

The research of Chinese property insurance companies' optimal reinsurance decision

——**An empirical analysis based on Option Dynamic Game Theory**

Abstract

The purpose of this paper: This article focuses on the Chinese property insurance companies' optimal reinsurance decision-making problem, through combining BS option pricing model and the dynamic game theory under complete information situation. This method not only takes both the time value and the risk value of insurance fund into consideration, but also combines the advantage of the dynamic game model to analyze the decision-making process of players' strategic behaviors. We develop the optimal reinsurance decision-making model based on option dynamic game theory, which can imitate accurate considerations of the two policy's parties when they are signing reinsurance policy, and simplify such decision-solving problems under complex dynamic financial conditions to the problem of solving optimization of reinsurers' expected utility, then taking the empirical data into function, we can achieve the optimal reinsurance decisions (namely the best amount of retention) under different situations.

The expected results of this paper: The author hopes the analysis in this paper, combining the advantages of BS option pricing model and the dynamic game theory under complete information situation to develop option dynamic game model, which is a more realistic optimal decision-making basis of Chinese property insurance companies, using historical loss data of insured of the original property insurance, then we can achieve the optimal reinsurance decision and make a contribution to the rational operations of Chinese property insurance companies.

The importance of this paper:

1. Taking the option dynamic game theory under the continuous-time financial framework into the use of reinsurance decision-making problems to do empirical analysis, there is no similar precedent in current research literatures. Although the option game theory based on physical assets and financial assets are developing dramatically, also the range of applications has expanded continuously, the empirical analysis in the field of property insurance companies' reinsurance decision-making is still blank. This paper applies option dynamic game theory in new research areas which are different from the traditional ones, we should say this is a new bold attempt.
2. The option dynamic game method combines the theoretical advantages of both BS option pricing model and game theory, which can help us analyze the people's dynamic decision-making problem under continuous-time situation and uncertainty, its essence is using option pricing techniques to determine the players' payment functions which are associated with uncertain benefits, then, in the case of the players action in order to solve the dynamic game. Option dynamic game theory chooses to maximize the value of the options instead of the maximization of the expected utility in common game model, which gives the no-arbitrage value of maximization of players' payment, the advantage of this method is that it takes the time value of money and the price of risk into account automatically.
3. This method makes further expansion of the option dynamic game theory's framework, and the risk-neutral option pricing techniques are introduced to the option game theory, which is parallel with option pricing techniques under the traditional BS option's framework, by solving the optimal

option value under the game framework, we simplify complex problems under uncertainty to looking for the first-order condition, by doing this we also can obtain optimal reinsurance decision and fair premiums of reinsurance based on the no-arbitrage pricing techniques.

Key words: optimal reinsurance decision; BS option pricing model; dynamic game theory

Introduction

Reinsurance refers to a reinsurance policy signed by two sides on the basis of original insurance policy which is established by the insured and the insurer, it is a process that the original insurance company transfers its risk to the reinsurer. So reinsurance is also known as "the insurance of the insurance". President of the American Insurance Information Association Robert • Hartwig treated that "insurance companies are better risk managers than banks, while reinsurance is the last risk protection for the risk managers after the financial crisis."

Insurance plays an increasingly important role in the modern financial system, and the importance of reinsurance is also easily to see, so how to make the accurate decision of the requirement of reinsurance has become a primary issue for Chinese property insurance companies who play a role of the undertaker of risks. Under this situation, the original insurance companies should not only make sure that the quantity of risk they can afford after transferring part of original insurance responsibility, but also guarantee do not transfer too much premium, otherwise they cannot earn enough profit.

In addition, in traditional insurance constructing process, we set all parties of the policy are rational people, but we have not seen the order of decision-makers in the model we have learnt, which in fact is not consistent with the insurance process in the practical operation, of course, reinsurance pricing process does not even treat it as a problem. However, the emergence of game theory can solve this problem effectively, game theory is a theory of decision-making tool to study more than one person, it takes the rational behavior of people who act in a given policy environment to ensure the maximization of their own benefits into account, and it is established under the consideration of maximizing the profits of others. So, this paper is based on the basic reinsurance principle, and uses a combination of option theory and game theory to develop a framework to maximize both parties' utilities.

1. Research Status

1.1 Research Status on optimal reinsurance decision

Our analysis of the optimal reinsurance decisions are initially focused mainly on qualitative research, quantitative research began relatively late, in 2006, Liu Bo applied knowledge of game theory and insurance theory to analyze the optimal retention model of non-cooperative game and cooperative game separately. In 2008, Yu Zhonghua made use of the method of optimization to analyze optimal reinsurance model in detail under mean-variance theory and utility theory. Since that, the use of mean-variance theory to analyze reinsurance decision-making problem has become more popular, the best representative of this method is the article of He Nannan in 2010, which used the information-entropy method to solve non-proportional retention problem of reinsurance. After that,

there has formed three research directions of optimal reinsurance decision, which includes mean-variance theory, utility theory and probability of bankruptcy, in 2010, Zang Wu used these three methods to construct the models to solve problems. In 2014, Wei Jiangsong used the expected value principle to calculate problems, then determined the specific forms of the optimal retention and the expression of VaR (value at the risk), which created a new idea for optimal reinsurance decision-making. However, by combining the option theory and game theory to analyze reinsurance decision has not been fully emerged. Only in 2011, Zhang Lanlan firstly used option game model by taking insurance as option and using options strategy game to seek the best primary premiums of insurance with the stochastic backward differential equations.

