Self-Protection, Insurance, and Risk Sharing – A Case of Catastrophe Risk

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Abstract

This paper studies how the catastrophe insurance and risk sharing plans affect national agents' emission decisions. We suppose that emissions contribute to production but also increase the probability of a major natural catastrophe. Insurance and risk sharing plans reduce such a risk at their respective costs. We show that, if the probability of a catastrophe is rather small, agents always make more emissions by employing a combination of insurance and risk sharing. This is also true if agents can only share risk between each other, but less as clear when only insurance is available. In this case, emissions may decrease with insurance when the cost of insurance is high.

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1 Introduction

In the Fifth Assessment Report of IPCC, scientists announce that "it is *extremely likely* [at 95 percent confidence] that more than half of the observed increase in global average surface temperature from 1951 to 2010 was caused by the anthropogenic increase in greenhouse gas concentrations and other anthropogenic forcings together."(IPCC, 2014) And as one of the many consequences of global warming, natural catastrophes, such as hurricanes, floods and earthquakes, are predicted to happen at an increasing frequency. For example, it is shown in Swiss Re's *sigma* report through a simulation exercise that, "if the sea level raises by 0.25 meters by 2050, the probability of extreme flood losses occurring will almost double." This means, e.g., "losses from an event currently reached or exceeded only once in every 250 years (a USD 20 billion insured loss event) would be incurred about every 140 years." (Swiss Re, 2013)

These findings confirm the direct link between emissions and the riskiness of major catastrophic events. The fundamental measure to reduce this risk is of course to cut the emission of each country, which already forms the core point of discussion of all the climate treaties. At the same time, the regular tools that have been developed to fight against risks, such as insurance and risk sharing, could also play important roles in this mission. Despite the currently low market penetration of catastrophe insurances, possibly due to low awareness/confidence of consumers and the high cost to write policies, the proposal that governments should promote the private catastrophe insurance market as a complement to public aids after catastrophes seems to be receiving more and more attention (see IPCC, 2014). In the meantime, various risk sharing schemes have already been founded within groups of countries. A good example is the European Union Solidarity Fund, which was established in 2002 to provide financial assistances to member countries that suffer from major national disasters.

With these observations, we feel necessary to find out how the introduction of catastrophe insurance and risk sharing plans would affect national agents' emission decisions. A part of this question has been answered by previous studies in the more generic theories. Since emissions affect the risky catastrophic events, the optimal choice of emission fits into Ehrlich and Becker (1972)'s story of self-insurance and self-protection. The interaction of self-insurance/-protection with the market provided insurance has been studied in their original paper, and also by following researchers. However, these studies mostly focus on actuarially fair insurances. We therefore go further in the current paper by introducing costly insurance from the very beginning. We believe that this captures better the current situation in the private insurance market. According to my knowledge, this study is also the first to consider risk sharing in the same problem.

We build our story in a simplified setting with only two homogenous national agents. Each agent makes decisions for his population about the level of emission and the purchase of insurance. They also decide together about the risk sharing scheme whenever it becomes relevant. We choose a static model, where information is public, and agents cannot make contracts on the level of emissions or insurance.

We find that the impact of insurance on emission is not deterministic, but rather depending on the probability of catastrophe and the cost of insurance. Since our concern is about major natural disasters, we tend to believe that a small prior probability of catastrophe is likely to be the case. Then we are able to claim that, acquiring insurance leads to lower levels of emissions, when the cost of insurance is relatively high; but instead leads to more emissions when the cost of insurance is relatively low. Comparing to the benchmark case where agents can only conduct self-protection, if one acquires a small coverage of insurance due to its high cost, he (the national agent) will at the same time reduce emission. But if one acquires nearly full insurance because insurance is offered at a rate near the actuarially fair level, then the emission levels will instead be higher than without insurance. This reversing effect highlights how an agent pays attention to the impact of emission on the riskiness of the catastrophe, and in turn on the cost of his insurance.

We also discover that perfect risk sharing will always promote emissions. The level of emission under risk sharing is always higher than that under self-protection, but lower than that when agents can purchase full insurance. When agents can at the same time acquire insurance from the private market and share risks between each other, the two mechanisms will to some extent substitute for each other. Unfortunately, now it is difficult to reach a clearcut conclusion about their impact on emissions. The best prediction that we can make is that, for the probability of catastrophe being *very small*, emissions will be always higher than when agents can only conduct self-protection.

The rest of this paper is organized as follows. In section 2 we briefly review the related literature. The model is then introduced in section 3, followed immediately by the benchmark analysis when agents only conduct self-protection. Section 4 examines the case when insurance is introduced, and section 5 is about risk sharing. The complete story in which agents can conduct self-protection, insurance and risk sharing is explored in section 6. Finally, section 7 concludes the study.

2 Related literature

Ehrlich and Becker (1972) introduced the concepts of self-protection and self-insurance, with the two differing in whether one can invest to reduce the probability or the damage of a risky event. In the case of self-protection, they show that, when insurance and self-protection are jointly available, one will acquire full insurance at the actuarially fair rate (when possible), at the meantime choose a higher self-protection effort provided that the probability of loss is not very small.[1] The driving force behind is that now increasing the investment in self-protection will also reduce the cost of insurance.

The perspective of self-insurance/-protection turns to be highly relevant for studying the environmental issues. Out of the many works that have since followed Ehrlich and Becker, Muermann and Kunreuther (2008) are the first to introduce the externality of self-protection, which makes the direct link to the problem of emission control. Under a setup where the risky events are perfectly correlated between agents, they show that the availability of costless insurance further reduces self-protection, given that the probability of the risk is low (i.e. opposite to Ehrlich and Becker's suggestion). A solution is to restrict the scope of insurance. This urges agents to invest more in self-protection, so that the public good problem can be partially solved. The similar conclusion about the substitutability between self-protection and insurance is also reached by Lohse, Robledo and Schmidt (2012). In that paper they suppose another form of externality, with which the aggregate investment in self-protection determines the probability of each agent's hazardous event, which is independently distributed.

The current study distinguishes from the above literature by introducing costly insurance and the possibility of risk sharing, while following the same setup of externality as used by Lohse, Robledo and Schmidt (2012). In this way, we bring the study one step closer to the case of catastrophe risks.

3 The model setup and the benchmark case

We look at a simple setup where there are two homogeneous agents. An agent should be considered as a national government who makes decisions on behalf of the residents in the country. The decisions concern the level of emission, which contributes to domestic consumption and the riskiness of a catastrophe, the coverage of insurance when there is a private catastrophe insurance market, and the level of risk sharing with the other agent. We assume that agents cannot contract on the level of emissions or the coverage of insurance, whereas they decide together the risk sharing scheme in terms of transfers between each other in each state of the world. There is a risk that a major natural catastrophe happens to either agent's country. The catastrophe event is assumed to be independent across agents, but the probability that it happens is affected by the aggregate emission of both agents. The model is static.

For notations, denote the emission of agent i by e_i, with $g(e_i)$ the pollutive production, which satisfies $g' > 0$ and $g'' \leq 0$. Ehrlich and Becker (1972) assume $g(e_i) = e_i$. This will not be essential for our results. Denote the agent's initial wealth by w_i. The

[1] They show that, if utility were a quadratic function of income, the probability being larger than 1/2 is sufficient for this result.

catastrophe causes a damage measured in wealth by D, and arrives at a probability of $p(E)$, where $E = \sum_i e_i$ represents the aggregate level of pollution. We would assume $p'(E) > 0$ and $p''(E) > 0$. We set $p(0) \geq 0$.

3.1 The self-protection-only benchmark

We start by deriving a benchmark case where an agent only determines his level of emission, and there exists neither insurances nor risk sharing plans. Call this benchmark case as *Autarky* (AU) for future references. Assume an agent's welfare can be represented by an expected utility function of the following:

$$U_i = p(E)u(w + g(e_i) - D) + (1 - p(E))u\left(w + g(e_i)\right) \tag{1}$$

where the utility index $u(\cdot)$ satisfies $u(\cdot) > 0$, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$. The agent's optimal choice needs to satisfy the following first order condition:

$$U'_{AU} = g'[pu'(w + g - D) + (1 - p)u'(w + g)] - p'[u(w + g) - u(w + g - D)] = 0 \tag{2}$$

We simply assume that the second order condition $U''_{AU} < 0$ is satisfied. The solution represented by (2) is denoted by e_{AU}.

