Organizational Form and Efficiency: The Coexistence of Takaful and Conventional Insurers

Abstract

This study analyzes the coexistence of two different organizational forms of insurance, namely Takaful and conventional insurance. We analyze the efficiency of different organizational forms in 13 countries for the period 2007-2011. Technical, allocative, cost, and revenue frontiers are estimated using data envelopment analysis. We test the expense preference hypothesis and efficient structure hypothesis. We find evidence for the efficient structure hypothesis and expense preference hypothesis in selected market segments. The results provide insight into the competitiveness of conventional and Takaful insurers from different countries.

Keywords: Data envelopment analysis, Efficiency, Takaful, Cross-frontier analysis, Organizational form, Expense preference hypothesis, Efficient structure hypothesis.

JEL Classification: G22, G32, D2

1. Introduction

Agency costs, in the modern theory of the firm, have been used to explain the structure of organizations with the organizations that succeed in any economic activity being the ones that deliver the preferred product at the lowest possible price while covering agency costs and the costs of production (e.g., Jensen and Meckling, 1976). The insurance industry provides a particularly interesting environment for studying agency-theoretic hypotheses as two main types of organizational forms coexist in the industry: stock insurers, owned by stockholders, and mutual insurers, owned by policyholders (Cummins, Rubio-Misas, and Zi, 2004). Several hypotheses have been developed that address the coexistence of these two types of organizations, the two most prominent being the expense preference hypothesis (Mester, 1991) and the efficient structure hypothesis (Cummins, Rubio-Misas, and Zi, 2004). The *expense preference hypothesis* states that mutual insurers are less cost efficient than stock insurers due to unresolved agency conflicts (e.g., higher perquisite consumption by mutual managers). The *efficient structure hypothesis* posits that the two organizational forms serve different market segments due to differences in managerial discretion and access to capital, such that there are not necessarily efficiency differences between organizaitonal forms.

Two types of organizational forms exist in Muslim countries as well as some western countries which have large Muslims populations, namely conventional and takaful insurers. Globally, takaful contributions grew by 22% per year from 2007-2011 and were estimated to grow by 16% in 2012 (Ernst and Young, 2013). World takaful insurance premiums were estimated at $19.0 billion in 2012 (Takaful Re, 2014). Conventional insurance has penetrated a small percentage of the 1.6B Muslims market globally. Whether this is due to religious inclinations, inadequate insurance distribution or lack of education about insurance products, the untapped segment provides a huge potential for takaful (Takaful Re, 2014).

The birth of takaful was due to the prevailing need of the Muslim population for an Islamic alternative to conventional insurance. It was also aimed to complement the operation of the Islamic banking system (Saad, 2012). While the first takaful insurance company was formed in Sudan in 1979, it has only been in the last few years that the industry has experienced rapid growth.[1] Increasing awareness and globalization will lead to takaful becoming a real and identifiable sector of the world insurance market in the next decade (ICMIF, 2009). Takaful has grown hand-in-hand in the same market as conventional insurance.

This paper focuses on two organizational forms: conventional and takaful insurers. The takaful firms are similar to the mutual form which is owned by policyholders, whereas the conventional insurance firms refer to the stock form owned by stockholders (see Saad, 2012). We use cross-frontier analysis, based on data envelopment analysis (DEA), to provide new evidence about the relationship between organizational form and efficiency in insurance markets where conventional and takaful firms coexist. This is the first study to empirically test the expense preference hypothesis and the efficient structure hypothesis in a large international study in the context of takaful and conventional insurers. We consider 666 firm-years covering 13 countries from 2007-2011.[2]

In addition to increasing public awareness of takaful insurance, our contributions include the following. First, it is the first study that explicitly estimates firm performance by computing the revenue efficiency of individual takaful and conventional firms over the time period 2007 to 2011 using data envelopment analysis (DEA). DEA measures efficiency by comparing each firm in an industry to a ‘‘best practice” frontier formed by the most efficient firms in the industry (Cooper, Seiford, and Zhu, 2004). Second, it is the first study that analyses organizational form

[1] However, some form of mutual or takaful-like insurance can be traced as far back as 622 B.C. (Billah, 1998). For general discussions of takafuls, see Maysami and Kwon (1999), Kwon (2007), White (2010), and Gonulal (2013).