1.2 Research status of option game theory

Option game theory's groundbreaking research is based on Smets in 1993, he first combined game theory and real options to establish a symmetry option game model under uncertainty condition. Dixit and Pindyck summarized Smets' model in 1994, and analyzed the cases in the incomplete competition case. Study in abroad have used option game theory in the insurance field in recent years, but they are all basic qualitative researches. Many scholars on the basis of foreign research on option game theory model studied deeply. In 2001, An Yinghui and Zhang Wei summarized the generalized analytical framework of option game method based on traditional enterprise project investment evaluation and decision-making problems in theory. In 2004, Shi Shanchong and Zhang Wei proposed ideas, basic framework and specific analytical steps of option games strategy, and pointed out the option game problems in the field of study and research.

Option game theory focuses on strategic and risky capital and real estate. In the insurance field, Sun Jiansheng applied option game theory under the financial framework to the insurance sector in 2006, but also at the stage of qualitative research. Overall, the use of option game model is mainly concentrated on real options, the use in the insurance field is still relatively seldom. Recently, there have been some use of option game model in the original insurance pricing model, but the study in optimal reinsurance decision is very seldom.

2. Introduction of the principle of option dynamic game model

2.1 Option features in the use of reinsurance

Reinsurance has the same principle of operation with the original insurance, premiums are paid by the insured to the insurer, the insured obtains the right of claims at the time of the accident has occurred by paying premiums. Under this definition, in fact, an insurance gives the insured contingent right in the future to claim and this right can be seen as options, the premium paid by the insured can be equivalent to the fee of the option. If the insurance accident occurred during the of insurance period, the insured can exercise the right to claim before the expiration of the policy, and if there is a deductible, which is equivalent to exercise price of American options. It is consistent with the principles of reinsurance, so in this article we will use option pricing techniques to decide reinsurance pricing model.

This paper focuses on reinsurance decision of Chinese property insurance companies under one-layer excess of loss reinsurance policy, which refers to the reinsurance decision is involved in the

process only once, the process also includes only an original insurer and a reinsurer. We set S as the original insurance policy claims, D is original insurer's retention, and our analysis supposes that we have a large enough number of similar policies of the original insurer. If S is not greater than the deductible D of reinsurance policy, the reinsurer fails to pay obligations; if S is greater than D, the reinsurer needs to bear part of the payment equal to the amount of S over D. Therefore, the maturity value of the original insurer can be described as:

$$V_I = \begin{cases} 0 & , S \leq D \\ S - D & , S > D \end{cases}$$

In the option contract S can be seen as the expiration price of the underlying asset, D is the option exercise price, the original insurer is holding an American call option with the exercise price of D. In our analysis, we choose the Modern option pricing model, Black-Shcholes model, which is developed by F • Black and M • Shcholes in 1973, BS model is an analytical tool that can effectively analyze options, and take the time value and risk factors of assets into consideration. Because of its superiority, BS model is widely used for hedging problem and pricing problem of financial products. After rapid development, BS model is now transformed into a general analytical framework, and be used in the field of real options more widely. Classical BS model is shown in following equation:

$$P_t = S_t N(d_{t_1}) - X e^{-rT} N(d_{t_2})$$

$$d_{t_1} = \frac{\ln\left(\frac{S_t}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_{t_2} = d_{t_1} - \sigma\sqrt{T-t}$$

Where, S_t represents the price of stock, in this article it means the claims of the original insurance policy; X represents the strike price of the option, in this article, we use the deductible of one-layer excess of loss D to substitute.

2.2 Complete information dynamic game theory in the use of reinsurance

Traditional insurance decision-making process is just the results of non-strategic rational people's behaviors, the policy's parties are not aware of the other party in this situation, based solely on their own optimal theory to make decisions. In fact, this decision-making process cannot be fully applicable in the insurance environment, especially in the reinsurance policy, due to the parties of reinsurance policy are insurance agencies with expertise knowledge, and such as a mutually agreed rate is developed after repeated game between the two sides of the policy, for instance, the rate, retention and the sum amount of the policy. So we use the theoretical framework of game theory in the analysis of this article to imitate decision-making process when they are signing the policy.

2.2.1 Complete information dynamic game theory's framework in the use of reinsurance

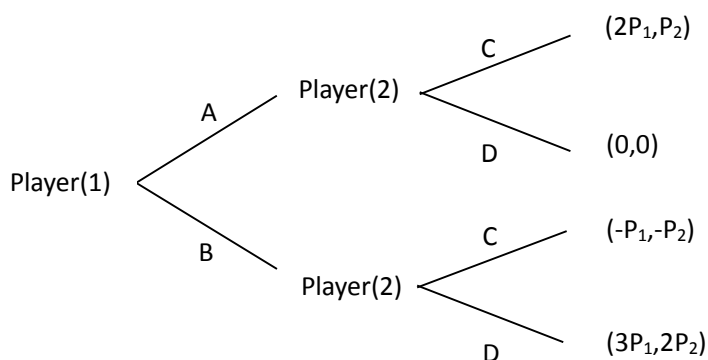
Game Theory is based on whether the two sides can affect each other when making decisions to separate into "cooperative game" and "non-cooperative game," cooperative game pays more attention to cooperation of two parties, non-cooperative game emphasizes rational individual decisions. Non-cooperative game is divided into static game and dynamic game by the order of decision-making parties, static game means both players simultaneously make decisions without knowing each other's decision when each individual makes decision, while dynamic game distinguishes the order, and the subsequent decision makers fully understand the behavior of the first decision makers. It can also be classified by the information, if both players are fully aware of

each other, it is the complete information game, on the other hand, it is incomplete information game. According to the difference of these two dimensions, game theory can be defined as the four forms, including: "complete information static game", "incomplete information static game", "complete information dynamic game", "incomplete information dynamic game." At the same time, these four games corresponding to four equilibrium states, namely "Nash equilibrium", "Bayesian Nash equilibrium", "sub-game perfect Nash equilibrium", "refined Bayesian equilibrium." In this paper, we will use complete information dynamic game theory for subsequent analysis.

