, with an optimum allocation across a wider spectrum of products.

Mother Nature can be unpredictable, but no one should be unprepared. Therefore, financing of catastrophic risks is increasingly becoming a public policy issue, as proper governmental intervention can potentially reduce risk-bearing costs and increase societal welfare. In this wider sense, the complex policy issues involved in optimum full risk sharing between providing *ex ante* opportunities to insure against extreme events and *ex post* government aid relief constitute another timely optimum-allocation research topic.

Appendix 1: Procedures of the Simulation Method

In this section we numerically assess the reinsurance value $PV_{R,0}$ using Equation (13a). Because the premium depends on the values of the reinsurer's assets and liabilities, we have to specify the stochastic processes for these variables (Equations (5) and (7)). Applying Ito's lemma to the logarithm of the value of a reinsurer's assets, Equation (5) becomes the following system:

$$d\ln(V_t) = (r_t - \frac{1}{2}\varphi_V \upsilon^2 r_t - \frac{1}{2}\sigma_V{}^2)dt + \varphi_V \upsilon\sqrt{r_t}dZ_t{}^* + \sigma_V dW_{V,t}{}^* \quad (14)$$

We can solve the above equation. Its solution, for any $0 \le q < 1$, is:

$$V_{t+q} = V_t \exp\left(\sigma_V\left(W_{V,t+q}{}^* - W_{V,t}{}^*\right)\right) * \exp[(1 - \frac{1}{2}\varphi_V{}^2\upsilon^2)\int_t^{t+q} r_s ds + \varphi_V \upsilon \int_t^{t+q} \sqrt{r_s}\, dZ_s{}^*] \quad (15)$$

Similarly, by Ito's lemma, equation (7) gives rise to:

$$d\ln(L_t) = (r_t - \frac{1}{2}\varphi_L \upsilon^2 r_t - \frac{1}{2}\sigma_L{}^2)dt + \varphi_L \upsilon\sqrt{r_t}dZ_t{}^* + \sigma_L dW_{L,t}{}^* \quad (16)$$

Its solution is:

$$L_{t+q} = L_t \exp\left(\sigma_L\left(W_{L,t+q}{}^* - W_{L,t}{}^*\right)\right) * \exp[(1 - \frac{1}{2}\varphi_L{}^2\upsilon^2)\int_t^{t+q} r_s ds + \varphi_L \upsilon \int_t^{t+q} \sqrt{r_s}\, dZ_s{}^*] \quad (17)$$

We set $q = \frac{1}{52}$, say, on a weekly basis, and simulate the risk-neutralized interest rate process as Equation (4) in order to approximate the whole sample path for the term of reinsurance coverage. This in turn allows us to compute two quantities of interest: $\int_t^{t+1} r_s ds$ and $\int_t^{t+1} \sqrt{r_s}\, dZ_s{}^*$. Second, we simulate $\left(W_{V,t+q}{}^* - W_{V,t}{}^*\right)$ and $\left(W_{L,t+q}{}^* - W_{L,t}{}^*\right)$ using the fact that they are independent of the path of r_t, and the coefficient of correlation between them is $\rho_{V\,L}$. Combining $(W_{V,t+q}{}^* - W_{V,t}{}^*)$ with $\int_t^{t+1} r_s ds$ and $\int_t^{t+1} \sqrt{r_s}\, dZ_s{}^*$ yields a simulated value of V_{t+1} as described in Equation (15). Similarly, combining $\left(W_{L,t+q}{}^* - W_{L,t}{}^*\right)$ with $\int_t^{t+1} r_s ds$ and $\int_t^{t+1} \sqrt{r_s}\, dZ_s{}^*$ yields a simulated value of L_{t+1} as described in Equation (17). Third, we generate $(N_{t+1} - N_t), (c_{t+1} - c_t)$ and $(c_{index,t+1} - c_{index,t})$. Since $(N_{t+1} - N_t)$ has a Poisson distribution with intensity parameter λ, it can be simulated easily. For a given value of $(N_{t+1} - N_t)$, we then simulate $\sum_{j=N_t}^{N_{t+1}} \ln c_j$ and $\sum_{j=N_t}^{N_{t+1}} \ln c_{index,j}$, knowing that $\ln c_j$ and $\ln c_{index,j}$ are normal random variables and the coefficient of correlation between them is ρ_c. After simulating these processes, $PV_{R,0}$ can be easily calculated via averaging over the contingent payoffs corresponding to the simulated values.

References

Aerts, J., W. J. W. Botzen, K. Emanuel, N. Lin, de Moel, H., Michel-Kerjan, E., 2014. Evaluating flood resilience strategies for coastal mega-cities. *Science* 344, 473-475.

Aon Benfiled, 2013. Annual Report, London.

Artemis, http://www.artemis.bm/.

Barrieu, P., Loubergé, H., 2009. Hybrid cat bonds. *Journal of Risk and Insurance* 76, 547-578.

Bodoff, N.M., 2009. Capital allocation by percentile layer. *Variance* 3, 13-24.

Bowers, N.L., Gerber, J.H.U., Hickman, J.C., Jones, D.A., Nesbitt, C.J., 1986. *Actuarial Mathematics*, Itasca, IL: Society of Actuaries.

Boyer, N.M., Nyce, C.M., 2013. An industrial organization theory of risk sharing, *North American Actuarial Journal* 17, 283-296.

Chang, C., Chang, J,.S.K., Wen, M.M., 2014. Optimum hurricane futures hedge in a warming environment: a risk-return jump-diffusion approach. *Journal of Risk and Insurance* 87, 199-217.