4 Self-protection and costly insurance

We now assume that catastrophe insurances are offered by private insurers in the market with a linear rate. For simplicity, we define *premium* as the amount that is paid to an insurer when no catastrophe happens to an agent, denoted by α, and *indemnity* the amount that the insurer pays to an agent when the latter suffers from a catastrophe, denoted by I The actuarially fair premium α for an indemnity I satisfies:

$$(1 - p(E))\alpha = p(E)I$$

or $\alpha = \frac{p}{1-p}I$. Different from the literature that overwhelming focuses on the relationship between self-protection and actuarially fair insurances, we base our study on the assumption that the catastrophe insurance is offered at a premium more expensive than the actuarially fair rate.[2] As Ehrlich and Becker (1972) suggest, we use a loading factor λ to capture this feature. An insurance coverage I is offered in the market at a premium of

$$\alpha = \frac{p(E)}{1 - p(E)}(1 + \lambda)I$$

with $\lambda > 0$.

[2] For example, Zanjani (2002) shows that an insurance firm charges high prices for overtaking the risk because it threatens the firm's financial solvency.

4.1 Optimal self-protection and insurance coverage

An agent's expected welfare now is the following:

$$U_i = p(E)u(w + g(e_i) - D + I_i) + (1 - p(E))u\left(w + g(e_i) - \frac{p(E)}{1 - p(E)}(1 + \lambda)I_i\right) \quad (3)$$

Whenever it does not cause confusions, we will simplify notations by using p for $p(E)$, g_i for $g(e_i)$, $\underline{u}_i$ for $u(w + g(e_i) - D + I_i)$, and $\bar{u}_i$ for $u(w + g(e_i) - \frac{p(E)}{1-p(E)}(1 + \lambda)I_i)$.

The agent chooses $\{e_i, I_i\}$ to maximize his expected welfare. The solution of this problem is given by the following first and second order conditions:

$$U'_I = p\underline{u}' - p(1 + \lambda)\bar{u}' = 0 \quad (4)$$

$$U'_e = g'[p\underline{u}' + (1 - p)\bar{u}'] - p'(\bar{u} - \underline{u}) - p'\frac{I(1 + \lambda)}{1 - p}\bar{u}' = 0 \quad (5)$$

$$U''_{II} = p\underline{u}'' + \frac{p^2(1 + \lambda)^2}{(1 - p)}\bar{u}'' < 0 \quad (6)$$

$$\begin{aligned} U''_{ee} &= g''[p\underline{u}' + (1 - p)\bar{u}'] + pg'^2\underline{u}'' + (1 - p)\bar{u}''[g' - \frac{p'(1 + \lambda)}{(1 - p)^2}I]^2 \\ &+p'[2g'(\underline{u}' - \bar{u}') + \frac{p'(1 + \lambda)}{(1 - p)^2}I\bar{u}'] - \frac{p'' - pp'' + p'^2}{(1 - p)^2}(1 + \lambda)I\bar{u}' - p''(\bar{u} - \underline{u}) < 0 \end{aligned} \quad (7)$$

$$\Sigma = U''_{II}U''_{ee} - (U_{Ie})^2 > 0 \quad (8)$$

where

$$U_{Ie} = g'p[\underline{u}'' - \bar{u}''(1 + \lambda)] + \frac{pp'(1 + \lambda)^2}{(1 - p)^2}I\bar{u}'' + p'[\underline{u}' - \bar{u}'(1 + \lambda)] \quad (9)$$

For any specific level of e_i, (4) determines the optimal choice of insurance coverage, given the cost of insurance λ, the probability of catastrophe that is determined by total production $\sum e_i$, and a nation's wealth level that is determined by e_i.[3] We may denote the choice of insurance coverage by $I(e_i, \sum e_i, \lambda)$. It is obvious that I decreases in λ, which can be directly observed in (4). This judgement is also rigorously obtained by making the total differential of (4):

$$\left[\underline{u}'' + \frac{p(1 + \lambda)^2}{1 - p}\bar{u}''\right] dI + \left[g'\underline{u}'' - g'(1 + \lambda)\bar{u}'' + \frac{p'(1 + \lambda)^2 I}{(1 - p)^2}\bar{u}''\right] de_i + \left[\frac{p(1 + \lambda)I}{1 - p}\bar{u}'' - \bar{u}'\right] d\lambda = 0$$

so that

$$dI = \frac{\bar{u}' - \frac{p(1+\lambda)I}{1-p}\bar{u}''}{\underline{u}'' + \frac{p(1+\lambda)^2}{1-p}\bar{u}''}d\lambda - \frac{g'\underline{u}'' - g'(1 + \lambda)\bar{u}'' + \frac{p'(1+\lambda)^2 I}{(1-p)^2}\bar{u}''}{\underline{u}'' + \frac{p(1+\lambda)^2}{1-p}\bar{u}''}de_i \quad (10)$$

[3]The latter two factors only take effect through $\overline{u}'$ and $\underline{u}'$.

which shows that $I'_\lambda < 0$.[4] So at any level of emission, the optimal coverage of insurance is decreasing in the cost of insurance. This implies that there is a minimum λ that renders zero insurance purchase at any level of emission. Specifically, it is $\bar{\lambda}(e_i)$ such that

$$u'(w+g-D) = u'(w+g)(1+\bar{\lambda}) \Rightarrow \bar{\lambda}(e_i) = \frac{u'(w+g-D)}{u'(w+g)} - 1$$

Here the probability of catastrophe, or the level of aggregate emission, does not play a role. One can also show that a richer nation (with a higher $w+g$) will have a smaller $\overline{\lambda}$, as long as the utility index follows non-increasing absolute risk aversion. In this case, with the same amount of loss, the ratio of marginal benefit over marginal cost of insurance is smaller for a richer nation.[5]

We conclude that, $\forall e_i$ and $\sum_i e_i$:

$$I(e_i, \sum e_i, 0) = (1-p)D, \quad I(e_i, \sum e_i, \bar{\lambda}(e_i)) = 0, \quad I'_\lambda(e_i, \sum e_i, \cdot) < 0$$

When taking a close look at the FOC of e_i, three parts are recognized in (5):

- $g'[p\underline{u}' + (1-p)\bar{u}']$, the marginal GDP contribution of increasing e_i, may be called the "production effect".
- $p'(\bar{u} - \underline{u})$, the marginal cost of increasing e_i as it increases the chance that the catastrophe loss happens, may be called the "risk effect".
- $p'\frac{I(1+\lambda)}{1-p}\bar{u}'$, the marginal cost of increasing e_i as it increases the premium for the same insurance coverage through increasing the probability of catastrophe, may be called the "insurance cost effect".

The production effect and the risk effect are equally identified in the benchmark case as shown in (2), and the insurance cost effect is new here. Through a quick comparison of (2) and (5), we can tell that introducing insurance will reduce the risk effect, because it brings the two outcomes closer to each other. Simply, the more insurance coverage is purchased, the smaller becomes the risk effect. At the same time, the insurance

[4] We can also calculate $\frac{dI(1+\lambda)}{d\lambda}$, which could help us to find if the *spending* on insurance is increasing or decreasing with λ. Unfortunately, this is not as clearcut as the choice of coverage, even holding e_i constant.

[5] One may be suspicious about this finding that a richer country is less likely to buy the same insurance. This is indeed brought by our setup. Alternatively, if we have assumed differently, the result could be reversed: e.g., 1.) we may assume that the same disaster causes more loss to a rich country than to a poor country, because the property value that is destroyed by the same disaster is larger in a rich country. Or 2.) using reference dependent preferences, the same loss may hurt different nations differently, depending on how we assume the same loss hurts differently at different reference points.

cost effect will increase with the acquired coverage of insurance.[6] But we cannot find directly how the production effect is affected by the amount of insurance purchased. Because I is a function of λ, we will look for the impact of λ on the production effect, which then indirectly determines the impact of insurance coverage. Our finding is the following:

Proposition 1. *Holding the level of emission, a lower λ, which renders a larger insurance coverage to be purchased, reduces the production effect of emission, if $u(\cdot)$ features non-increasing absolute risk aversion (i.e. $u(\cdot)$ is either CARA or DARA).*

Proof. We first combine (4) and (5) to rewrite the FOC of e_i under insurance as

$$g'\frac{1+p\lambda}{1+\lambda}\underline{u}' - p'(\bar{u}-\underline{u}) - \frac{p'I\underline{u}'}{1-p} = 0 \tag{11}$$

It can be found that given e_i, the production effect under insurance is increasing in λ, as long as $u(\cdot)$ features non-increasing absolute risk aversion. This is because:

$$\frac{d\left(\frac{1+p\lambda}{1+\lambda}\underline{u}'\right)}{d\lambda} = \frac{p-1}{(1+\lambda)^2}\underline{u}' + \frac{1+p\lambda}{1+\lambda}\underline{u}''I'_\lambda$$

By inserting I'_λ that is found in (10) and rearranging the terms, we get

$$sign[\frac{d\left(\frac{1+p\lambda}{1+\lambda}\underline{u}'\right)}{d\lambda}] = -sign[\bar{u}'\underline{u}''-\underline{u}'\bar{u}''-\frac{1+p\lambda}{1-p}I\bar{u}''\underline{u}''] = sign[\underline{RA}-\overline{RA}+\frac{1+p\lambda}{1-p}I\underline{RA}\overline{RA}]$$

where $RA = -\frac{u''}{u'}$ is the Arrow-Pratt measure of absolute risk aversion. We know that $\bar{u} > \underline{u}$ as long as there is no over insurance (which is true given that $\lambda > 0$, check (4)). So if we assume the utility index $u(\cdot)$ features non-increasing absolute risk aversion (i.e. CARA or DARA), then $\underline{RA} \geq \overline{RA}$, and $\frac{d\left(\frac{1+p\lambda}{1+\lambda}\underline{u}'\right)}{d\lambda} > 0$.