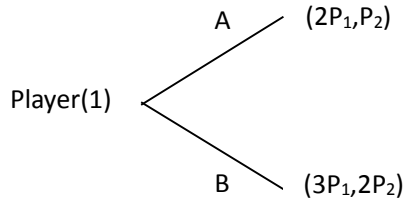
The basic elements of game theory, including "the players", in the reinsurance it is equivalent to the original insurer and reinsurer, "action" is the original insurer's reinsurance decisions, "strategy" is the plans of insurer and reinsurer, including deductible, rate and other reinsurance element, "payment" is the an individual payment function, here we use BS pricing model in the first section to analyze. Of course, the game follows the full information disclosure and the order of the both sides when they making decisions are taken into consideration, the result is the end of the game when the reinsurance policy is signed, at that time, all the players can achieve the optimal equilibrium strategy.

2.2.2 Complete information dynamic game theory's solving process in the use of reinsurance

The following example is explained in detail, gamble A has two the players, Player (1) and Player (2), they have the same judgment of their income, and each faces two decisions, the former faces the decision-A and decision-B, the latter one faces decision-C and decision-D, income portfolio (X, Y) represents the return of Player (1) and Player (2), as the result $(2P_1, P_2)$ represents that if the Player (1) selects decision-A and Player (2) selects decision-B, then Player (1) will get $2P_1$, Player (2) will get benefits of P_2 , where P_1, P_2 represents a positive wealth randomly. So in this case, how do Player (1) and Player (2) make decision to achieve the optimal outcome.



This in fact describes a complete information dynamic game process, in this game, players have full disclosure of information and the players have decision-making order, so the result of this game is equivalent to solving sub-game perfect Nash equilibrium, we can adopt common backwards induction. Starting with the second phase of the game to get Player (2)'s the optimal decision, firstly, comparing the two states of Strategy C, in the result 1 $(2P_1, P_2)$ and result 3 $(-P_1, -P_2)$, P_2 is larger than $-P_2$, so when Player (2) selects the strategy C, the optimal result is $(2P_1, P_2)$. Under the same process, when the Player (2) selects the strategy D, the optimal results is $(3P_1, 2P_2)$, as the follows Fig.



So, when the Player (1) makes decision, he clearly knows when he chooses strategy-A, Player (2) of course will choose strategy-C, when chooses strategy-B, Player (2) selects strategy-D. Setting Player (1)'s payment function $V_1 = (S_1, S_2, S)$, and Player (2)'s the payment function $V_2 = (S_1, S_2, S)$ (S_1 represents the decision of Player (1), S_2 represents the decision of Player (2), S represents the state variables), the Player (2)'s the optimal decision V_2^* satisfies the following formula:

$$\frac{\partial(S_1, S_2, S)}{\partial S_2} = 0$$

So, $V_2^* = V_2^*(S_1, S)$, namely, Player (2)'s the optimal decision V_2^* is a function of Player (1)'s decision, and when Player (1) makes decisions, he can fully predict the decisions of Player (2), so Player (1) the optimal decision V_1^* satisfies the following formula:

$$\frac{d(S_1, S_2^*, S)}{dS_1} = \frac{\partial(S_1, S_2^*, S)}{\partial S_1} + \frac{\partial(S_1, S_2^*, S)}{\partial S_2} \cdot \frac{\partial S_2^*}{\partial S_1} = 0$$

So, $V_1^* = V_1^*(S)$ and $\frac{\partial(S_1, S_2^*, S)}{\partial S_2} \cdot \frac{\partial S_2^*}{\partial S_1}$ indicates the decisions of Player (1) affect the decisions of Player (2), Player (1)'s the optimal decision is based on Player (2)'s the optimal decision, which is essence of the backwards induction as described above. We will use this approach to estimate the insurance companies' reinsurance decisions, it is possible for reinsurer to understand insurer's the optimal decision, namely, the reinsurer knows the insurer's insurance option pricing formula and optimal result, so the reinsurer can decide the best deductible on the basis of the observed results.

3. The basic framework of Option Dynamic Game Model

3.1 The elements' definition in the basic framework of Option Dynamic Game Model

Option dynamic game model is a combination of BS option pricing model and dynamic game theory, due to BS model is built on the premise of no arbitrage, it is possible by solving the Nash equilibrium to obtain the theoretical price of one-layer excess of loss reinsurance and the optimal reinsurance decisions. The essence of this approach is using pricing techniques to define the players' payment function, in the case of the two parties make decisions in order, using dynamic game approach to solve the problems. The advantage of this approach is that in the decision-making process if both players are dynamic decision maker, the payment of both sides are not fixed, which is variable determined by many factors, at this time the expected utility model which is used in Game Theory cannot measure the risk of reinsurance correctly, however, the BS option pricing model described in the previous section is an effective tool for risk pricing, so the combination of the two theories

will be able to effectively analyze the game process based on the two's decisions.