Chang, J.S.K., Cheung, C.S., Krinsky, I., 1989. On the derivation of reinsurance premiums. *Insurance: Mathematic and Economics* 8, 137-144.

Collin-Dufresne, P., Goldstein, R.S., Yang, F., 2012. On the relative pricing of long-maturity index options and collateralized debt obligations. *The Journal of Finance* 68, 1983-2014.

Coval, J.D., Jurek, J.W., Stafford, E., 2009. Economic catastrophe bonds. *American Economic Review* 99, 628-666.

Croson, D.C., Kunreuther, H.C., 2000. Customizing indemnity contracts and indexed cat bonds for natural hazard risks. *Journal of Risk Finance* 1, 24-41.

Cox, J., Ingersoll, J., Ross, S., 1985. The term structure of interest rates. *Econometrica* 53, 385-407.

Cox, S.H., Schwebach, R.G., 1992. Insurance and hedging insurance price risk. *Journal of Risk and Insurance* 59, 628-644.

Cummins, J.D., Doherty, N., Lo, A., 2002. Can insurers pay for the "big one"? Measuring the capacity of insurance market to respond to catastrophic losses. *Journal of Banking and Finance* 26, 557-583.

Cummins, J.D., Lalonde, D., Phillips, R.D., 2004. The basis risk of catastrophe-loss index securities. *Journal of Financial Economics* 71, 77-111.

Cummins, J.D., Trainar, P., 2009. Securitization, insurance, and reinsurance. *Journal of Risk and Insurance* 76, 463-492.

Cummins, J.D., Weiss, M., 2009. Convergence of insurance and financial markets: hybrid and securitized risk-transfer solutions. *Journal of Risk and Insurance* 76, 493–545.

Denault, M., 2001. Coherent allocation of risk capital. *Journal of Risk* 4, 1-34.

Deutsche Bank, 2013. Annual Report, Frankfurt.

Dieckmann, S., 2010. By force of nature: explaining the yield spread on catastrophe bonds. Working Paper, Wharton Finance Department, University of Pennsylvania.

Demiroglu, C., 2012. How important is having skin in the game? Originator-sponsor affiliation and losses on mortgage-backed securities. *Review of Financial Studies*, first published online September 10, 2012.

Dobbs, R., Lund, S., Koller, T., Shwayder, A., 2013. QE and ultra-low interest rates: distributional effects and risks. Discussion Paper, McKinsey Global Institute.

Duan, J.C., Moreau, A., Sealey, C.W., 1995. Deposit insurance and bank interest rate risk: pricing and regulatory implications. *Journal of Banking and Finance* 19, 1091-1108.

Duan, J.C., Yu, M.T., 2005. Fair insurance guaranty premia in the presence of risk-based capital regulations, stochastic interest rates and catastrophe risk. *Journal of Banking and Finance* 29, 2435-2454.

Elsner, J.B., Kossin J.P., and T.H. Jagger, 2008. The increasing intensity of the strongest tropical cyclones. *Nature* 455: 92-94.

Emanuel, K.A., 2005. Increasing destructiveness of tropical cyclones over the past 30 years. *Nature* 436, 686-688.

Finken, S., Laux, C., 2009. Cat bonds and reinsurance: the competitive effect of information-insensitive triggers. *Journal of Risk and Insurance* 76, 579-605.

Froot, K.A., 2001. The market for catastrophe risk: a clinical examination. *Journal of Financial Economics* 60, 529-571.

Froot, K.A., 2008. The intermediation of financial risks: evolution in the catastrophe reinsurance market. *Risk Management and Insurance* 11, 281-294.

Froot, K.A., O'Connell, P.G.J., 2008. On the pricing of intermediated risks: theory and application to catastrophe reinsurance. *Journal of Banking and Finance* 32, 69-85.

Fu, L.Y., Khury, C.K., 2010. Optimal layers for catastrophe reinsurance. *Variance* 4, 191-208.

GC Securities, 2013. Convergence capitals impact on the reinsurance market, 2013/10/02, New York.

Gürtler, M., Hibbelin, M., Winkelvos, C., 2012. The impact of the financial crisis and natural catastrophes on cat bonds. Working paper IF40V1, Technische Universität Braunschweig.

Härdle, W.K., Lopez-Cabrera, B., 2010. Calibrating cat bonds for Mexican earthquakes. *Journal of Risk & Insurance* 77, 625-650.

He, Z., Krishnamurthy, A., 2013. Intermediate asset pricing. *American Economic Review* 103, 732-770.

Ibragimov, R., Jaffee, D., Walden, J., 2008. Nondiversification traps in catastrophe insurance markets. *Review of Financial Studies* 22, 959-993.

Jaffee, D., Russel, T., 2013. The welfare economics of catastrophe losses and insurance. *Geneva Papers on Risk & Insurance* 38, 469-494.

Lakdawalla, D., Zanjani, G., 2012. Catastrophe bonds, reinsurance, and the optimal collateralization of risk transfer. *Journal of Risk and Insurance* 79, 449-476.

Lane Financial, 2014. Quarterly Market Performance Report, Q2 2014, Wilmette, IL.

Lane, M., Mahul, O., 2008. Catastrophe risk pricing: an empirical analysis. Policy Research Working Paper 4765, World Bank.

Lee, J.P., Yu, M.T., 2002. Pricing default-risky cat bonds with moral hazard and basis risk. *Journal of Risk and Insurance* 69, 49-82.

Lee, J.P., Yu, M.T., 2007. Valuation of catastrophe reinsurance with cat bonds. *Insurance: Mathematics and Economics* 41, 264-278.