Furthermore, we have (using our conclusion about I found above):

- when $\lambda = 0$, $\frac{1+p\lambda}{1+\lambda}\underline{u}' = u'(w+g-pD) < pu'(w+g-D)+(1-p)u'(w+g)$, given that $u(\cdot)$ follows CARA or DARA, either of which implies $u'''(\cdot) > 0$;
- when $\lambda = \bar{\lambda}(e_i)$, $\frac{1+p\lambda}{1+\lambda}\underline{u}' = pu'(w+g-D)+(1-p)u'(w+g)$

Therefore we can conclude that the production effect increases with λ, or decreases with insurance because $I'_\lambda < 0$, and reaches the highest level when λ is so high that no insurance is purchased. In another word, the more insurance is purchased by an agent, the lower will be the production effect of emission.

□

[6]It is straightforward to verify that, when everything else is held constant, $\frac{dp'\frac{I(1+\lambda)}{1-p}\bar{u}'}{dI} > 0$.

When λ decreases and an agent purchases a larger insurance coverage, the two outcomes that he faces get closer to each other. Now when the worse outcome is realized, increasing emission has a reduced utility contribution through production than before, and the opposite when the better outcome is realized. The two combine to form the production effect, and with CARA or DARA utility, the former overwhelms the latter.

The impacts of λ on the other two effects, the risk effect and the insurance cost effect, can be studied in the similar manner. However, it turns out that there is no clear prediction for their signs. For the risk effect, a derivation of $p'(\bar{u} - \underline{u})$ with regard to λ returns

$$\frac{d(p'(\bar{u} - \underline{u}))}{d\lambda} = -I'_\lambda \left[\underline{u}' + \frac{p(1+\lambda)}{1-p}\bar{u}'\right] - \frac{pI}{1-p}\bar{u}'$$

in which the bracketed term indicates how the risk effect increases with λ, as less insurance coverage implies that the two outcomes tend to get farther apart from each other, hence increases the risk; the term out of the bracket reveals a countervailing impact, capturing that due to insurance becoming more expensive, the better outcome tends to move closer to the bad outcome, due to more wealth being paid for the same coverage. The net effect is then unclear.[7] Likely, for the impact of λ on the insurance cost effect, one will find

$$\frac{d\frac{p'I\underline{u}'}{1-p}}{d\lambda} = \frac{p'}{1-p}I'_\lambda\underline{u}'(1 - I\underline{RA})$$

the sign of which is again undetermined, due to the two countervailing impacts as λ increases: an agent now purchases less insurance coverage, meanwhile the expenditure of unit coverage increases.

Knowing the impact of insurance on the different marginal effects of emission is however not sufficient to tell us, whether an agent chooses more or less emission when he purchases insurances. In order to draw any insights on this question, we examine the sign of U'_e at the optimal emission level that is chosen when insurance is not available. This is done by taking (2) into U'_e, meanwhile let I be chosen to maintain (4). In this way, we would be able to claim that insurance increases emission if and only if it could be confirmed that:

$$\frac{u(w+g) - u(w+g-D)}{pu'(w+g-D) + (1-p)u'(w+g)}\frac{1+p\lambda}{1+\lambda} > \frac{u(w+g-\frac{p(1+\lambda)I}{1-p}) - u(w+g-D+I)}{u'(w+g-D+I)} + \frac{I}{1-p} \tag{12}$$

[7]What we can obtain in the end is the following expression, which is of an ambiguous sign:

$$\frac{d(p'(\bar{u} - \underline{u}))}{d\lambda} = -\frac{\bar{u}'\underline{u}'/(1-p)}{\underline{u}'' + \frac{p(1+\lambda)^2}{1-p}\bar{u}''}[1 + p(1+\lambda)I\overline{RA} - pI\underline{RA}]$$

where

$$I \text{ satisfies } : u'(w+g-D+I) = (1+\lambda)u'(w+g-\frac{p(1+\lambda)I}{1-p});$$
$$e_i \text{ satisfies } : g'[pu'(w+g-D)+(1-p)u'(w+g)] = p'[u(w+g)-u(w+g-D)].$$

(12) says that, at the optimal level of emission that is obtained when no insurance is available, if the beneficial production effect (represented by $LHS * p'u'(w+g-D+I)$) is larger than the damaging risk effect together with the insurance cost effect (represented by $RHS * p'u'(w+g-D+I)$), then an agent will want to increase emission at the meantime of purchasing an insurance.

When $\lambda = 0$, (12) becomes

$$\frac{u(w+g)-u(w+g-D)}{D} > pu'(w+g-D)+(1-p)u'(w+g) \tag{13}$$

which is basically Ehrlich and Becker (1972)'s equation (35). Since we are considering catastrophe risks – which are known to be associated with small prior p's, (13) is likely to be satisfied.[8] In words, if actuarially fair catastrophe insurance can be purchased, it is likely that agents increase emissions. To get the intuition, we restore the complete form of (13) as

$$\frac{p'[u(w+g)-u(w+g-D)]}{Dp'u'(w+g-pD)} > \frac{pu'(w+g-D)+(1-p)u'(w+g)}{u'(w+g-pD)},$$

in which the LHS is the ratio of the marginal cost of emission under no insurance (its risk effect) to that under full insurance (the zero risk effect + insurance cost effect), and the RHS is the ratio of the marginal benefit of emission under no insurance (its production effect) to that under full insurance (also the production effect). As p takes a smaller value, the production effect either with or without insurance is reduced, due to now there being a smaller chance that marginal emission will improve the outcome in the most needed case (i.e. when the catastrophe happens). The risk effect under no insurance is constant, meanwhile the insurance cost effect under full insurance is also reduced. As a result, when p is small, emission becomes more "cost efficient" to generate production under full insurance than under no insurance, so that we expect more emissions to be generated in the former case.

When $\lambda \neq 0$, (12) is rearranged into a similar form as (13):

$$\frac{u(w+g)-u(w+g-D)}{\left(\frac{\overline{u}-\underline{u}}{\underline{u}'}+\frac{I}{1-p}\right)\frac{1+\lambda}{1+p\lambda}} > pu'(w+g-D)+(1-p)u'(w+g) \tag{14}$$

Notice that:

[8] Ehrlich and Becker (1992) mention that with a quadratic utility function, $p > 1/2$ is sufficient to ensure (13) is violated. Equivalently, in that case $p < 1/2$ ensures that (13) is satisfied.

- When $\lambda = 0$, (14) reduces to (13), which guarantees the ">" sign;
- When $\lambda = \bar{\lambda}(e_i)$, (14) ends with the equal sign;
- When $\lambda \in (0, \bar{\lambda}(e_i))$, the denominator of the *LHS* features

$$\frac{d\left(\frac{\bar{u}-\underline{u}}{\underline{u}'}+\frac{I}{1-p}\right)}{d\lambda} = \frac{\underline{u}'}{\underline{u}''+\frac{p(1+\lambda)^2}{1-p}\bar{u}''}\left\{\left[\frac{\bar{u}-\underline{u}}{\underline{u}'}\frac{p(1+\lambda)}{1-p} - I\left(\frac{1}{1+p\lambda}-\frac{p}{1-p}\right)\right]\underline{RA}\right.$$
$$\left.-\left[\frac{\bar{u}-\underline{u}}{\underline{u}'}\frac{p(1+\lambda)}{1-p}+I\left(\frac{1}{1+p\lambda}-\frac{p}{1-p}\right)\frac{p(1+\lambda)}{1-p}\right]\overline{RA}+I\frac{\bar{u}-\underline{u}}{\underline{u}'}\frac{p(1+\lambda)}{1-p}\overline{RA}\,\underline{RA}\right\}$$

which is negative when $\lambda = \bar{\lambda}(e_i)$, and positive when $\lambda = 0$. So when λ increases from 0, (14) obtains a smaller *LHS* than (13), indicating a decreasing chance that the inequity being satisfied. More interestingly, when λ reduces from $\bar{\lambda}$, an agent starts to buy a little bit insurance, now (14) is violated, implying that at this low level of insurance coverage, an agent will *reduce* instead of *increase* emission.