Prior to the framework set up, we need to understand the basic concepts of the game, including the players, actions, information, strategies, benefits, balanced and the results. The players, strategies and benefits are the most basic elements, the players, actions and results are collectively referred to as the rules of the game.

So, based on the definition of game's elements, the elements of reinsurance option dynamic game process can be described as: the first stage is the reinsurer's decision-making process, the second stage is the original insurer's decision-making process, the reinsurer works out a "deductible" of the reinsurance policy in the first stage, which in fact is the original insurer's retention (D), the original insurer observes reinsurer's D, then, buys a reinsurance policy with retention of D*, which is fairly same as purchasing an American call option, and exercises the option until obtaining maximization of the option. The final returns VR of reinsurer by reinsurance policy can be described as the following formula:

$$V_R = \begin{cases} P & , 0 \leq S \leq D \\ P - S + D & , M < S < \infty \end{cases}$$

Where, P represents premiums, S represents the original insurance payments, D represents the original insurer's retention. According to the formula, the reinsurer's expected utility is defined as the following formula (assuming original insurers and reinsurers are risk-averse, their utilities fulfill the Von-Neumann Morgenstern expected utility function form, where, $U' > 0; U'' < 0$; distribution function for the payment amount S is F(S), the density function is f(S)):

$$EU_R = EU_{V_R} = \left\{ \int_0^D U(P) dF(S) + \int_D^\infty U(P + S - D) dF(S) \right\}$$

Original insurer's final return V_I can be described as the following formula:

$$V_I = \text{Max}\{-P, S - D^* - P\}$$

$$EU_I = EU_{V_I} = EU(\text{Max}\{-P, S - D^* - P\})$$

Where D* represents optimal retention, which can reach the largest amount of the option value at the time of option is executed.

For setting the utility function of original insurance companies, because the insurance companies have mechanism of risk distribution, and insurance companies are regarded as the last protection for risk in financial system and the organizations for pricing the risk, they have a large amount of risky units. So, we usually recognize insurance companies' attitudes of risk are risk-neutral, and we usually assume the utility function as $U(X) = a + bX$, where "a" and "b" are constant.

For setting the distribution function of original insurance loss, in this article, we select the Weibull distribution function (About why does Weibull fit for this situation are described in detail in the

appendix), the expression for the Weibull distribution is $F(X) = 1 - e^{-\left(\frac{X}{\lambda}\right)^k}$. In 1937, professor Waloddi Weibull (1887-1979) proposed this creative distribution, it is one of the most widely used distributions to analyze the data for the failure. Weibull distribution's advantages are its function to

provide more accurate failure prediction and analysis of small data samples, providing the solutions of problems as soon as possible, and describe a simple and useful chart for a single failure mode, so when the data is not sufficient, it is still easy to understand. The analysis of Weibull distribution is generally used for analyzing failure data, including the development, production and services, quality control and design defects, which also includes the loss distribution analysis of natural disasters (lightning strikes, storms, strong winds, snow, etc.). Because of the risk of the reinsurance primarily exposed to natural disasters just as described above, so we use the Weibull distribution in this paper to estimate the loss of the original distribution of the insured is very property.

3.2 To build the basic framework of Option Dynamic Game Model

Through the above analysis, we know that during the process of reinsurance-signing, the original insurer and reinsurer how to make optimal decisions can affect each other, so we use a two-stage dynamic game to describe this decision-making process of reinsurance policy. In the first stage, the reinsurer develops a retention D ; in the second stage, the original insurer chooses whether to exercise the option. The payment function is determined by BS option pricing model, so we can get the optimal reinsurance decision of Chinese property insurance companies and the fair price of one-layer of excess of loss reinsurance. Then describe the game process as following three steps:

First, we need to define the elements of the game, including the order of actions and payment functions;

Secondly, we use BS option pricing model to determine the player's uncertain future payment;

Finally, we use backwards induction from the beginning of the second stage to solve the equilibrium outcome.

Through building this analytical framework, the original complex dynamic analysis process has been simplified and easy to solve, this framework not only has the advantage of option pricing model which contains the time value of money and risky factors, but also combines the strategy of dynamic game theory analysis, which actually imitates the consideration of two parties when they are signing the reinsurance policy. And it also simplifies decision-making process in this complex and dynamic financial condition to just solve simple calculations.

4. Solving the results under Option Dynamic Game's framework of one-layer excess of loss reinsurance without arbitrage

Previous chapters have already solved dynamic game under complete information, which is backwards induction. Firstly, we need to determine the optimal decision of original insurer in the second phase after obtaining retention D . We know only at the time of signing and expiration of reinsurance policy there can have cash flows, which is equivalent to two parties signed an American call option without dividends. From the standpoint of the original insurer, exercise option ahead of the expiration is meaningless, because the minimum value of American call option and European call option in the absence of dividends can be expressed as $C_t \geq \max \left[0, S_t - X / (1 + r_f)^{T-t} \right]$,

so American call option without paying dividends is same as the European call option with same

underlying asset, expiration date and strike price, previous analysis shows that the option value P_0 at the time of signing the policy for the original insurer just as the following formula:

$$P_0 = S_t N(d_{t_1}) - D e^{-rT} N(d_{t_2})$$

The reinsurer knowing $P_0(\bullet)$ when he develops D , so we can see $P = P_0$ in the reinsurer's utility function, the reinsurer's utility maximum problem can be expressed as

$$\text{Max}(EU_R) = \text{Max} \left\{ \int_0^D U(P_0) dF(S) + \int_D^\infty U(P_0 + S - D) dF(S) \right\}$$