Mango D., 2003. Capital assumption: an alternative methodology for pricing reinsurance. *CAS Winter Forum*, 251-378,

Merton, R., 1974. On the pricing of corporate debt: the risk structure of interest rates. *The Journal of Finance* 29, 449-470.

McKinsey & Company, 2014. Could third-party capital transform the reinsurance markets? New York.

Munich Re, 2013. Market Review, Munich.

National Science Board, 2007. Hurricane warning: the critical need for a national hurricane research initiative, January 12, 2007, Arlington, Virginia.

Schiermeier, Q., 2011. Increased flood risk linked to global warming. *Nature* 470, 316.

Webster, P.J., G.J. Holland, J.A. Curry, and H.R. Chang, 2005, Changes in Tropical Cyclone Number, Duration, and Intensity in a Warming Environment, *Science* 309:1844-1846.

Wall Street Journal, 2014. Investors embrace 'cat bonds', 4/23/2014, New York.

Zanjani, G., 2002. Pricing and capital allocation in catastrophe insurance. *Journal of Financial Economics* 65, 283-305.

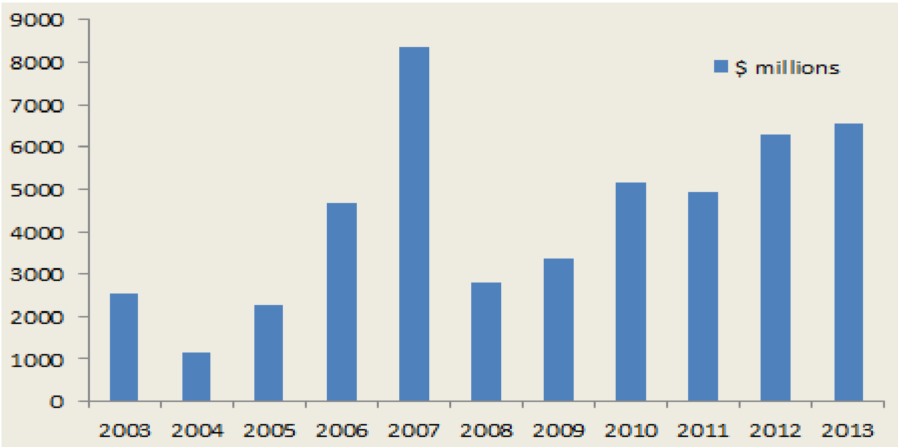

Figure 1. Cat bond growth 2003-2013.
This chart indicates that the growth dried up during the heyday of the 2007-2009 subprime crisis, but has recovered since.
Source: Munich Re 2013 Market Review

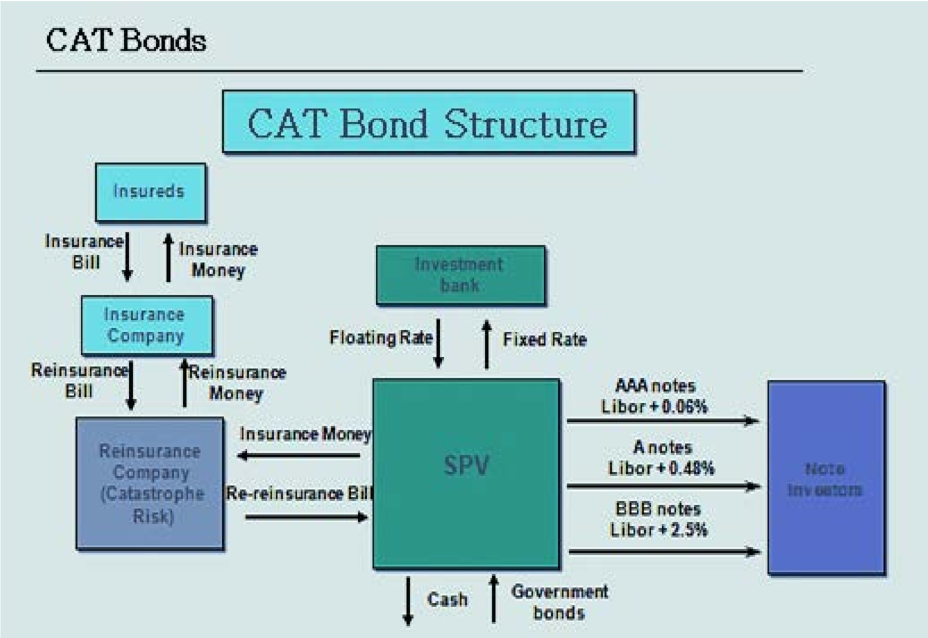

Figure 2. A typical reinsurer-sponsored cat bond structure.
This structure shows that cat bonds are issued by a Special Purpose Vehicle (SPV) set up by a sponsor, normally a (re)insurer. The SPV has three counterparties: the sponsor, the investor/fund manager, and the Interest Rate Swap (IRS hereafter) counterparty. Potential insurance payments by the SPV to the sponsor, a reinsurance company here, are pre-funded by the proceeds of the bond issue that are invested in a collateral account for a fixed return. Investors receive periodic interest payments reset to LIBOR plus a fixed spread as the "premium". To immunize the interest rate exposure, the SPV typically enters into an IRS arrangement with a counterparty to exchange the fixed return from the collateral account for floating payments of LIBOR minus a "spread" to match the payment schedule to the investor. Any shortfall, i.e. the premium plus the spread, is then captured by the re-reinsurance payment made to the SPV by the sponsor as a "pure play" in catastrophic risk. In other words, by issuing cat bonds, (re)insurers actually enter into a long position in re-reinsurance with the SPV free from default and interest rate risks under a parity relationship in which the reinsurance premium paid is tied to "the bond premium plus the IRS spread". An insurer-sponsored cat bond structure is similar, except that the insurer has a long default-risky reinsurance–long default-free re-reinsurance position.
Source: Swiss Re