[2] The countries included are Bahrain, Bangladesh, Jordan, Kuwait, Malaysia, Morocco, Oman, Pakistan, Qatar, Saudi Arabia, Tunisia, Turkey, and the United Arab Emirates.

in the context of the coexistence of takaful firms and conventional insurers in 13 countries. We consider different types of insurance in this study: life, non-life, and multi-line insurance firms.

This study shows the operating status of the takaful industry in the presence of conventional insurers in terms of efficiency. Hence, it would be useful for managers and regulators in taking steps to improve the overall insurance industry market. For example, conventional and takaful insurers can be ranked by their efficiency to determine the most efficient suppliers of insurers. Managers of insurers with low efficiency rankings can use this knowledge to develop ways to improve performance.

The paper is organized as follows. Section 2 presents the literature review followed by the efficiency concepts in section 3. The hypotheses are presented in section 4. Section 5 presents the discussion of the DEA methodology, and data are discussed in Section 6. Section 7 presents the empirical results and analysis, and finally, Section 8 concludes.

2. Literature Review

Prior literature can be categorized as focusing on organizational form in the insurance industry and organizational form in the takaful industry. The rest of this section discusses these categories.

2.1 Prior Literature on Organizational Form in Insurance Industry

Organizational form and efficiency has been extensively studied in the insurance literature. Early studies include Fecher et al. (1993; French life and non-life insurers), Cummins and Weiss (1993; U.S. non-life insurance), Gardner and Grace (1993; U.S. life insurance), Fukuyama (1997; Japanese life insurance), and Cummins and Zi (1998; U.S. life insurance). More recent studies include Cummins, Weiss, and Zi (1999; U.S. non-life insurance. More details on recent insurance efficiency studies are provided in Eling and Luhnen (2010a) and Cummins and Weiss (2013).

Recent research has shown mixed evidence for the expense preference hypothesis for the U.S., Europe, and other insurance markets. For example, Cummins, Weiss, and Zi (1999) find evidence supporting the expense preference hypothesis for the U.S. property-liability insurance industry. However, Greene and Segal (2004) find that both U.S. life mutual companies and stock companies are operating with comparable levels of efficiency, which is inconsistent with the expense preference hypothesis. Cummins, Rubio-Misas, and Zi (2004) conclude that the expense preference hypothesis does not hold for the Spanish insurance industry. Diboky and Ubl (2007) and Wende, Berry-Stölzle, and Lai (2008) find mixed results for the German insurance market. Wende, Berry-Stölzle, and Lai (2008) find evidence both for expense preference and managerial discretion in non-life whereas Diboky and Ubl (2007) reject the expense preference hypothesis for German life insurers. Eling and Luhnen (2010b), in a broad efficiency comparison of insurers from 36 countries, did not find support for the expense preference hypothesis.

The first study to empirically test the expense preference hypothesis and the efficient structure hypothesis in a large cross-country sample was Biener and Eling (2012). They provide new insight into the relationship between organizational form and efficiency in international insurance markets. The study covers 23,807 firm-years for 21 countries from North America and the European Union. They find evidence supporting the efficient structure hypothesis in selected market segments, but they find no support for the expense preference hypothesis.