Proposition 2. *When the loading factor is so large that agents only buy minimal insurance, their emissions are reduced with regard to the levels under no insurance. In the contrary, when the loading factor is small enough, so that agents purchases nearly full insurance, they make more emissions than when insurance is not available.*

4.2 The impact of λ

Now we examine exactly how emission decisions vary with λ. In order to pin down the sign of $de/d\lambda$, we solve simultaneously the two total differential equations that are obtained from the two respective first order conditions, (4) and (5). Specifically, based on $U'_e = 0$ and $U'_I = 0$, one gets

$$U''_{ee}de + U''_{eI}dI + U''_{e\lambda}d\lambda = 0; \quad U''_{II}dI + U''_{Ie}de + U''_{I\lambda}d\lambda = 0$$

which solve to give[9]

$$de = \frac{U''_{Ie}U''_{I\lambda} - U''_{e\lambda}U''_{II}}{U''_{ee}U''_{II} - U''^2_{Ie}}d\lambda, \quad dI = \frac{U''_{Ie}U''_{e\lambda} - U''_{I\lambda}U''_{ee}}{U''_{ee}U''_{II} - U''^2_{Ie}}d\lambda$$

By (8), the sign of $de/d\lambda$ is the same as that of $U''_{Ie}U''_{I\lambda} - U''_{e\lambda}U''_{II}$, which we call as the *signing index* of $de/d\lambda$. We have already derived U''_{II} and U''_{Ie} as in (6) and (9), and the other two cross derivatives are also easily obtained by differentiating (4) and

[9]A minor hint: even though dI and de are not variables, and they may well take different values in either of the total differentials, they must take the same value across the two differentials at any equilibrium point. This is because the equilibrium solution is the fixed point obtained by solving the two FOCs simultaneously, and the solution must satisfy both total-derivative equalities at the same time. Therefore at this point, de and dI and $d\lambda$ are equal across the two equalities.

(5) with respect to λ. We then introduce these expressions into the signing index, meanwhile imposing the two first order conditions (4) and (5). After collecting items and rearrangements, we obtain

$$U''_{Ie}U''_{I\lambda} - U''_{e\lambda}U''_{II} = \frac{p'\underline{u}'}{1+p\lambda} \cdot A(\lambda) \tag{15}$$

where

$$A(\lambda) = p(1-p)(\bar{u}-\underline{u})\left[(1-p)(\underline{RA}-\overline{RA}) + (1+p\lambda)I\underline{RA}\overline{RA}\right] - I(1-2p-p^2\lambda)[p\underline{u}'\overline{RA}+(1-p)\bar{u}'\underline{RA}]$$

The sign of $\frac{de}{d\lambda}$ is therefore the same as $A(\lambda)$, which consists of two parts. The first part of $A(\lambda)$ is always non-negative, given that we have assumed non-increasing absolute risk aversion. The second part however depends its sign on that of $1 - 2p - p^2\lambda$.

We have argued earlier that p as the probability that a catastrophe happens should take small values. We therefore tend to believe that $1 - 2p - p^2\lambda$ is positive.[10] This means that the second part of $A(\lambda)$ is non-positive, which then makes the sign of $de/d\lambda$ ambiguous. As we check the extreme values, there are:

- when $\lambda = 0$, $I = (1-p)D$, $\underline{u} = \bar{u}, \Rightarrow \frac{de}{d\lambda} < 0$;
- when $\lambda = \bar{\lambda}, I = 0, \Rightarrow \frac{de}{d\lambda} > 0$.

Since $A(\lambda)$ is continuous, we can expect that in the vicinity of $\lambda = 0$, as λ increases so that an agent cuts his optimal insurance coverage from full insurance, he will at the same time reduce his level of emission. Interestingly, when λ decreases from a high level so that an agent starts to buy some insurance, he will also reduce his emission level. Nevertheless, the level of emission at full insurance is higher than that at zero insurance.

We can imagine that, if the implicit function of λ (obtained from $A(\lambda) = 0$):

$$\lambda = \frac{1-2p}{p^2} - \frac{p(1-p)(\bar{u}-\underline{u})\left[(1-p)(\underline{RA}-\overline{RA}) + (1+p\lambda)I\underline{RA}\right]}{p^2 I[p\underline{u}'\overline{RA} + (1-p)\bar{u}'\underline{RA}]}$$

has only one solution λ^* within the range of $(0, \bar{\lambda})$, then it would be

$$\frac{de}{d\lambda} < 0, \text{ if } \lambda < \lambda^*; \text{ and } \frac{de}{d\lambda} > 0, \text{ if } \lambda^* < \lambda < \bar{\lambda}$$

In this case, we would be able to claim that

[10] In contrast, Ehrlich and Becker (1972) allow p to take large values, based on which they predict that there will be less emission (more pollution abatement as in their original text) under full insurance than under no insurance. In their case, it is also likely that $1 - 2p - p^2\lambda < 0$. Then $A(\lambda)$ would have an unambiguously positive sign, which would lead to the conclusion that $\frac{de}{d\lambda} > 0$. Then as λ increases from 0 to $\bar{\lambda}$, the insurance coverage would decrease from $(1-p)D$ to 0, meanwhile the level of emission would constantly increase, and reach its highest level when no insurance is purchased.

Proposition 3. *If $A(\lambda) = 0$ has only one solution within the range of $(0, \bar{\lambda})$, then in the value range of λ that allows nearly full insurance, an agent makes more emission as he acquires more insurance. In the contrary, in the value range of λ that implies low insurance coverages, one will cut his emission when acquiring more insurance. The level of emission is higher under full insurance than under no insurance, but the minimum emission is realized when the optimal insurance is somewhere between the two extremes.*

5 Self-protection and risk sharing

In this section, we introduce risk sharing between agents and exclude market insurances for a moment. A risk sharing scheme is a transfer plan $\{\alpha_i^s\}$ that defines the transfers between agents in any state of the world, denoted by s. Here $s \in S = \{0,1\} \times \{0,1\}$, where we use 0 to indicate that no catastrophe happens to an agent and 1 that a catastrophe happens. There are in total four possible state of the world, with $\{s\} = \{00, 10, 01, 11\}$. To give an example, $(\alpha_1^{01}, \alpha_2^{01})$ represent a transfer of value α_1^{01} to agent 1 and α_2^{01} to agent 2, when only agent 2 receives a catastrophe.

We assume that risk sharing is perfect, so that the agents make state-contingent transfers to each other in any realized outcome, with the transfer scheme chosen to maximize the expected aggregate welfare. The risk sharing problem is therefore:

$$\begin{aligned} Max_{\{\alpha_i\}} \quad & p_1 p_2[u(w + g_1 - D + \alpha_1^{11}) + u(w + g_2 - D - \alpha_1^{11})] \\ + \quad & p_1(1 - p_2)[u(w + g_1 - D + \alpha_1^{10}) + u(w + g_2 - \alpha_1^{10})] \\ + \quad & (1 - p_1)p_2[u(w + g_1 + \alpha_1^{01}) + u(w + g_2 - D - \alpha_1^{01})] \\ + \quad & (1 - p_1)(1 - p_2)[u(w + g_1 + \alpha_1^{00}) + u(w + g_2 - \alpha_1^{00})] \end{aligned} \tag{16}$$

In which we have incorporated the statewise budget constraints:

$$\alpha_1^s + \alpha_2^s = 0$$

The first order conditions with regard to each α_1 return:

$$1 = \frac{u'(w + g_1 - D + \alpha_1^{11})}{u'(w + g_2 - D - \alpha_1^{11})} = \frac{u'(w + g_1 - D + \alpha_1^{10})}{u'(w + g_2 - \alpha_1^{10})} = \frac{u'(w + g_1 + \alpha_1^{01})}{u'(w + g_2 - D - \alpha_1^{01})} = \frac{u'(w + g_1 + \alpha_1^{00})}{u'(w + g_2 - \alpha_1^{00})}$$

By assuming symmetry, we have:

$$\alpha_1^{11} = \alpha_1^{00} = 0, \alpha_1^{10} = \frac{D}{2}, \alpha_1^{01} = -\frac{D}{2} \tag{17}$$

That is, a scheme always equally redistributes wealth between agents in any outcome.