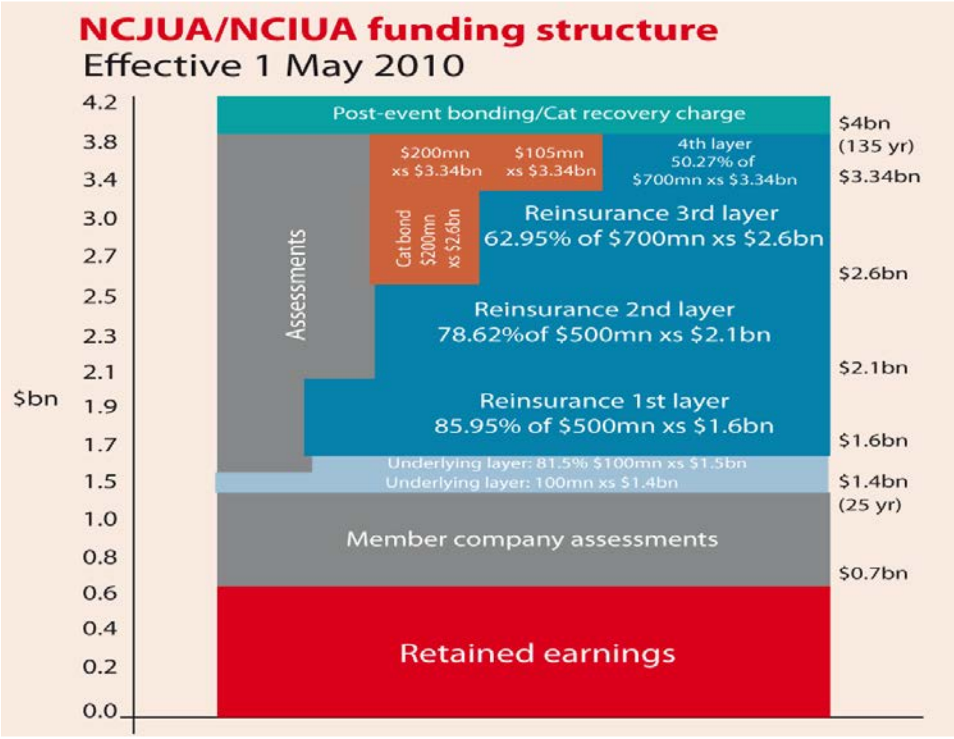

Figure 3. NCJUA/NCIUA's (North Carolina Joint Underwriting Association/North Carolina Insurance Underwriting Association) funding structure as of May 2010.
This figure shows that NCJUA/NCIUA's funding structure as of May 2010 was composed of four excess-of loss reinsurance layers and three cat bond issuances in addition to other sources. For example, for the 4th layer to cover a 135-year event with attachment point $3.34bn and detachment point $4bn and a funding limit of $700mn, the allocation is composed of 50.27% of excess-of-loss reinsurance purchased, 43.57% of cat bond issued with respective sizes $200mn and $105mn, and 6.16% of member company assessments. For both the 1st and 2nd layers, however, the decisions were not to issue cat bonds.
Source: Aon Benfield

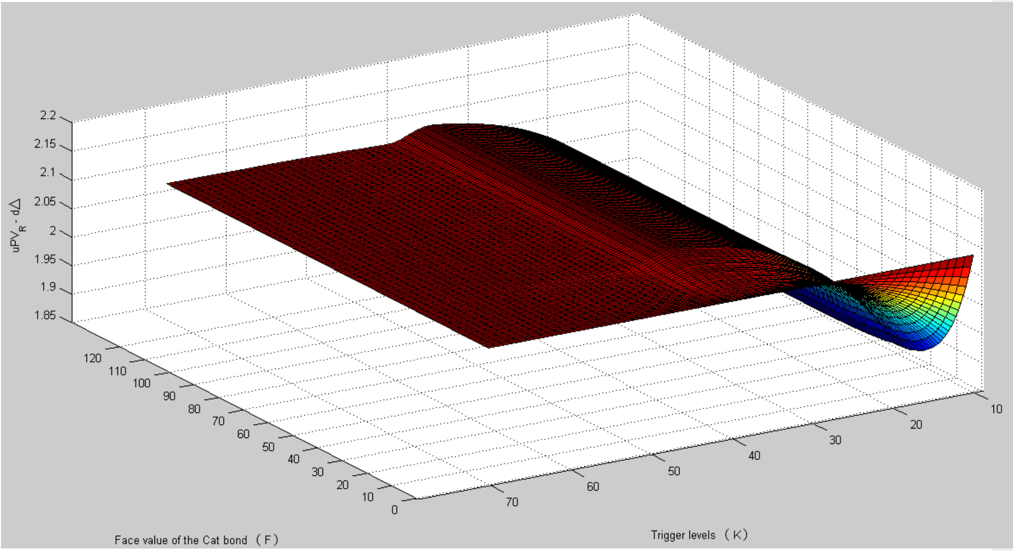

Figure 4. A 3-D representation of the reinsurer's maximum NPV for a coverage layer (70,10) as a function of a cat bond issuance' face value and trigger level**.**