2.2 Prior Literature on Organizational Form in Takaful Industry

Unlike conventional insurance, takaful research is very limited in scope, especially on organizational form. This is partially due to the limited availability of data, the recent existence of takaful insurers in many countries, and the limited number of experts in the field. Most of the takaful organizational form research has been conducted in the Malaysia market. The seven main studies on organizational form in the takaful insurance context are Saad et al. (2006), Ismail

Alhabshi, and Bacha (2011), Yusop et al. (2011), Saad (2012), Yakob et al. (2012), Singh and Zahran (2013), and Kahn and Noreen (2014).[3]

Saad et. al. (2006) investigates the efficiency of eleven life insurers and one takaful firm, Takaful Nasional, over the period 2002 to 2005. They found that pure efficiency for Takaful Nasional is below the industry average whereas scale efficiency change is equivalent to the industry average. This study is not the final word on evaluating takaful efficiency because only one takaful operator is studied compared to eleven life insurers. Ismail et al. (2011) provides an empirical study of the relationship between efficiency and organizational structure for takaful operators in the Malaysian market over the period 2004-2009.The findings indicate that there is a significant difference in technical efficiency between the takaful industry and the conventional insurance industry. It is found that takaful insurers have lower technical efficiency than conventional insurers. The study found that conventional insurers have higher scale efficiency than takaful insurers.

Saad (2012) examines the efficiency of general or non-life takaful and the conventional insurance industry in Malaysia during the period 2007 to 2009. The author uses data envelopment analysis (DEA) and Malmquist indices. A significant difference in efficiency is found between the takaful industry and the conventional insurance industry. The takaful insurance industry is less efficient than the conventional insurance industry under both constant returns to scale (CRS) and variable returns to scale (VRS). In addition, the takaful industry has lower pure technical efficiency (PTE) and scale efficiency (SE) than the conventional insurance industry. Therefore, it seems that the organizational form has direct implications for efficiency. This is consistent with the managerial discretion hypothesis and the expense preference hypothesis as stated by Cummins, Weiss and Zi (1999).

[3] There have also been several takaful efficiency studies that do not analyze organizational form. E.g., Kader, Adams, and Hardwick (2010), Kader, et al., (2014), and Miniaoui and Chaibi (2014).

In summary, the literature review shows that that mutual and stock ownership forms coexist in most developed and emerging economies worldwide. takaful and conventional insurers coexist also, but this coexistence has never been analyzed in an international context.

3. Efficiency concepts

This section first discusses frontier efficiency concepts and cross-frontier analysis. The disccuss then sets forth our estimation strategy.

Frontier efficiency techniques measure a company's performance relative to the ''best practices'' of the most efficient companies in the same industry and integrate inputs and outputs into a single performance measure that differentiates between companies based on a multidimensional framework (Cooper, Seiford, and Zhu, 2004). We use DEA to estimate ''best practice'' technical, cost, revenue, and allocative frontiers. Efficiency scores can be used in different ways to provide managerial insight, both at the firm and the industry levels.

One significant application of DEA at the industry level is the efficiency analysis of different organizational forms (Biener and Luhnen, 2012). In this framework, Cummins, Weiss and Zi (1999) introduced *cross-frontier analysis* (CFA), an innovative technique that allows direct tests of hypotheses with regard to organizational form in insurance markets such as the efficient structure and the expense preference hypotheses. In our work, we follow Cummins, Weiss, and Zi (1999), and Cummins et al. (2004) by estimating technical, cost, revenue, and allocative efficiencies with respect to different frontiers for takaful and conventional insurers from the life, non-life, and the multi-line insurance industry.

We modified the simple example used by Biener and Eling (2012) to illustrate CFA in the context of conventional and takaful insurers. The example contained just one input and one output. In Table 1, we consider 10 firms; the first five are conventional, the second five are takaful insurers. The efficiency score of insurer *i* on the pooled frontier, i.e., consisting of both conventional and takaful insurers, is calculated as the output/input ratio for insurer *i* divided by the maximum output/input ratio of all insurers in the sample. The efficiency of insurer *i* on its own frontier, i.e., only insurers belonging to its own group, is obtained as the output/input ratio of insurer *i* in relation to the maximum output/input ratio of all conventional insurers (if insurer *i* is a conventional insurer) or takaful insurers (if insurer *i* is a takaful insurer) in the sample. The cross-frontier efficiency scores are calculated as the output/input ratio of insurer *i* in relation to the maximum output/input ratio of all conventional insurers (if insurer *i* is a takaful insurer) or takaful insurers (if insurer *i* is a conventional insurer). Based on these results, cross-to-own efficiency scores can be obtained by dividing the cross-frontier efficiency score of insurer *i* by its own frontier efficiency score.