If we want to solve the above equation, we need to know how to solve equation of $\frac{\partial EU_R}{\partial D} = 0$, the solution process is as follows:

$$\frac{\partial EU_R}{\partial D} = U'(P_0) \frac{\partial P_0}{\partial D} F(D) + \int_D^\infty U'(P_0 - X + D) \frac{\partial P_0}{\partial D} f(S) dX = 0$$

The EU_R changes to:

$$\begin{aligned} &= U(P_0)F(S)|_{0 \rightarrow D} - \int_0^D F(S) U'(P_0) \frac{\partial P_0}{\partial S} dS + U(P_0 + D - S)F(S)|_{D \rightarrow \infty} \\ &\quad - \int_D^\infty U'(P_0 + D - S) \frac{\partial P_0 + D - S}{\partial S} F(S) dS \end{aligned}$$

According to the features of loss distribution, we know $F(0) = 0$; $F(\infty) = 1$

$$\begin{aligned} &= U(P_0)F(D) - \int_0^D F(S) bN(d_{t_1}) dS + U(P_0 + D - S) - U(P_0 + D - S)F(D) \\ &\quad - \int_D^\infty F(S) b(N(d_{t_1}) - 1) dS \end{aligned}$$

So, $\frac{\partial EU_R}{\partial D}$ changes to (where, $U(X) = a + bX$; $F(X) = 1 - e^{-\left(\frac{X}{\lambda}\right)^k}$):

$$\begin{aligned} &= U'(P_0) \frac{\partial P_0}{\partial D} F(D) + U(P_0)F'(D) - bN(d_{t_1}) F(D) \\ &\quad + b \left(1 - e^{-rT} N(d_{t_2}) \right) (1 - F(D)) + U(P_0 + D - S) \left((1 - F(D)) \right)' \\ &\quad - b(N(d_{t_1}) - 1)(1 - F(D)) \\ &= F(D) \left[-be^{-rT} N(d_{t_2}) - bN(d_{t_1}) + be^{-rT} N(d_{t_2}) - b + bN(d_{t_1}) - b \right] \\ &\quad + F'(D) \left[U(P_0) - U(P_0 + D - S) \right] + b \left(2 - e^{-rT} N(d_{t_2}) - N(d_{t_1}) \right) \end{aligned}$$

$$\begin{aligned}
&= -2b \left(1 - e^{-\left(\frac{D}{\lambda}\right)^k} \right) + b(S - D) \frac{k}{\lambda} \left(\frac{D}{\lambda}\right)^{k-1} e^{-\left(\frac{D}{\lambda}\right)^k} + 2b + be^{-rT} N(d_{t_2}) - bN(d_{t_1}) \\
&= e^{-\left(\frac{D}{\lambda}\right)^k} \cdot \left(2b + b(S - D) \cdot \frac{k}{\lambda} \left(\frac{D}{\lambda}\right)^{k-1} \right) + be^{-rT} N(d_{t_2}) - bN(d_{t_1})
\end{aligned}$$

From calculation we get the following equation:

$$e^{-\left(\frac{D}{\lambda}\right)^k} \cdot \left(2b + b(S - D) \cdot \frac{k}{\lambda} \left(\frac{D}{\lambda}\right)^{k-1} \right) + be^{-rT} N(d_{t_2}) - bN(d_{t_1}) = 0$$

So, at last we get an equation which describes the relationship of S and D, where, $k = 2.0207$; $\lambda = 2691.5$ (a detailed solution process is in the appendix); setting $b = 1$ (the attitude of insurance companies are risk-neutral).

By calculating the equation, we can get a D equals D^* , D^* is the Player (1)'s namely reinsurer's optimal retention based on Player (2) namely the insurer can achieve the best P_0 . Game process can be described as in the first stage of the game reinsurer determines D^* , and then in the second stage, the original insurer chooses the best time to maximize his own effectiveness and exercises the option on this basis, the two parties in the policy can achieve relative maximum utility. Then the even solution of the Game is $(D^*, P_0(D^*))$, where D^* is the best reinsurance decision namely the retention of the original insurer (in this article we set the original insurers are the Chinese property insurance companies), and $P_0(D^*)$ is a fair price of one-layer excess of loss reinsurance based on no-arbitrage pricing theory. Since the solving process combines both the advantages of the European call option and dynamic game theory, the time value and risk of price are all included in the analysis, in the absence of arbitrage we can get a fair price for reinsurance.

5. Some conclusions about the Option Dynamic Game Model

5.1 The problem of functions setting in the process of solving Option Dynamic Game

Model

In fact, in the process of solving the D^* we also need to define the reinsurer's utility function $U(\bullet)$ and the distribution function $F(X)$ of original insurance payments, in this article, the game sets both original insurer and reinsurer are risk-neutral, their utility functions are linearity and in accordance with Von-Neumann Morgenstern theory. But in fact, there are many paradoxes of expected utility theory in the academic search for a long time, such as Allias Paradox, etc., in addition, RDEU (random expected utility) and other theories are constantly revealing the shortcomings of expected utility theory in analyzing issues, but in general analysis, the use of $EU = \sum_{i=1}^n x_i p_i$ can meet analytical needs. Otherwise, we can attempt to transform the utility model in the analysis, it is possible to get more reliable results.