Table 1: Parameter Definitions and Base Values

Asset parameters		
V	Reinsurer's assets	V/L=1.1
μ_V	Drift due to credit risk	Irreverent
ϕ_V	Interest rate elasticity of asset	-3
σ_v	Volatility of credit risk	5%
$W_{V,t}$	Wiener process for credit shocks	
Liability parameters		
L	Reinsurer's liabilities	100
μ_L	Drift due to idiosyncratic risk	0
ϕ_L	Interest rate elasticity of liability	-3
σ_L	Volatility of idiosyncratic risk	2%
$W_{L,t}$	Wiener process for idiosyncratic shocks	
Interest rate parameters		
r	Initial instantaneous interest rate	2%
κ	Magnitude of mean-reverting force	0.2
m	Long-run mean of interest rate	5%
ν	Volatility of interest rate	10%
λ_r	Market price of interest rate risk	-0.01
Z	Wiener process for interest rate shocks	
Catastrophe loss parameters for both C_t and C_{index}		
N(t)	Poisson process for the arrival of catastrophes	
λ	Catastrophe arrival intensity	0.5
μ_C	Mean of the logarithm of the losses per arrival	2
σ_C	Standard deviation of the logarithm of the losses per arrival	0.5
ρ_c	The correlation coefficients of the logarithms of C_j and $C_{index,j}$	1
Other parameters		
K	Trigger level of a cat bond	$M \le K \le A$
F	Face value of a cat bond	0~90
A	Attachment level of a reinsurance contract	10~30
M	Cap level of loss paid by a reinsurance contract	60~90
R	Total hedging capital needs	80
T	Maturity	3 years

Table 2.1: The Reinsurer's Optimum Allocation for Coverage Layer (*M*, *A*)=(70,10)

This table presents the reinsurer's sell-side optimum allocation result for coverage layer (70,10) in the base case scenario as summarized in Table 1, with markups: u=0.4 for the reinsurer and d=0.05 for the insurer. The optimum allocation (NPV_{Max},F_R,K_R) is (2.100952,33,37) as shown below. That is, the reinsurer's optimum allocation for coverage layer (M,A)=(70,10) is to issue cat bond with face value 33 and trigger 37, which would maximize the reinsurer's NPV to 2.100952.

	F=0	F=10	F=20	F=30	F=33	F=40	F=50	F=70	F=130
K=10	2.087526	1.932766	1.877060	1.863365	1.862492	1.857624	1.854605	1.853341	1.853165
K=20	2.087526	2.028887	2.015934	2.017540	2.018853	2.017095	2.016101	2.015704	2.015655
K=30	2.087526	2.071641	2.073988	2.080890	2.083019	2.082470	2.082200	2.082025	2.082024
K=35	2.087526	2.081200	2.086854	2.095014	2.097367	2.097076	2.096904	2.096808	2.096808
K=36	2.087526	2.082596	2.088742	2.096910	2.099281	2.099033	2.098869	2.098784	2.098784
K=37	2.087526	2.083707	2.090223	2.098559	2.100952	2.100736	2.100580	2.100505	2.100505
K=38	2.087526	2.084407	2.090845	2.098751	2.100943	2.100752	2.100606	2.100541	2.100541
K=39	2.087526	2.083934	2.088725	2.095366	2.097582	2.097412	2.097275	2.097219	2.097219
K=40	2.087526	2.082846	2.085836	2.090647	2.092453	2.092302	2.092174	2.092126	2.092126
K=50	2.087526	2.084507	2.083513	2.083642	2.084678	2.084600	2.084553	2.084551	2.084551
K=60	2.087526	2.086532	2.086262	2.086333	2.086529	2.086498	2.086496	2.086496	2.086496
K=70	2.087526	2.087256	2.087128	2.087280	2.087278	2.087278	2.087278	2.087278	2.087278

Table 2.2: The Reinsurer's Global Optimum Allocation across All Layers of Reinsurance Coverage

This table presents the reinsurer's optimum allocation results for all coverage layers (with M ranging from 60 to 90 and A ranging from 10 to 30) in the base case scenario as summarized in Table 1, with markups: u=0.4 for the reinsurer and d=0.05 for the insurer. It also identifies the reinsurer's sell-side global optimum allocation as the optimum allocation with the largest NPV across all coverage layers. The global optimum reinsurance coverage layer is (M, A) = (90, 10) with NPV 2.10374 and corresponding optimum cat bond issuance (F, K) of (52, 37).

Maximum NPV across All Layers of coverage							
Coverage	M=60	M=65	M=70	M=75	M=80	M=85	M=90
A=10	2.09445	2.09879	2.10095	2.10221	2.10286	2.10342	2.10374
A=15	1.32104	1.32546	1.32766	1.32893	1.32957	1.33014	1.33043
A=20	0.80600	0.81049	0.81274	0.81400	0.81465	0.81521	0.81550
A=25	0.47884	0.48335	0.48564	0.48690	0.48755	0.48811	0.48837
A=30	0.27942	0.28366	0.28597	0.28723	0.28788	0.28844	0.28871
F_R: Optimum Amount of Cat Bond across All Layers of coverage							
Coverage	M=60	M=65	M=70	M=75	M=80	M=85	M=90
A=10	23	28	33	38	43	48	52
A=15	18	23	28	33	38	43	47
A=20	13	18	23	28	33	38	42
A=25	8	13	18	23	28	33	37
A=30	0	8	13	18	23	28	32
K_R: Optimum Trigger Level of Cat Bond across All Layers of coverage							
Coverage	M=60	M=65	M=70	M=75	M=80	M=85	M=90
A=10	38	38	37	38	38	38	37
A=15	43	43	43	43	43	43	42
A=20	48	48	48	48	48	48	47
A=25	55	53	53	53	53	53	52
A=30	60	60	58	58	58	58	57

Table 2.3: Sell-Side Reinsurance Pricing across All Coverage Layers

This table presents the sell-side reinsurance pricing $(1 + u)PV_{R,0}$ for each of the optimum allocation results in Table 2.2. The pricing is highest at the global optimum allocation layer of (*M*,*A*) = (90,10).