In Example 1 (Table 1, Panel A), the cross-to-own efficiency scores are consistently larger than 1 for the conventional insurers and consistently lower than 1 for the takaful insurers. This can be interpreted as meaning that the conventional production technology dominates the takaful production technology. In the single input/single output example, the company with the best output/input ratio will obtain the maximum efficiency score of 1 and thus will always constitute the pooled frontier, the own frontier of the dominating technology, and the cross-frontier of the technology that is dominated. If, e.g., Company 10 needs only five (instead of seven) input units to produce its output, it would obtain the maximum output/input ratio (1.20) and the maximum efficiency score of 1, and thus constitute the pooled frontier and its own frontier (see Table 1, Panel B).

[Insert Table 1 Here]

Computing CFA for a single input/single output situation is simple. However, real life is seldom that simple and calculating CFA when there is more than one input and output is more complex. An example of cross-frontier analysis with two input units and one output is presented in Figure 1. It shows the production isoquants subject to inputs x_1 and x_2, i.e., the combinations of x_1 and x_2 that could be used to produce the same output quantity. The isoquants represent the best available technology for producing a fixed output quantity. The isoquants for the conventional and for the takaful are given by $L^C(y)$ and $L^K(y)$, respectively. The own frontier efficiency of the conventional company represented by point B can be obtained as $0A/0B$. The own frontier efficiency of the takaful company represented in F can be obtained as $0E/0F$. The cross-frontier efficiency of the conventional (takaful) company represented in B (F) can be obtained as $0G/0B$ $(0D/0F)$. The cross-frontier efficiency for the conventional insurer is larger than 1, for the takaful it is lower than 1. In this example, the conventional technology again dominates the takaful technology.

Note that the isoquants might intersect, meaning that the conventional technology is optimal for some operating points and the takaful technology is best for other operating points.[4] Under such conditions, we cannot conclude that one technology is strictly dominant in producing the other technology. Under the efficient structure hypothesis, we would expect that conventional firms dominate takaful firms in some areas (where conventional firms have advantages) and that takaful firms dominate conventional firms in other areas (where takaful firms have advantages). Such an example is illustrated in the right part of Figure 1. Left of points D and E, the conventional firms' technology dominates the takaful firms' technology; to the right of points D and E, the takaful technology dominates the conventional technology.

[Insert Figure 1 Here]

4. Hypotheses

To examine the efficient structure and the expense preference hypotheses, we depend on the cross-to-own efficiency ratios. The takaful firms are similar to the mutual form which is

[4] For an example involving stocks and mutuals, see Cummins, Weiss, and Zi (1999).

owned by policyholders whereas the conventional insurance firms are akin to the stock form owned by stockholders. The efficient structure hypothesis predicts that conventional and takaful insurers produce different insurance outputs and each will be relatively successful in producing its own insurance outputs efficiently. This suggests that input usage or production costs are on average less than would be incurred if the same insurance outputs were produced by the other type of insurer (Cummins, Rubio-Misas, and Zi, 2004).

In the first step of the analysis, we test the hypothesis that conventional and takaful insurers operate on the same frontier. Rejection of this hypothesis would provide the grounds for estimating efficiency with separate frontiers. To this end, we estimate efficiency for conventional and takaful insurers based on the pooled and the individual own frontiers. Rejection of this hypothesis would also be consistent with the efficient structure hypothesis in that it indicates that conventional and takaful insurers employ different technologies and that measuring efficiency based on the pooled frontier leads to biased conclusions.