In the article, the original insurance claims' (X) distribution function $F(X)$ is defined as the Weibull distribution, although Weibull distribution is widespread in loss pricing for natural disasters, there is not yet a very universal conclusion for how to determine the loss distribution function, including

the most common loss distribution function can only be able to set up with many constraints. And because of the big differences of loss distribution under different contractual liability items, just the risk classification of the part of property insurance risks is more than dozens, also because there exists many factors such as the region, the environment, etc. can affect the risks, so the loss distribution function is not very easy to determine. In a subsequent case studies we can try other loss distributions.

5.2 The promotion of Options Dynamic Game Model

We constructed the Game options dynamic model in this paper to analyze the Chinese property insurance companies' optimal reinsurance decision in one-layer excess of loss reinsurance policy, of course, we can extend this basic theory based on the model to multi-layer excess of loss optimal reinsurance decision, its essence is developing the two-stage game process to three stages, four stages, etc.. The players from two and then have been expanded. And we can also use P to analyze the fair price of the reinsurance. Meanwhile, this model can not only analyze the case of reinsurance, but also of course apply to the original insurance policy to analyze optimal decisions for the insured and the insurer.

References

- [1] Sun Jiansheng, Insurance Option Game Analysis [D]. Capital University of Economics, 2006.04.
- [2] Zhang Lanlan, Option game method in insurance pricing application [D]. Hua Zhong University of Science and Technology, 2011.05.
- [3] Sun Jiansheng, Wang He. Optimal reinsurance Option Game Analysis [J]. Capital University of Economics and Trade, 2006.01.
- [4] Duan Jingjing, The study of Game Review of abroad options [J]. Development Research, 2007.09.
- [5] Liu Yinguo, Insurance Innovation –The study of Insurance options[J] study economic issues, 2000.06...
- [6] Li Xuan. Reflections on Chinese Insurance Option System [J]. Shanghai Insurance, 2005.10.
- [7] Deng Zhimin, Zhang Runchu. Reinsurance pricing formulas based on investment [J]. Applied Mathematics journal, 2006.10.
- [8] K.A.Froot, P.G.J.O'Connell, US catastrophe reinsurance pricing [J]. World Earthquake Renditions, 2005.04.
- [9] Jiang Lishang, Mathematical pricing models and methods of options [M] Higher Education Press, 2003.
- [10] Karl • Borch. Insurance Economics [M] Commercial Press, 1999.
- [11] Wang Wenju, etc. Application of Game Theory and Economic Development [M]. Capital Economic and Trade University Press, 2003.
- [12] Wang Wenju, Xia Longmei. The analysis of insurers' game behavior. [J]. Quantity Technical Economics Study, 2003.08.
- [13] Zhang Yishan, Ma Hongmei. Optimal insurance limits and optimal insurance rates [J].

Quantity Technical Economics Study, 1997.08.

[14] Botch, K. The Economics of Uncertainty [M]. Princeton: Princeton University Press, 1968.

[15] Goovaerts, M J., deVlyder F. and Haezendonck [J].Insurance Premium: Theory and Application. North Holland, Amsterdam, 1984.

APPENDIX

Table 1: Direct economic losses from natural disasters (1990-2013)

YEAR	Direct economic losses
1990	616
1991	1215
1992	854
1993	993
1994	1876
1995	1863
1996	2882
1997	1975
1998	3007. 4
1999	1962
2000	2045. 3
2001	1942. 2
2002	1717. 4
2003	1884. 2
2004	1602. 3
2005	2042. 1
2006	2528. 1
2007	2363
2008	11752. 4
2009	2523. 7
2010	5339. 9
2011	3096. 4
2012	4185. 5
2013	5808. 4

(Data Source: Ministry of Civil Affairs of People's Republic of China)

Using software easy-fit to find the most appropriate distribution for direct economic losses amount of natural disasters, fitting results are in the table2 below, Weibull loss distribution is in fifth place under Kolmogorov goodness of fit test and Chi-Squared Goodness of fit test, so it is a better distribution fitting to some extent, and it is appropriate to use the Weibull distribution in this paper.