Now agent 1 solves the following problem (for agent 2 the problem is analogous):

$$\max_{e_1} p_1 p_2 u(w + g - D) + [p_1(1 - p_2) + (1 - p_1)p_2]u(w + g - \frac{D}{2}) + (1 - p_1)(1 - p_2)u(w + g)$$

Assuming the second order condition is satisfied[11], the optimal level of emission must satisfy the following first order condition:

$$
\begin{aligned}
& g'(e)\left[p_1p_2u'(w+g-D)+[p_1(1-p_2)+p_2(1-p_1)]u'(w+g-\frac{D}{2})+(1-p_1)(1-p_2)u'(w+g)\right] \\
- \;& p_1'\left[(1-p_2)u(w+g)-(1-2p_2)u(w+g-\frac{D}{2})-p_2u(w+g-D)\right] \\
- \;& p_2'\left[(1-p_1)u(w+g)-(1-2p_1)u(w+g-\frac{D}{2})-p_1u(w+g-D)\right]=0 \qquad (18)
\end{aligned}
$$

where, disregard of symmetry between agents, we have maintained the subscripts for probabilities. This is only to highlight that an agent's concern of emission indeed includes three parts:

- the *production effect*, as in the first line leading by g';
- the *risk effect*, as in the second line leading by p_1', representing the marginal effect of e_i through affecting an agent's own probability of catastrophe;
- the *shared risk effect*, as in the third line leading by p_2', showing that the agent recognizes how his emission decision affects the other agent's probability of catastrophe, which in turn affects his own welfare due to the risk sharing mechanism.

We now impose symmetry between agents and simplify (18) into

$$
\begin{aligned}
U'_{RS} &= g'[p^2\underline{u}'+2p(1-p)\tilde{u}'+(1-p)^2\bar{u}'] \\
&\quad -2p'[(1-p)\bar{u}-(1-2p)\tilde{u}-p\underline{u}]=0 \qquad (19)
\end{aligned}
$$

where the notations are $\bar{u}=u(w+g)$, $\underline{u}=u(w+g-D)$, $\tilde{u}=u(w+g-\frac{D}{2})$. And the subscript RS indicates risk sharing.

In order to find if agents make more or less emission when they can share the risk with each other, we examine again the sign of U'_{RS} at the emission level when agents behave alone, the first order condition in which case has been given earlier in (2). Subtract U'_{AU} from U'_{RS} (using the (18) version) returns:

$$
\begin{aligned}
\Delta FOC &= g'\left[p_1(1-p_2)(\tilde{u}'-\underline{u}')+p_2(1-p_1)(\tilde{u}'-\bar{u}')\right] \\
&- p_1'[p_2(\tilde{u}-\bar{u})+(1-p_2)(\underline{u}-\tilde{u})] \\
&- p_2'[p_1(\tilde{u}-\underline{u})+(1-p_1)(\bar{u}-\tilde{u})] \qquad (20)
\end{aligned}
$$

We realize that risk sharing:

- reduces the production effect of emission, as is indicated by the negative first line of ΔFOC. This becomes obvious after imposing symmetry between p_1 and p_2, and using $u''' > 0$ that is implied by non-increasing absolute risk aversion. The benefit of increasing emission is hence reduced;

[11] In the appendix we show how restrictive this SOC assumption is.

- reduces the risk effect of emission, as is represented by the positive second line. This is because under risk sharing, the wealth gap between catastrophe and no catastrophe is sometimes eliminated, thanks to the transfer scheme. The cost of increasing emission is hence also reduced;

- introduces a new cost of increasing emission, as is captured by the negative shared risk effect in the third line.

These three impacts of risk sharing correspond to the three impacts of insurance that we have earlier looked at. This similarity between the two schemes is not a surprising finding as we recall that risk sharing is often referred to as mutual insurance.

When we suppress the subscripts under symmetry, (20) rewrites into

$$\Delta FOC = g'p(1-p)[2\tilde{u}' - (\underline{u}' + \bar{u}')] - p'(1-2p)[\underline{u} + \bar{u} - 2\tilde{u}]$$

which is the first order derivative with regard to e of the following value function:

$$V(e) := p(1-p)\left[2\tilde{u} - (\underline{u} + \bar{u})\right] \tag{21}$$

(21) is positive for $u(\cdot)$ being concave. This $V(e)$ signifies the positive expected gain from joining the risk sharing scheme for each individual agent (i.e. $V(e) = U_{RS} - U_{AU}$). Obviously, at the optimal level of emission under autarky, all the first order effects of varying emission is captured by the variation of $V(e)$.[12] Suppose $V(e)$ is increasing at this point (i.e. $\Delta FOC > 0$), then one will want to increase his level of emission, whereas the reverse is true if $V(e)$ is decreasing at this point.[13]

Replace g' in ΔFOC by $p'\frac{\bar{u}-\underline{u}}{p\underline{u}'+(1-p)\bar{u}'}$ according to $U'_{AU} = 0$, we can find that

$$\begin{aligned} & sign[\Delta FOC] \\ = \; & sign\left[p(1-p)(\bar{u} - \underline{u})[2\tilde{u}' - (\underline{u}' + \bar{u}')] - (1-2p)[p\underline{u}' + (1-p)\bar{u}'][\underline{u} + \bar{u} - 2\tilde{u}]\right] \end{aligned}$$

This new expression to be signed is a continuous and quadratic function in p. More specifically, there are

$$\Delta FOC > 0, \text{ when } p = 0; \quad \Delta FOC < 0, \text{ when } p \geq \frac{1}{2}$$

[12] Because now $U'_{AU} = 0$, a marginal emission has no first order impact on U_{AU}.

[13] Explicitly writing down $V(e)$ may facilitates our analyses in the cases where there are more than two agents. Based on our findings about the regularity of risk sharing, the gain from risk sharing increases with n, and we believe that the gain can be written in the form of $V_2(e) + V_3(e) + ...V_n(e)$, where $V_2(e)$ is the one we found here. Like in the 2 agents case, an agent then chooses his e to maximize all these $V's$.

We can be sure that there exits a unique $p^* \in (0, \frac{1}{2})$, which satisfies $\Delta FOC(p^*) = 0$, so that

$$\Delta FOC > 0, \text{ when } p < p^*; \quad \Delta FOC \leq 0, \text{ when } p \geq p^*$$

The sign of ΔFOC allows us to tell if agents make more or less emissions when the risk sharing scheme is implemented. This is concluded in the following proposition:

Proposition 4. *When agents share risk perfectly between each other, their level of emission may be higher or lower than that when the risk sharing scheme is not available. Roughly speaking, for p small enough, risk sharing encourages emission. The opposite would be true when p takes relatively large values.*[14]

And a quick comparison to the full insurance case reveals that, under perfect risk sharing, the two agents' emissions are still below that under full insurance. This is easily shown by introducing $U'_e = 0$, at the point of $\lambda = 0$, into $U'_{_{RS}}$, and finding that $U'_{_{RS}} < 0$ because at this level of emission the following condition is satisfied, given p is small:

$$(1-p)[p\tilde{u}' + (1-p)\bar{u}'] + p[p\underline{u}' + (1-p)\tilde{u}'] < (1-p)\frac{\bar{u} - \tilde{u}}{D/2} + p\frac{\tilde{u} - \underline{u}}{D/2}$$

6 Self-protection, insurance, and risk sharing in one problem

We now allow insurance and risk sharing to be chosen at the same time. The timing is the following:

- agents talk with each other to determine the risk sharing scheme;
- Each agent chooses own level of emission and insurance coverage, taking into consideration of the risk sharing scheme;
- Nature throws the dice, catastrophes happen or not to either agent, and transfers as well as insurance indemnities are implemented.

In this setting, the price of insurance is assumed to be not affected by risk sharing. The argument is that catastrophe insurance is usually index-based and purchased by individual families. The indemnity is paid when the catastrophe happens, whereas if the nation receives transfers from the other nation shall not affect directly the compensation that individual insured families receive.

[14]Interestingly, we again find that, if one follows Ehrlich and Becker (1972)'s argument that p is not too small, he will be able to find unambiguous conclusion that risk sharing always reduces emission, just like catastrophe insurance does. Assuming p to be not too small seems to be a technically easier scenario for this kind of study.

On the other hand, the equilibrium risk sharing scheme takes full consideration of the emission and insurance that each agent chooses in the equilibrium. We simply assume that all the choices are public information. Our argument is, in the case of catastrophes, the development of a nation's insurance market is sometimes published information, especially for developed countries. For this reason, even though the amount of insurance that an individual person holds may well be hidden information to the other individual, we do not assume this in the current study with national agents.