To gain more insight into whether conventional and takaful are sorted into market segments where they have comparative advantages, we use size-ownership interaction terms in the frontier distance regressions. The size-ownership interaction terms can be interpreted as the cross-to-own efficiency ratios of firms in the respective group, i.e., small, medium, and large conventional and takaful insurers, while controlling for other important variables.

To validate the efficient structure hypothesis, we test whether each group's output vectors could be produced with equal efficiency using the other group's production technology. For values of a cross-to-own efficiency ratio significantly greater than 1, we can reject the hypothesis and conclude that insurers in the respective group have a comparative efficiency advantage (technical, cost, revenue, allocative) in producing their own outputs. Such a result would support the efficient structure hypothesis for this group of insurers.

Even if conventional and takaful insurers have comparative technical efficiency advantages in producing their own outputs, i.e., the efficient structure hypothesis holds, these advantages may be corroded if they fail to choose cost minimizing input combinations, which would be consistent with expense preference behavior. The expense preference hypothesis suggests that takaful insurers are less cost efficient than conventional insurers due to unresolved agency conflicts (e.g., higher perquisite consumption by takaful insurer managers). Also, service charges are very high because an Islamic investment portfolio is not adequately well diversified to reduce investment risk (Choudhury, 2013). For the expense preference hypothesis to hold, we would have to observe comparative technical efficiency advantages, i.e., size-ownership interaction terms for takaful significantly larger than 1, and comparative cost efficiency disadvantages, i.e., cost and allocative size-ownership interaction terms for takaful significantly less than 1.

5. Methodology

5.1 Cross Frontiers Analysis (CFA)

CFA can be modified to multiple inputs and outputs utilizing the concept of distance functions introduced by Shephard (1970). For analyzing production frontiers, we use input-oriented distance functions assuming constant returns to scale (CRS). Following Cummins, Rubio-Misas, and Zi (2004), an input-oriented distance function D(y,x) for an insurer producing outputs $\mathrm{y} = (\mathrm{y}_1, \ldots, \mathrm{y}_\mathrm{n})^\mathrm{T} \in \mathbb{R}_+^\mathrm{n}$ with inputs $\mathrm{x} = (\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{k})^\mathrm{T} \in \mathbb{R}_+^\mathrm{k}$ is defined as:

$$D_p(y,x) = \sup\left\{\theta\colon \left(y, \frac{x}{\theta}\right) \in V(y)\right\}, \tag{1}$$

With subscript P indicating the measurement of the output/input set (y, x) for the pooled frontier consisting of conventional and takaful insurers.

The distance function estimates the largest θ for which $\left(y, \frac{x}{\theta}\right)$ is attainable in set $V(y)$, where $V(y)$ denotes the subset of all input vectors $x \in \mathbb{R}_+^k$. The resulting θ can be interpreted as the distance between the operation point (y, x) and the efficient frontier. By applying this definition, we assume a production technology that transforms inputs into outputs. The relation $y \rightarrow V(y) \subseteq \mathbb{R}_+^k$ models this transformation in that $V(y)$ constitutes the subset of all input vectors $x \in \mathbb{R}_+^k$ yielding at least y for any $y \in \mathbb{R}_+^n$. From the definition of the distance function, we can derive Farrell's (1957) measure of input technical efficiency. The input technical efficiency is$T_P(y, x) = 1/D_P(y, x)$ where $D_P(y, x)$ is defined on the interval [0;∞[, the Farrell measure $T_P(y, x)$is defined on [0;1]. This input-oriented distance function can be applied to different reference sets, e.g., production frontiers. To test the efficient structure and expense preference hypotheses, we need an estimate of the dominance of one specific technology (conventional, takaful) over another. For this purpose, we estimate the distance of insurers belonging to a specific group (conventional, takaful) to the production frontier of their own and the opposing group.