Table 2: Distribution of goodness fit test table

Goodness of Fit - Summary

	Distribution	Kolmogorov		Anderson		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.2268	29	3.3126	41	13.708	44
2	Burr	0.13581	1	0.41122	1	0.45676	4
3	Burr (4P)	0.3943	48	6.5569	48	10.357	39
4	Cauchy	0.16359	15	0.96779	20	0.65759	7
5	Chi-Squared	0.70739	57	788.81	58	38.76	52
6	Chi-Squared (2P)	0.41667	51	80.899	56	13.5	43
7	Dagum	0.64419	55	19.354	54	45.822	53
8	Dagum (4P)	0.67437	56	22.765	55	22.906	50
9	Erlang	0.40168	50	5.2015	44	1.8417	14
10	Error	0.27316	35	2.2802	30	5.3233	29
11	Error Function	0.60597	54	17.186	53	46.627	54
12	Exponential	0.27455	38	2.4067	33	3.827	24
13	Exponential (2P)	0.203	24	2.6328	37	13.846	45
14	Fatigue Life	0.15092	11	0.70176	16	1.7659	13
15	Fatigue Life (3P)	0.14445	4	0.6805	15	2.9878	15
16	Frechet	0.2324	31	0.99481	22	6.0127	33
17	Frechet (3P)	0.147	6	0.49128	3	1.4553	11
18	Gamma	0.21306	28	1.5107	24	1.5617	12
19	Gamma (3P)	0.1637	16	0.82835	18	10.248	38
20	Gen. Extreme Value	0.18278	21	0.70984	17	3.5416	23
21	Gen. Gamma	0.20344	25	1.2122	23	3.0193	16
22	Gen. Gamma (4P)	0.15269	14	0.65708	14	7.3076	35
23	Gen. Pareto	0.21067	27	11.384	50	N/A	
24	Gumbel Max	0.20413	26	1.6558	26	0.79694	6
25	Gumbel Min	0.33974	46	5.188	43	7.3075	34
26	Hypersecant	0.25911	33	2.206	28	14.244	46
27	Inv. Gaussian	0.20262	23	0.93834	19	11.354	41
28	Inv. Gaussian (3P)	0.14915	8	0.64233	13	4.8449	26
29	Johnson SB	0.25283	32	15.041	51	N/A	
30	Kumaraswamy	0.59964	53	16.979	52	38.76	51
31	Laplace	0.27316	36	2.2802	31	5.3233	30
32	Levy	0.39612	49	5.5079	46	0.27411	1
33	Levy (2P)	0.2997	44	3.214	38	1.4375	10
34	Log-Gamma	0.14752	7	0.57544	8	3.0629	18
35	Log-Logistic	0.17411	19	0.54381	7	5.7337	31
36	Log-Logistic (3P)	0.14229	3	0.44435	2	0.44399	3
37	Log-Pearson 3	0.16815	17	0.58598	11	5.282	27

38	Logistic	0.26581	34	2.2704	29	14.533	47
39	Lognormal	0.1386	2	0.59438	12	3.0407	17
40	Lognormal (3P)	0.1513	13	0.5808	10	7.3356	36
41	Normal	0.27379	37	2.5306	36	15.109	48
42	Pareto	0.35796	47	5.2156	45	10.592	40
43	Pareto 2	0.28046	41	2.4712	34	5.9945	32
44	Pearson 5	0.1711	18	0.57811	9	5.2834	28
45	Pearson 5 (3P)	0.14958	9	0.51951	4	3.082	21
46	Pearson 6	0.15101	12	0.52439	6	3.0742	19
47	Pearson 6 (4P)	0.14976	10	0.52131	5	3.0802	20
48	Pert	0.18053	20	2.5087	35	1.2083	9
49	Power Function	0.27553	39	3.2358	40	3.8634	25
50	Rayleigh	0.23159	30	1.7839	27	3.4262	22
51	Rayleigh (2P)	0.27592	40	2.361	32	15.875	49
52	Reciprocal	0.28569	42	3.6505	42	1	8
53	Rice	0.31076	45	3.2323	39	12.191	42
54	Student's t	1	58	346.91	57	N/A	
55	Triangular	0.44424	52	8.4913	49	0.375	2
56	Uniform	0.29008	43	6.4776	47	N/A	
57	Weibull	0.14648	5	1.5178	25	0.81246	5
58	Weibull (3P)	0.18926	22	0.98555	21	10.187	37
59	Erlang (3P)	No fit					
60	Johnson SU	No fit					
61	Nakagami	No fit					

Using weibull distribution to fit the loss distribution of the original insurance, then we get the probability density distribution in the following figure1, where, $\alpha=2.0207$ $\beta=2691.5$ from table3.

Figure 1: Weibull probability density distribution of the loss of original insurance