Coverage	M=60	M=65	M=70	M=75	M=80	M=85	M=90
A=10	7.38127	7.39848	7.41306	7.41203	7.41461	7.41684	7.42419
A=15	4.65181	4.66930	4.67804	4.68301	4.68559	4.68782	4.69254
A=20	2.83578	2.85353	2.86242	2.86739	2.86997	2.87220	2.87544
A=25	1.68134	1.70048	1.70951	1.71448	1.71706	1.71929	1.72162
A=30	0.97797	0.99590	1.00583	1.01080	1.01338	1.01561	1.01740

Table 3: The Insurer's Global Optimum Allocation in each of the Simulation Stages

This table presents the insurer's global optimum allocation result in the base case scenario as summarized in Table 1 in each of the simulation stages. We search for the global optimum (*M*, *A*) coverage layer that minimizes the insurer's total hedging cost across all layers of coverage offered by the reinsurer. Panel I shows the result in the initial stage when *u*=0.4 and *d*=0.05. Panel II shows the result in the intermediate stage when the reinsurance markup decreased to 0.1 from 0.4. Panel III shows the result when *u*=*d*=0 in the final stable stage.

Panel I: the Initial Stage with u=0.4, d=0.05					
Coverage	M=60	M=65	M=70	M=75	M=80
A=10	12.68857	12.50621	12.17674	11.63300	10.74934
A=15	8.08177	8.03535	7.92784	7.73049	7.40975
A=20	4.96457	4.95992	4.92900	4.86493	4.75000
A=25	2.96594	2.97738	2.97289	2.95475	2.91403
A=30	1.73904	1.75557	1.76121	1.75915	1.74789

Panel II: the Intermediate Stage with u=0.1, d=0.05					
Coverage	M=60	M=65	M=70	M=75	M=80
A=10	11.10572	10.92270	10.59731	10.05688	9.17712
A=15	7.08440	7.03403	6.92604	6.73124	6.41270
A=20	4.35666	4.34806	4.31513	4.25088	4.13707
A=25	2.60532	2.61280	2.60631	2.58710	2.54647
A=30	1.52935	1.54192	1.54555	1.54243	1.53061

Panel III: the Final Stage with u=0, d=0					
Coverage	M=60	M=65	M=70	M=75	M=80
A=10	10.32563	10.15122	9.84197	9.32781	8.49055
A=15	6.58874	6.54013	6.43722	6.25209	5.94907
A=20	4.05271	4.04389	4.01221	3.95099	3.84278
A=25	2.42401	2.43051	2.42401	2.40555	2.36687
A=30	1.42324	1.43458	1.43772	1.43458	1.42323

Table 4: The Impacts of Interest Rate Risk, Leverage, and Markup on Optimum Allocations for Insurers and Reinsurers
Across Combinations of (*M, A, F, K*).

We extend the base value scenario as reported in Table 1 to consider three interest rate elasticity scenarios: (ϕ_V, ϕ_L) = (0, 0), (-3×(*L/V*), -3), and (-5, -3), three leverage scenarios: *V/L* = (1,1, 1.3, 1.5), and three markup scenarios: (*u*,*d*) = (0.4,0.05), (0.1, 0.05) and (0,0). Under the case of (ϕ_V, ϕ_L) = (0, 0), both assets and liabilities are interest rate insensitive, whereas the scenario of (-3×(*L/V*), -3) has a zero size-adjusted interest rate elasticity gap and can constitute an interest risk management scenario that strives to eliminate the interest risk confronting the reinsurer by adjusting the asset-liability mix. The case of (-5, -3) represents the interest rate risk scenarios with a positive duration gap on the asset-liability structure. THC_{min} denotes the minimum total hedging cost. NPV_{max} denotes the maximum NPV. We determine THC_{min} and NPV_{max} across combinations of (*M, A, F, K*).

(u, d)	Insurer: Buy Reinsurance and Issue Cat Bonds $THC_{min} = Min_{M,A}\left((1+u)PV_{R,0} + (1+d)\Delta_{I0}\right)$ (*M, A, F, K, THC_{min}*)			Reinsurer: Sell Reinsurance and Issue Cat Bonds $NPV_{max} = Max_{F,K}\left(uPV_{R,0} - d\Delta_0\right)$ (*M, A, F, K, NPV_{max}*)		
	V/L=1.1	V/L=1.3	V/L=1.5	V/L=1.1	V/L=1.3	V/L=1.5
.4, 0.05)	(80,30,30,80,0.9437)	(80,30,30,80,1.0078)	(60,30,50,60,1.0088)	(80,10,62,18,1.9906)	(80,10,43,37,2.1028)	(80,10,23,58,2.1186)
.1, 0.05)	(80,30,30,80,0.7423)	(80,30,30,80,0.7927)	(60,30,50,60,0.7991)	(80,10,0,80,0.4855)	(80,10,0,80,0.5221)	(80,10,0,80,0.5292)
), 0)	(80,30,30,80,0.6749)	(80,30,30,80,0.7208)	(60,30,50,60,0.7282)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
.4, 0.05)	(80,30,30,80,0.9436)	(80,30,30,80,1.0078)	(60,30,50,60,1.0088)	(80,10,62,18,1.9907)	(80,10,43,38,2.1029)	(80,10,23,58,2.1186)
.1, 0.05)	(80,30,30,80,0.7422)	(80,30,30,80,0.7926)	(60,30,50,60,0.7991)	(80,10,0,80,0.4855)	(80,10,0,80,0.5221)	(80,10,0,80,0.5292)
), 0)	(80,30,30,80,0.6749)	(80,30,30,80,0.7207)	(60,30,50,60,0.7278)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
.4, 0.05)	(80,30,30,80,0.8431)	(80,30,30,80,1.0074)	(60,30,50,60,1.0088)	(80,10,63,17,1.9732)	(80,10,44,37,2.0993)	(80,10,25,56,2.1180)
.1, 0.05)	(80,30,30,80,0.7418)	(80,30,30,80,0.7923)	(60,30,50,60,0.7991)	(80,10,0,80,0.4852)	(80,10,0,80,0.5220)	(80,10,0,80,0.5291)
), 0)	(80,30,30,80,0.6745)	(80,30,30,80,0.7204)	(75,30,35,75,0.7278)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)