The problem is solved backwardly. First the nations determine together the risk sharing scheme. Given e_i and I_i, they choose $\{\alpha_i^s\}$. Equivalently, we solve the problem by letting them choose c_i^s.

$$\begin{aligned}
\max_{\{c_i\}} \quad & p_1p_2[u_1(c_1^{11})+u_2(c_2^{11})]+p_1(1-p_2)[u_1(c_1^{10})+u_2(c_2^{10})] \\
& +(1-p_1)p_2[u_1(c_1^{01})+u_2(c_2^{01})]+(1-p_1)(1-p_2)[u_1(c_1^{00})+u_2(c_2^{00})] \\
s.t. \quad & c_1^{11}+c_2^{11} \leq w+g(e_1)-D+I_1+w+g(e_2)-D+I_2 \\
& c_1^{10}+c_2^{10} \leq w+g(e_1)-D+I_1+w+g(e_2)-\frac{p(1+\lambda)}{1-p}I_2 \\
& c_1^{01}+c_2^{01} \leq w+g(e_1)-\frac{p(1+\lambda)}{1-p}I_1+w+g(e_2)-D+I_2 \\
& c_1^{00}+c_2^{00} \leq w+g(e_1)-\frac{p(1+\lambda)}{1-p}I_1+w+g(e_2)-\frac{p(1+\lambda)}{1-p}I_2
\end{aligned}$$

We get:

$$c_1^{11}=c_2^{11}=w+g-D+\tfrac{1}{2}(I_1+I_2), \qquad c_1^{10}=c_2^{10}=w+g-\frac{D}{2}+\frac{1}{2}(I_1-\frac{p+p\lambda}{1-p}I_2),$$

$$c_1^{01}=c_2^{01}=w+g-\tfrac{D}{2}+\tfrac{1}{2}(I_2-\tfrac{p+p\lambda}{1-p}I_1), \qquad c_1^{00}=c_2^{00}=w+g-\frac{1}{2}\frac{p(1+\lambda)}{1-p}(I_1+I_2)$$

Now an agent chooses his insurance coverage given $\{c_i^s\}$. We show agent 1's maximization problem, keeping in mind that agent 2's is symmetric[15].

$$\begin{aligned}
\max_{I_1} \quad & p^2u(w+g-D+\frac{1}{2}(I_1+I_2))+p(1-p)u(w+g-\frac{D}{2}+\frac{1}{2}\left(I_1-\frac{p(1+\lambda)}{1-p}I_2\right)) \\
+ \quad & p(1-p)u(w+g-\frac{D}{2}+\frac{1}{2}\left(I_2-\frac{p(1+\lambda)}{1-p}I_1\right))+(1-p)^2u(w+g-\frac{1}{2}\frac{p(1+\lambda)}{1-p}(I_1+I_2))
\end{aligned}$$

After obtaining the first order conditions, we impose symmetry between I_1 and I_2 to have:

$$\begin{aligned}
& p\left[u'(w+g-D+I)-(1+\lambda)u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p})\right] \\
& +(1-p)\left[u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p})-(1+\lambda)u'(w+g-\frac{p(1+\lambda)}{1-p}I)\right]=0
\end{aligned} \tag{22}$$

[15]Note that here it is again important to distinguish I_1 from I_2.

In this equity, the first line captures the distance between marginal utilities of the 11 and 01 case, and the second line that of the 10 and 00 case. Using contradiction we can show that:

$$w + g - D + I \leq w + g - \frac{D}{2} + \frac{I}{2}\frac{1-2p-p\lambda}{1-p} \leq w + g - \frac{p(1+\lambda)}{1-p}I, \tag{23}$$

with the two equal signs only take at the same time. In the appendix, we show that if $u(\cdot)$ features non-increasing absolute risk aversion, there are two possibilities for the optimal insurance coverage, each satisfying either:

$$\begin{aligned} u'(w+g-D+I) &= (1+\lambda)u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p}) \\ &= (1+\lambda)^2 u'(w+g-\frac{p(1+\lambda)}{1-p}I) \end{aligned} \tag{24}$$

if $u(\cdot)$ satisfies CARA, or the combination of (22) together with

$$\begin{aligned} u'(w+g-D+I) &> (1+\lambda)u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p}); \\ u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p}) &< (1+\lambda)u'(w+g-\frac{p(1+\lambda)}{1-p}I) \end{aligned} \tag{25}$$

if $u(\cdot)$ satisfies DARA.

If $\lambda = 0$, (24) becomes the only candidate of solution, which leads to $I = (1-p)D$. In this case, full insurance is acquired at the actuarially fair rate, and agents have the same wealth in any state of outcome before risk sharing is carried out. In consequence, risk sharing is not relevant.

When $\lambda > 0$, the equal signs in (23) cannot be taken (otherwise (22) is violated), therefore there is $I < (1-p)D$, i.e. partial insurance is acquired. In this case, we return to the optimal risk sharing scheme to find that $\{\alpha_i^s\}$ involves:

$$\alpha_1^{11} = \alpha_1^{00} = 0, \quad \alpha_1^{10} = -\alpha_1^{01} = \frac{D}{2} - \frac{1}{2}\frac{1+p\lambda}{1-p}I > 0$$

Furthermore, it can be shown that $u'(w+g-D+I) > (1+\lambda)u'(w+g-\frac{p(1+\lambda)}{1-p}I)$. When comparing this to (4), where an equal sign is reached when agents can only purchase insurance, we can conclude that risk sharing leads to a smaller coverage of insurance being purcahsed.[16]

[16]To show that $\underline{u}' > (1+\lambda)\overline{u}'$, use contradictions. Suppose instead $\underline{u}' \leq (1+\lambda)\overline{u}'$. Then replace $(1+\lambda)\overline{u}'$ by $\underline{u}'$ in (22) to have: $(1-2p)\underline{u}' \leq (1-2p-p\lambda)\tilde{u}'$, where $\tilde{u}' = u'(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p})$. This reaches a contradiction, because $\underline{u}' > \tilde{u}'$, and p is assumed to be small so that $(1-2p) > (1-2p-p\lambda) > 0$.

Meanwhile the minimum λ that leads to zero insurance purchase satisfies [17]:

$$\bar{\lambda}(e_i)_{RI} = \frac{pu'(w+g-D) + (1-p)u'(w+g-D/2)}{pu'(w+g-D/2) + (1-p)u'(w+g)} - 1 \tag{26}$$

Compare to $\bar{\lambda}(e_i)$ found for the insurance-only case, we have $\bar{\lambda}_{RI}(e_i) < \bar{\lambda}(e_i)$, because for $u(\cdot)$ following either CARA or DARA, there are:

$$\bar{\lambda}_{RI}(e_i) - \bar{\lambda}(e_i) = \frac{p\underline{u}'[\bar{u}' - \tilde{u}'] + (1-p)\bar{u}'[\tilde{u}' - \underline{u}']}{p\tilde{u}'\bar{u}' + (1-p)\bar{u}'^2} < 0$$

where $\bar{u} = u(w+g)$, $\tilde{u} = u(w+g-D/2)$, $\underline{u} = u(w+g-D)$. In words, holding everything else equal, introducing risk sharing reduces the chance that an insurance is purchased. Together with the above finding that introducing risk sharing also reduces the acquired insurance coverage, we claim that risk sharing and insurance are substitutes, when insurance is costly.

We now look at the agent's emission decisions. Denote now: $\bar{u} = u(w+g-\frac{p(1+\lambda)}{1-p}I)$, $\underline{u} = u(w+g-D+I)$, $\tilde{u} = u(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p})$, it is necessary to have:

$$U'_{RIe} = \left[p^2\underline{u}'g' + 2p(1-p)\tilde{u}'(g' - \frac{1}{2}\frac{p'(1+\lambda)I}{(1-p)^2}) + (1-p)^2\bar{u}'(g' - \frac{p'(1+\lambda)I}{(1-p)^2})\right]$$
$$-2p'\left[(1-p)\bar{u} - (1-2p)\tilde{u} - p\underline{u}\right] = 0$$

where subscript RI indicates risk sharing and insurance. Above can be rewritten into the form similar to what we have before:

$$\begin{aligned} & g'[p^2\underline{u}' + 2p(1-p)\tilde{u}' + (1-p)^2\bar{u}'] \\ & - \left[p\frac{p'(1+\lambda)I}{1-p}\tilde{u}' + (1-p)\frac{p'(1+\lambda)I}{1-p}\bar{u}'\right] \\ & -2p'\left[(1-p)\bar{u} - (1-2p)\tilde{u} - p\underline{u}\right] = 0 \end{aligned} \tag{27}$$

Comparing (27) to (5) and (19), we can see that (27) is a kind of combination of the other two – for example, with regard to (19), now the second line represents an extra cost from insurance.

We first test if risk sharing may again push emissions high, after agents have already acquired insurances.[18] Following the method used in the risk-sharing-only section, we

[17] This is obtained by solving (22) with regard to λ, holding $I = 0$.