The own-frontier distance functions measure the distance of a firm to the own technologies' efficient frontier, e.g., a conventional firm is evaluated against all efficient conventional firms. For conventional insurers $D_C(y_c, x_c)$ and takaful insurers $D_K(y_k, x_k)$, the respective own-frontier distance functions are defined as:

$$D_C(y_c, x_c) = Sup\left\{ \theta: \left(y_c, \frac{x_c}{\theta}\right) \in V^C(y_c) \right\}, c = 1, 2, \dots, C, \text{ and} \tag{2a}$$

$$D_K(y_k, x_k) = Sup\left\{ \theta: \left(y_k, \frac{x_k}{\theta}\right) \in V^K(y_k) \right\}, k = 1, 2, \dots, K, \tag{2b}$$

$D_c(y_c, x_c)$ measures the distance of conventional firms $c = 1, 2, \dots, C$ from the conventional frontier. $V^C(y_c)$ is the conventional firms' input correspondence for the output vector y_c, and C is the total number of conventional firms. $D_K(y_k, x_k)$ is defined accordingly for $k = 1, 2, \dots, K$ with $V^K(y_k)$ being the takaful firms' input correspondence and K the total number of takafuls.

The cross-frontier distance functions were introduced by Cummins, Weiss and Zi (1999) and measure the distance of a firm from the opposing technologies' efficient frontier. For conventional $D_K(y_c, x_c)$ and takaful $D_C(y_k, x_k)$, the respective cross-frontier distance functions are defined as:

$$D_K(y_c, x_c) = Sup\left\{\theta:\left(y_c, \frac{x_c}{\theta}\right) \in V^K(y_c)\right\}, c = 1, 2, \dots, C, \text{ and} \tag{3a}$$

$$D_C(y_k, x_k) = Sup\left\{\theta:\left(y_k, \frac{x_k}{\theta}\right) \in V^C(y_k)\right\}, k = 1, 2, \dots, K \tag{3b}$$

$D_K(y_c, x_c)$ measures the distance of a conventional firm $c = 1, 2, \dots, C$ to the takaful frontier where $V^K(y_c)$ is the takaful firms' input correspondence for the output vector y_c and K is the total number of takaful firms. $D_C(y_k, x_k)$ is defined accordingly for $k = 1, 2, \dots, K$ where $V^C(y_k)$ is the conventional firms' input correspondence and C is the total number of conventional firms.

We estimate DEA efficiency with linear optimization procedures described in Cummins Weiss and Zi (1999) and in Cummins and Weiss (2012). Estimation of cross-frontier distance functions allows me to measure the performance of firms belonging to a specific group (conventional, takaful) against the technology of the other group, which also implies that values of cross-frontier efficiencies are not bounded by 1. Since firms are not included in the subset of $V(y)$ (the subset of all input vectors $x \in \mathbb{R}_+^k$), they can perform better than the efficient frontier firms of the opposing technology, and thus their cross-frontier distance functions and efficiency values can range between 0 and infinity.

To test the hypotheses with respect to the superiority of one technology over the other, we need a ratio that constitutes the distance between two opposing frontiers. Following Cummins, Rubio-Misas, and Zi (2004), we use cross-to-own efficiency ratios, $D_{T\{C:K\}}(y_c, x_c)$ and $D_{T\{K:C\}}(y_k, x_k)$, which measure the distance between the conventional and takaful frontiers at each operation point. T characterizes the distance measure as related to the production frontier (technical) and the terms $\{C\colon K\}$ and $\{K\colon C\}$ describe which frontiers are measured against each other, i.e. the conventional frontier against the takaful frontier with $\{C\colon K\}$, and the takaful frontier against the conventional frontier with $\{K\colon C\}$. The cross-to-own efficiency ratios are defined as:

$$D_{T\{C:K\}}(y_c, x_c) = \frac{D_C(y_c, x_c)}{D_K(y_c, x_c)} = \frac{T_K(y_c, x_c)}{T_C(y_c, x_c)} \quad (4a)$$

$$D_{T\{K:C\}}(y_k, x_k) = \frac{D_K(y_k, x_k)}{D_C(y_k, x_k)} = \frac{T_C(y_k, x_k)}{T_K(y_k, x_k)}, \quad (4b)$$