Table 5: The Impact of Basis Risk on Optimum Allocations for Insurers and Reinsurers across Combinations of (*M*, *A*, *F*, *K*).

We extend the base value scenario as reported in Table 1 to consider three basis risk level: ρ_c = (0.8, 0.5, 0.3), three leverage scenarios: *V/L* = *V/L* = (1,1, 1.3, 1.5), and three markup scenarios: (*u*,*d*) = (0.4,0.05), (0.1, 0.05) and (0,0). THC_{min} denotes the minimum total hedging cost. NPV_{max} denotes the maximum NPV. We determine THC_{min} and NPV_{max} across combinations of (*M, A, F, K*).

ρ	(u, d)	Insurer: Buy Reinsurance and Issue Cat Bonds $THC_{min} = Min_{M,A}\left((1+u)PV_{R,0} + (1+d)\Delta_{I0}\right)$ (*M, A, F, K*, THC_{min})			Reinsurer: Sell Reinsurance and Issue Cat Bonds $NPV_{max} = Max_{F,K}\left(uPV_{R,0} - d\Delta_0\right)$ (*M, A, F, K*, NPV_{max})		
		V/L=1.1	V/L=1.3	V/L=1.5	V/L=1.1	V/L=1.3	V/L=1.5
0.8	(0.4, 0.05)	(80,30,30,80,0.9437)	(80,30,30,80,1.0078)	(60,30,50,60,1.0088)	(80,10,62,18,1.9906)	(80,10,43,37,2.1028)	(80,10,23,58,2.118
	(0.1, 0.05)	(80,30,30,80,0.7423)	(80,30,30,80,0.7927)	(60,30,50,60,0.7991)	(80,10,0,80,0.4855)	(80,10,0,80,0.5221)	(80,10,0,80,0.5292
	(0, 0)	(80,30,30,80,0.6749)	(80,30,30,80,0.7208)	(60,30,50,60,0.7282)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
0.5	(0.4,0.05)	(80,30,30,80,0.9436)	(80,30,30,80,1.0078)	(60,30,50,60,1.0088)	(80,10,62,18,1.9907)	(80,10,43,38,2.1029)	(80,10,23,58,2.118
	(0.1,0.05)	(80,30,30,80,0.7422)	(80,30,30,80,0.7926)	(60,30,50,60,0.7991)	(80,10,0,80,0.4855)	(80,10,0,80,0.5221)	(80,10,0,80,0.5292
	(0,0)	(80,30,30,80,0.6749)	(80,30,30,80,0.7207)	(60,30,50,60,0.7278)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
0.3	(0.4,0.05)	(80,30,30,80,0.8431)	(80,30,30,80,1.0074)	(60,30,50,60,1.0088)	(80,10,63,17,1.9732)	(80,10,44,37,2.0993)	(80,10,25,56,2.118
	(0.1,0.05)	(80,30,30,80,0.7418)	(80,30,30,80,0.7923)	(60,30,50,60,0.7991)	(80,10,0,80,0.4852)	(80,10,0,80,0.5220)	(80,10,0,80,0.5291
	(0,0)	(80,30,30,80,0.6745)	(80,30,30,80,0.7204)	(75,30,35,75,0.7278)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)

Table 6: The Impacts of Catastrophe Intensity and Loss Volatility on Optimum Allocations for Insurers and Reinsurers across Combinations of (*M*, *A*, *F*, *K*).

We extend the base value scenario as reported in Table 1 to consider three levels of catastrophe arrival intensity: λ = (0.5, 1, 2), and three levels of loss volatility: σ_c = (0.5, 1, 2), three leverage scenarios: *V/L* = (1,1, 1.3, 1.5), and three markup scenarios: (u,d) = (0.4,0.05), (0.1, 0.05) and (0,0). THC_{min} denotes the minimum total hedging cost. NPV_{max} denotes the maximum NPV. We determine THC_{min} and NPV_{max} across combinations of (*M*, *A*, *F*, *K*).