[18] Here we have no clear prediction to start with. Because risk sharing makes agents acquire less insurance, and we have discovered that less insurance may cause emissions to either increase or decrease, depending on how costly the insurance is. Even though risk sharing may unambiguously increase emission again, it is hard to tell if the final level of emission is below or above that before risk sharing is imposed.

subtract $U'_e = 0$ (i.e. (5)) from U'_{RIe} and obtain

$$\Delta FOC = g'p(1-p)[2\tilde{u}' - \underline{u}' - \bar{u}'] - p'\left[\frac{p(1+\lambda)I}{1-p}(\tilde{u}' - \bar{u}')\right] - p'(1-2p)[\bar{u} + \underline{u} - 2\tilde{u}] \quad (28)$$

This is the first order derivative to e_i of the risk sharing value function $V_{RI}(e)$:

$$V_{RI}(e) := p(1-p)\left[2\tilde{u} - (\underline{u} + \bar{u})\right] \quad (29)$$

The difference between (28) and (20) originates from the fact that now both $\bar{u}$ and $\tilde{u}$ are functions of $p(E)$. To find the sign of ΔFOC, we notice that it consists of three parts that have clear signs each:

- $g'p(1-p)[2\tilde{u}' - \underline{u}' - \bar{u}'] < 0$, by the non-increasing absolute risk aversion assumption.
- $-p'\left[\frac{p(1+\lambda)I}{1-p}(\tilde{u}' - \bar{u}')\right] < 0$;
- $-p'(1-2p)[\bar{u} + \underline{u} - 2\tilde{u}] > 0$, when $p < \frac{1}{2}$.

We can clearly see that $\Delta FOC < 0$ for $p > 1/2$. That is, if p were large, allowing risk sharing would reduce emissions, if agents already acquire insurance by themselves. This is similar to the case when insurance is not available. However, for $p < 1/2$, it becomes difficult to reach a clearcut conclusion about how the sign of ΔFOC varies with p. We only observe that, for $p \in (0, 1/2)$:

- $\Delta FOC > 0$ for $p = 0$;
- $\Delta FOC < 0$ for $p = 1/2$.

And since ΔFOC is continuous in p, we feel safe to claim that, when p is very small, introducing risk sharing will increase emission, when agents have already purchased catastrophe insurances.

We now study the combined impact of insurance and risk sharing on emissions. As we have done couple of times, if we find U'_{RIe} positive at the optimal level of emission in the autarky, then the combined mechanism of insurance and risk sharing promotes emission. Otherwise, the reverse would be true.

For notations, we now use $u(w+g) = \bar{u}_0$ and $u(w+g-D) = \underline{u}_0$. $\bar{u}, \underline{u}$ and $\tilde{u}$ follow previous definition under the combined scheme. We obtain, after some arrangements,

$$\begin{aligned}
\Delta FOC &= g'[p^2(\underline{u}' - \underline{u}'_0) + p(1-p)(2\tilde{u}' - \underline{u}'_0 - \bar{u}'_0) + (1-p)^2(\bar{u}' - \bar{u}'_0)] \\
&- p'\left[p\frac{(1+\lambda)I}{1-p}\tilde{u}' + (1-p)\frac{(1+\lambda)I}{1-p}\bar{u}'\right] \\
&- p'[2(1-p)(\bar{u} - \bar{u}_0) - (1-2p)(2\tilde{u} - \bar{u}_0 - \underline{u}_0) - 2p(\underline{u} - \underline{u}_0)]
\end{aligned}$$

Then we replace g' by $p'\frac{\bar{u}_0-\underline{u}_0}{p\underline{u}_0'+(1-p)\bar{u}_0'}$ according to $U'_{AU}=0$. We finally get

$$\begin{aligned}\Delta FOC &= \frac{p'}{p\underline{u}_0'+(1-p)\bar{u}_0'}\left\{[p^2\underline{u}'+2p(1-p)\tilde{u}'+(1-p)^2\bar{u}'](\bar{u}_0-\underline{u}_0)\right. \\ &- \left.(p\underline{u}_0'+(1-p)\bar{u}_0')\left[2p(\tilde{u}-\underline{u})+2(1-p)(\bar{u}-\tilde{u})+\frac{I}{1-p}[p\underline{u}'+(1-p)\tilde{u}']\right]\right\}\end{aligned} \tag{30}$$

It can be checked that:

- When $I=0$, $\Delta FOC>0$ for small p values. This is the previous case of only risk sharing;
- When $I=(1-p)D$, $\Delta FOC>0$ for small p values. This is the previous case of full insurance case.

Furthermore, we can also confirm that

- When p is very small, $\frac{d\Delta FOC}{dI}>0$.

Recall that we have found previously that the emission under full insurance is higher than that under risk sharing. We can now firmly claim that:

Proposition 5. *When both risk sharing and insurance are available, agents always make more emissions than in the benchmark case, provided that p is small enough.*

7 Conclusions

In this study we have looked at how emissions are affected, when national agents have the opportunities to acquire catastrophe insurance from the private market, or/and to share the catastrophe risk between themselves. We assume that an agent's emission contributes to his consumption because it is productive, but at the same time also increases the probability that a catastrophe happens to any other agents. An agent then have three tools to reduce the catastrophe risk, namely reducing emission, purchasing insurance, and sharing the risk with the other agent.

Insurance and risk sharing are similar mechanisms in the sense that they both affect agents' emission decisions in three aspects. They reduce emission's impact on the riskiness of catastrophe, reduce emission's marginal contribution to production, and meanwhile generate a cost of insurance/risk sharing that is directly related to the catastrophe risk. Nevertheless, the two mechanisms are distinguished in practice due to their extent of availability (including the level of costs) and in turn their different effectiveness. Specifically, in the current study we allow insurance to be costly by introducing a non-negative loading factor to the premium, and on the other hand we assume perfect risk sharing between agents. At the meantime, we suppose the probability that a natural catastrophe happens is fairly low, even after considering the impact of emissions. Based on these assumptions, we reach the following findings.

Firstly, introducing insurance may either cause emissions to increase or to decrease, depending on the cost of insurance. Specifically, when the cost is high, acquiring a bit insurance will reduce emission; whereas when the cost is low, acquiring insurance makes the emission increase. In the extreme, when insurance can be acquired at the actuarially fair rate (i.e. at zero cost), an agent chooses full insurance, and he makes more emission than in the case when insurance is not available. In the contrary, perfect risk sharing always pushes higher the emission levels, although the emission level under perfect risk sharing is somewhere below that under full insurance. And when the two mechanisms are available at the same time, agents will always employ both, as long as the insurance is costly, and the two shall be considered as substitute for each other. Furthermore, if the probability of catastrophe is close to zero, we can predict with confidence that emission will be unambiguously higher than when neither mechanism is available. The maximal level of emission is reached only when the agents acquire full insurance at the actuarially fair rate.

Having found these results, we also recognize the limits of the current study due to its simplified setup. To list some, we have only two agents who may share risk between each other. However in practice, a risk sharing group easily consists of more members. We also force complete symmetry between agents, however differences in wealth as well as in catastrophe risks are more common among countries. Both these characters would make the risk sharing arrangement more complicated. We also assume that all information is public, which is probably ideal concerning the insurance that each agent acquires. Some of these observations, we believe, could make interesting extensions of the current study in the future.

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Appendices

A The $SOC < 0$ condition for risk sharing

First we reproduce the FOC of the RS problem:

$$\begin{aligned} & g'(e)\left[p_1p_2u'(w+g-D)+[p_1(1-p_2)+p_2(1-p_1)]u'(w+g-\frac{D}{2})+(1-p_1)(1-p_2)u'(w+g)\right] \\ = \;& p_1'\left[p_2u_1(w+g-\frac{D}{2})+(1-p_2)u_1(w+g)-p_2u(w+g-D)-(1-p_2)u(w+g-\frac{D}{2})\right] \\ + \;& p_2'\left[p_1u_1(w+g-\frac{D}{2})+(1-p_1)u_1(w+g)-p_1u(w+g-D)-(1-p_1)u(w+g-\frac{D}{2})\right] \end{aligned} \tag{31}$$

The SOC is formed by the following entries, after some reorganization (symmetry is only applied after making the derivative):

$$\begin{aligned} SOC = \;& u''(w+g-D)g'^2p^2+u''(w+g-\frac{D}{2})g'^2 2p(1-p)+u''(w+g)g'^2(1-p)^2 \\ & +g''\left[p^2u'(w+g-D)+2p(1-p)u'(w+g-\frac{D}{2})+(1-p)^2u'(w+g)\right] \\ & +4p'g'\left[p\left(u'(w+g-D)-u'(w+g-\frac{D}{2})\right)+(1-p)\left(u'(w+g-\frac{D}{2})-u'(w+g)\right)\right] \\ & +2p''\left[p\left(u(w+g-D)-u(w+g-\frac{D}{2})\right)+(1-p)\left(u(w+g-\frac{D}{2})-u(w+g)\right)\right] \\ & +2p'^2\left[u(w+g-D)+u(w+g)-2u(w+g-\frac{D}{2})\right] \end{aligned}$$

It is very clear that, except for the 3rd line, all the other four lines are negative. Our assumption that $SOC < 0$ is therefore a restriction about $u'(\cdot)$ being not too different from each other, so that the 3rd line is not dominating all the other entries.