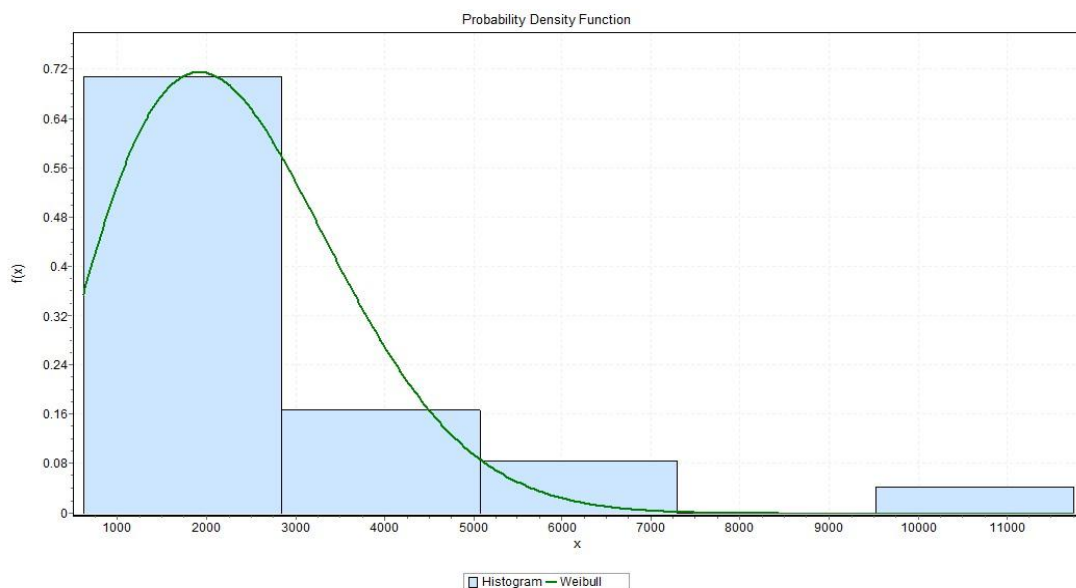


Table 3: Parameters' table of fitting results

#	Distribution	Fitting Results	
		Parameters	
1	Beta	$\alpha_1=0.59015$ $\alpha_2=2.2489$ $a=616$ $b=11752.0$	
2	Burr	$k=0.65767$ $\alpha=3.6635$ $\beta=1815.9$	
3	Burr (4P)	$k=0.26063$ $\alpha=0.72363$ $\beta=13.579$ $\gamma=616.0$	
4	Cauchy	$\sigma=476.94$ $\mu=1995.7$	
5	Chi-Squared	$v=2753$	
6	Chi-Squared (2P)	$v=1821$ $\gamma=235.55$	
7	Dagum	$k=333.96$ $\alpha=1.7955$ $\beta=26.386$	
8	Dagum (4P)	$k=209.42$ $\alpha=1.4479$ $\beta=11.675$ $\gamma=31.032$	
9	Erlang	$m=1$ $\beta=1907.2$	
10	Error	$k=1.0$ $\sigma=2291.4$ $\mu=2753.1$	
11	Error Function	$h=3.0859E-4$	
12	Exponential	$\lambda=3.6323E-4$	
13	Exponential (2P)	$\lambda=4.6792E-4$ $\gamma=616$	
14	Fatigue Life	$\alpha=0.65379$ $\beta=2273.4$	
15	Fatigue Life (3P)	$\alpha=0.7195$ $\beta=2063.1$ $\gamma=160.89$	
16	Frechet	$\alpha=1.9922$ $\beta=1589.7$	
17	Frechet (3P)	$\alpha=3.4604$ $\beta=3344.4$ $\gamma=-1558.8$	
18	Gamma	$\alpha=1.4435$ $\beta=1907.2$	
19	Gamma (3P)	$\alpha=1.2535$ $\beta=1727.3$ $\gamma=587.92$	
20	Gen. Extreme Value	$k=0.41652$ $\sigma=820.77$ $\mu=1712.0$	
21	Gen. Gamma	$k=1.1671$ $\alpha=1.6661$ $\beta=1907.2$	
22	Gen. Gamma (4P)	$k=0.43936$ $\alpha=7.7887$ $\beta=18.167$ $\gamma=433.41$	
23	Gen. Pareto	$k=0.27399$ $\sigma=1263.2$ $\mu=1013.2$	
24	Gumbel Max	$\sigma=1786.6$ $\mu=1721.8$	
25	Gumbel Min	$\sigma=1786.6$ $\mu=3784.4$	
26	Hypersecant	$\sigma=2291.4$ $\mu=2753.1$	
27	Inv. Gaussian	$\lambda=3974.2$ $\mu=2753.1$	
28	Inv. Gaussian (3P)	$\lambda=4811.8$ $\mu=2637.2$ $\gamma=115.85$	
29	Johnson SB	$\gamma=2.0721$ $\delta=0.60436$ $\lambda=19109.0$ $\xi=1255.9$	
30	Kumaraswamy	$\alpha_1=1.05$ $\alpha_2=1.15$ $a=616.0$ $b=11752.0$	
31	Laplace	$\lambda=6.1717E-4$ $\mu=2753.1$	
32	Levy	$\sigma=1868.9$	

33	Levy (2P)	$\sigma=1015.4$ $\gamma=475.62$
34	Log-Gamma	$\alpha=150.72$ $\beta=0.05115$
35	Log-Logistic	$\alpha=2.873$ $\beta=2072.3$
36	Log-Logistic (3P)	$\alpha=2.5926$ $\beta=1865.7$ $\gamma=287.1$
37	Log-Pearson 3	$\alpha=15.514$ $\beta=0.15942$ $\gamma=5.2355$
38	Logistic	$\sigma=1263.3$ $\mu=2753.1$
39	Lognormal	$\sigma=0.61469$ $\mu=7.7087$
40	Lognormal (3P)	$\sigma=0.69973$ $\mu=7.5721$ $\gamma=231.71$
41	Normal	$\sigma=2291.4$ $\mu=2753.1$
42	Pareto	$\alpha=0.77792$ $\beta=616$
43	Pareto 2	$\alpha=101.31$ $\beta=2.7312E+5$
44	Pearson 5	$\alpha=3.0033$ $\beta=5612.9$
45	Pearson 5 (3P)	$\alpha=3.9446$ $\beta=8856.9$ $\gamma=-287.53$
46	Pearson 6	$\alpha_1=12.484$ $\alpha_2=3.9451$ $\beta=642.48$
47	Pearson 6 (4P)	$\alpha_1=19.254$ $\alpha_2=3.9707$ $\beta=434.19$ $\gamma=-93.693$
48	Pert	$m=1037.5$ $a=616$ $b=11752.0$
49	Power Function	$\alpha=0.33296$ $a=616.0$ $b=12054.0$
50	Rayleigh	$\sigma=2196.7$
51	Rayleigh (2P)	$\sigma=2730.3$ $\gamma=-389.68$
52	Reciprocal	$a=616.0$ $b=11752.0$
53	Rice	$v=0.69679$ $\sigma=2511.1$
54	Student's t	$v=2$
55	Triangular	$m=739.2$ $a=616$ $b=11752.0$
56	Uniform	$a=-1215.8$ $b=6722.0$
57	Weibull	$\alpha=2.0207$ $\beta=2691.5$
58	Weibull (3P)	$\alpha=1.0426$ $\beta=2177.0$ $\gamma=611.91$
59	Erlang (3P)	No fit
60	Johnson SU	No fit
61	Nakagami	No fit