(u, d)	Insurer: Buy Reinsurance and Issue Cat Bonds $THC_{min} = Min_{M,A}\left((1+u)PV_{R,0} + (1+d)\Delta_{I0}\right)$ (*M, A, F, K, THC_{min}*)			Reinsurer: Sell Reinsurance and Issue Cat Bonds $NPV_{max} = Max_{F,K}\left(uPV_{R,0} - d\Delta_0\right)$ (*M, A, F, K, NPV_{max}*)		
	V/L=1.1	V/L=1.3	V/L=1.5	V/L=1.1	V/L=1.3	V/L=1.5
(0.4,0.05)	(80,30,30,80,0.9265)	(80,30,30,80,0.9853)	(60,30,50,60,0.9872)	(80,10,0,80,1.9457)	(80,10,0,80,2.0922)	(80,10,23,66,2.1184)
(0.1,0.05)	(80,30,30,80,0.7287)	(80,30,30,80,0.7750)	(65,30,45,65,0.7817)	(80,10,0,80,0.4864)	(80,10,0,80,0.5231)	(80,10,0,80,0.5295)
(0, 0)	(80,30,30,80,0.6626)	(80,30,30,80,0.7047)	(75,30,35,75,0.7111)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,5.0511)	(60,30,50,60,5.7000)	(60,30,50,60,5.7240)	(80,10,63,24,3.2283)	(80,10,43,44,3.6586)	(80,10,23,60,3.8734)
(0.1,0.05)	(80,30,30,80,4.0868)	(80,30,30,80,4.6237)	(60,30,50,60,4.7847)	(80,10,0,80,0.8023)	(80,10,0,80,0.9067)	(80,10,0,80,0.9645)
(0, 0)	(80,30,30,80,3.7391)	(80,30,30,80,4.2272)	(80,30,30,80,4.3994)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,16.8482)	(60,30,50,60,19.2319)	(60,30,50,60,19.2426)	(80,10,63,15,5.9501)	(80,10,43,30,6.6201)	(80,10,23,49,7.1307)
(0.1,0.05)	(80,30,30,80,14.2414)	(80,30,30,80,16.5956)	(60,30,50,60,16.9611)	(80,10,0,80,1.3334)	(80,10,0,80,1.5520)	(80,10,0,80,1.7376)
(0,0)	(80,30,30,80,13.1495)	(80,30,30,80,15.2897)	(60,30,50,60,15.7913)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,5.4578)	(80,30,30,80,5.9833)	(60,30,50,60,6.0099)	(80,10,62,12,5.1078)	(80,10,43,35,5.6989)	(80,10,23,59,5.9022)
(0.1,0.05)	(80,30,30,80,4.3027)	(80,30,30,80,4.7156)	(70,30,40,70,4.8073)	(80,10,0,80,1.2665)	(80,10,0,80,1.4198)	(80,10,0,80,1.4745)
(0,0)	(80,30,30,80,3.9144)	(80,30,30,80,4.2898)	(70,30,40,70,4.3774)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,14.2684)	(60,30,50,60,16.3107)	(60,30,50,60,16.3198)	(80,10,63,10,7.1597)	(80,10,43,30,8.0593)	(80,10,23,57,8.6182)
(0.1,0.05)	(80,30,30,80,11.6379)	(80,30,30,80,13.3266)	(60,30,50,60,13.8160)	(80,10,0,80,1.7057)	(80,10,0,80,1.9629)	(80,10,0,80,2.1334)
(0,0)	(80,30,30,80,10.6662)	(80,30,30,80,12.2013)	(60,30,30,80,12.7356)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,32.2840)	(60,30,50,60,36.8984)	(60,30,50,60,36.9129)	(80,10,63,10,10.9158)	(80,10,43,18,11.9808)	(80,10,23,38,12.8098)
(0.1,0.05)	(80,30,30,80,27.4117)	(80,20,20,80,31.8826)	(60,30,50,60,32.6993)	(80,10,0,80,2.3396)	(80,10,0,80,2.7378)	(80,10,0,80,3.0892)
(0,0)	(80,30,30,80,25.3330)	(80,20,20,80,29.2693)	(60,30,50,60,30.4734)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,23.3491)	(80,30,30,80,26.7818)	(60,30,50,60,26.8094)	(80,10,63,10,12.2276)	(80,10,43,29,13.2282)	(80,10,23,52,13.9395)
(0.1,0.05)	(80,30,30,80,18.6751)	(80,30,30,80,21.3722)	(60,30,50,60,22.2195)	(80,10,0,80,2.6802)	(80,10,0,80,3.1266)	(80,10,0,80,3.4338)
(0,0)	(80,30,30,80,17.0439)	(80,30,30,80,19.4959)	(60,30,50,60,20.4329)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,36.6331)	(60,30,50,60,42.1994)	(60,30,50,60,42.2137)	(80,10,63,10,14.4157)	(80,10,43,15,15.7680)	(80,10,23,43,16.6841)
(0.1,0.05)	(80,30,30,80,30.3907)	(80,30,30,80,35.4995)	(60,30,50,60,36.5729)	(80,10,0,80,3.0690)	(80,10,0,80,3.5983)	(80,10,0,80,4.0421)

(u, d)	Insurer: Buy Reinsurance and Issue Cat Bonds $THC_{min} = Min_{M,A}\left((1+u)PV_{R,0} + (1+d)\Delta_{I0}\right)$ (*M, A, F, K, THC_{min}*)			Reinsurer: Sell Reinsurance and Issue Cat Bonds $NPV_{max} = Max_{F,K}\left(uPV_{R,0} - d\Delta_0\right)$ (*M, A, F, K, NPV_{max}*)		
	V/L=1.1	V/L=1.3	V/L=1.5	V/L=1.1	V/L=1.3	V/L=1.5
(0, 0)	(80,30,30,80,27.9527)	(80,30,30,80,32.5970)	(60,30,50,60,33.9359)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)
(0.4,0.05)	(80,30,30,80,54.8104)	(60,30,50,60,62.6977)	(60,30,50,60,62.7099)	(80,10,63,10,17.5266)	(80,10,43,10,18.8046)	(80,10,24,20,19.7943)
(0.1,0.05)	(80,10,10,80,44.5909)	(80,10,10,80,51.3851)	(60,30,50,60,56.0414)	(80,10,0,80,3.4818)	(80,10,0,80,4.0994)	(80,10,0,80,4.6801)
(0, 0)	(80,10,10,80,40.8096)	(80,10,10,80,46.9861)	(60,30,50,60,52.3143)	(60,10,0,60,0)	(60,10,0,60,0)	(60,10,0,60,0)