B Analyzing the optimal condition for insurance coverage under risk sharing

In this section we show that, when agents can both purchase insurance and share risk between each other, the optimal insurance coverage satisfies the first order condition

$$p[\underline{u}'-(1+\lambda)\tilde{u}']+(1-p)[\tilde{u}'-(1+\lambda)\bar{u}']=0 \tag{32}$$

with

- either: $\underline{u}'=(1+\lambda)\tilde{u}'=(1+\lambda)^2\bar{u}'$;

- or: $\underline{u}' > (1+\lambda)\tilde{u}'$ and $\tilde{u}' < (1+\lambda)\bar{u}'$,

where $\underline{u} = u(w+g-D+I)$, $\tilde{u} = u(w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p})$, $\bar{u} = u(w+g-\frac{p(1+\lambda)}{1-p}I)$.

To start, recognize that (32) can accommodate the following three possibilities:

1.) $\underline{u}' = (1+\lambda)\tilde{u}' = (1+\lambda)^2\bar{u}'$

2.) $\underline{u}' > (1+\lambda)\tilde{u}'$; and $\tilde{u}' < (1+\lambda)\bar{u}'$

3.) $\underline{u}' < (1+\lambda)\tilde{u}'$; and $\tilde{u}' > (1+\lambda)\bar{u}'$

To study which of the three cases are possible, we first rewrite everything in λ. Use (32) to get:

$$\lambda = \frac{p\underline{u}' - (1-p)\bar{u}' + (1-2p)\tilde{u}'}{p\tilde{u}' + (1-p)\bar{u}'}$$

And the three cases rewrite respectively into:

$$1.)\lambda = \frac{\underline{u}' - \tilde{u}'}{\tilde{u}'} = \frac{\tilde{u}' - \bar{u}'}{\bar{u}'} \Rightarrow \underline{u}'\bar{u}' = \tilde{u}'^2, \text{ or } \frac{\bar{u}'}{\tilde{u}'} = \frac{\tilde{u}'}{\underline{u}'};$$
$$2.)\lambda \in \left(\frac{\tilde{u}' - \bar{u}'}{\bar{u}'}, \frac{\underline{u}' - \tilde{u}'}{\tilde{u}'}\right) \Rightarrow \underline{u}'\bar{u}' > \tilde{u}'^2, \text{ or } \frac{\bar{u}'}{\tilde{u}'} > \frac{\tilde{u}'}{\underline{u}'};$$
$$3.)\lambda \in \left(\frac{\underline{u}' - \tilde{u}'}{\tilde{u}'}, \frac{\tilde{u}' - \bar{u}'}{\bar{u}'}\right) \Rightarrow \underline{u}'\bar{u}' < \tilde{u}'^2, \text{ or } \frac{\bar{u}'}{\tilde{u}'} < \frac{\tilde{u}'}{\underline{u}'}.$$

It is not necessary to bring λ from (32) into the three ranges for checking, because by construction the value always fits into the ranges. To proceed, recognize that:

$$\int_w^x -\frac{u''(c)}{u'(c)}dc = -\log u'(c)|_w^x = -\log\frac{u'(x)}{u'(w)}$$

Therefore if 2.) is true, then

$$\frac{\bar{u}'}{\tilde{u}'} > \frac{\tilde{u}'}{\underline{u}'} \iff -\log\frac{\bar{u}'}{\tilde{u}'} < -\log\frac{\tilde{u}'}{\underline{u}'} \iff \int_{c2}^{c1} -\frac{u''(c)}{u'(c)}dc < \int_{c3}^{c2} -\frac{u''(c)}{u'(c)}dc$$

where $c1 = w+g-\frac{p(1+\lambda)}{1-p}I$, $c2 = w+g-\frac{D}{2}+\frac{I}{2}\frac{1-2p-p\lambda}{1-p}$, $c3 = w+g-D+I$, with $c1 > c2 > c3$ and $c1-c2 = c2-c3$. The above inequity can therefore be considered as the integral (summation) of infinitely many pairwise comparisons between the coefficient of absolute risk aversion at a higher consumption level to that at a lower consumption level. When we follow our assumption that $u(\cdot)$ is of non-increasing absolute risk aversion, we must then have

$$\frac{\bar{u}'}{\tilde{u}'} \geq \frac{\tilde{u}'}{\underline{u}'} \tag{33}$$

That is, case 1.) and 2.).

C Risk sharing in a large group

First we show how the conclusion of proposition **??** is obtained. An agent's expected utility in an $n-$sized group is

$$
\begin{aligned}
U_i(n) &= \sum_{k=0}^{n} C_n^k p^k (1-p)^{n-k} u(w+g-\frac{k}{n}D) \\
&= C_n^0 p^0 (1-p)^n u(w+g) \\
&+ C_n^1 p^1 (1-p)^{n-1} u(w+g-\frac{1}{n}D) \\
&... \\
&+ C_n^{n-1} p^{n-1} (1-p)^1 u(w+g-\frac{n-1}{n}D) \\
&+ C_n^n p^n (1-p)^0 u(w+g-D)
\end{aligned}
$$

And we reorganize that of an agent in an $n-1$ group as:

$$
\begin{aligned}
U_i(n-1) &= \sum_{k=0}^{n-1} C_{n-1}^k p^k (1-p)^{n-1-k} u(w+g-\frac{k}{n-1}D) \\
&= C_{n-1}^0 p^0 (1-p)^n u(w+g) + C_{n-1}^0 p^1 (1-p)^{n-1} u(w+g) \\
&+ C_{n-1}^1 p^1 (1-p)^{n-1} u(w+g-\frac{1}{n-1}D) + C_{n-1}^1 p^2 (1-p)^{n-2} u(w+g-\frac{1}{n-1}D) \\
&... \\
&+ C_{n-1}^{n-2} p^{n-2} (1-p)^2 u(w+g-\frac{n-2}{n-1}D) + C_{n-1}^{n-2} p^{n-1} (1-p)^1 u(w+g-\frac{n-2}{n-1}D) \\
&+ C_{n-1}^{n-1} p^{n-1} (1-p)^1 u(w+g-D) + C_{n-1}^{n-1} p^n (1-p)^0 u(w+g-D)
\end{aligned}
$$

Now we collect terms for $U_i(n-1)$ as following:

$$
\begin{aligned}
U_i(n-1) &= C_{n-1}^0 p^0 (1-p)^n u(w+g) \\
&+ C_n^1 p^1 (1-p)^{n-1} \left[\frac{C_{n-1}^0}{C_n^1} u(w+g) + \frac{C_{n-1}^1}{C_n^1} u(w+g-\frac{1}{n-1}D) \right] \\
&+ C_n^2 p^2 (1-p)^{n-2} \left[\frac{C_{n-1}^1}{C_n^2} u(w+g-\frac{1}{n-1}D) + \frac{C_{n-1}^2}{C_n^2} u(w+g-\frac{2}{n-1}D) \right] \\
&... \\
&+ C_n^{n-1} p^{n-1} (1-p)^1 \left[\frac{C_{n-1}^{n-2}}{C_n^{n-1}} u(w+g-\frac{n-2}{n-1}D) + \frac{C_{n-1}^{n-1}}{C_n^{n-1}} u(w+g-D) \right] \\
&+ C_{n-1}^{n-1} p^n (1-p)^0 u(w+g-D)
\end{aligned}
$$

For the terms in the square brackets, we pick the k^{th} line as an example, and we find the following:

$$
\begin{aligned}
& C_n^k p^k (1-p)^{n-k} \left[\frac{C_{n-1}^{k-1}}{C_n^k} u(w+g-\frac{k-1}{n-1}D) + \frac{C_{n-1}^{k}}{C_n^k} u(w+g-\frac{k}{n-1}D) \right] \\
= \; & C_n^k p^k (1-p)^{n-k} \left[\frac{k}{n} u(w+g-\frac{k-1}{n-1}D) + \frac{n-k}{n} u(w+g-\frac{k}{n-1}D) \right] \\
\leq \; & C_n^k p^k (1-p)^{n-k} u(w+g-\frac{k}{n}D)
\end{aligned}
$$

where the inequity is obtained based on u being concave. The distance between the second and the last line in above is exactly one partial insurance that is offered at the actuarially fair rate. Specifically, the insurance charges $\frac{n-k}{n(n-1)}D$ in the $\frac{k}{n}$ probability event, and pays $\frac{k}{n(n-1)}$ in the $\frac{n-k}{n}$ probability event. Notice that the last line in the above is exactly the k^{th} line of $U_i(n)$. We can therefore claim, like in proposition **??**, that the distance between risk-sharing in a $n-1$ member group and that in a n member group is just $n-1$ such fair insurances.