Cross-to-own efficiency ratios larger than 1 indicate that the own production technology dominates the opposing technology at the considered operating point. If we perform this calculation for all insurers in our sample, we can determine whether the own technology dominates the opposing technology. Finding average values of cross-to-own efficiency ratios larger than 1 for each type of insurer (conventional, takaful) would provide evidence in support of the efficient structure hypothesis; i.e., the respective type of insurer (conventional, takaful) has a comparative advantage in producing its own output. If the own production technology is dominated by the opposing technology, average cross-to-own efficiency ratios are smaller than 1; such a finding would not support the efficient structure hypothesis. The cross-to-own efficiency ratios are thus the basis of deriving conclusions as to the competition between conventional and takaful insurers in insurance markets. Our DEA estimation methodology for cost, revenue, and allocative efficiency are specified in the *Appendix*.

5.2 Frontier Distance Regressions

We further analyze the cross-frontier efficiencies by conducting multiple regression analyses with cross-to-own efficiency ratios as dependent variables and firm-specific characteristics as independent variables (frontier distance regressions). We follow Cummins, Rubio-Misas, and Zi (2004) and Biener and Eling (2012) by applying a Tobit regression model with omitted intercept and the dependent variable censored at 0. We calculate different regression models for technical, cost, revenue, and allocative cross-to-own efficiency ratios, for life, non-life, and multiple lines insurers, separately.

The regression model for technical cross-to-own efficiency ratios is:

$$D_{Tit} = \beta_{\alpha} INTERACT_{i\alpha} + \beta_{Den} DENSITY_{it} + \beta_{Pen} PENETRATION_{it} + \beta_{\gamma} LAW_{i\gamma} + \beta_{Cor} CORRUPTION_{it} + \beta_{HII} HII\_Country_{it} + \beta_{\delta} YEAR_{i\delta} + \beta_{\varepsilon} COUNTRY_{i\varepsilon} + u_{it}, \quad (5)$$

where D_{Tit} is the cross-to-own efficiency ratio for firm i in year t, $INTERACT_{i\alpha}$ is a vector of six size-ownership interaction terms used to control for firm size with $\alpha = Q1 * K, Q2 * K, Q3 * \mathrm{K}, \mathrm{Q1} * \mathrm{C}, \mathrm{Q2} * \mathrm{C}, \mathrm{Q3} * \mathrm{C}$. We interact three size quartile dummy variables ($Q1$ = small, $Q2$ = medium, $Q3$ = large) with two dummy variables K (1 for takaful, 0 otherwise) and C (1 for conventional, 0 otherwise). Consequently, a large conventional insurer i would receive the value 1 for the interaction term $INTERACT_{iQ3*C}$ and 0 for the other five interaction terms.

$DENSITY_{it}$ indicates the density of the insurance market (premiums in U.S. dollar per capita) for firm i in year t. Insurance density represents the average spent on insurance by each person and shows the depth of insurance coverage in an economy. A high insurance density implies that insurance market is highly developed. $PENETRATION_{it}$ measures the degree of market penetration (premiums in U.S. dollar/ GDP) in the market in which firm i is operating in year t. Insurance penetration is a rough indicator of growth potential. $LAW_{i\gamma}$ are two dummy variables reflecting legal systems: γ = Islamic, MIXED represent Islamic law or mixed legal systems, respectively. A third legal system variable (civil law) is excluded to avoid singularity. $CORRUPTION_{it}$ indicates the degree of corruption in year t and country of firm i.

$HHI\_Country$ is the Herfindahl index based on percentages of premiums written within countries. It serve as a measure for competition within countries. $YEAR_{i\delta}$ are dummy variables for each year with one year excluded to avoid singularity. $COUNTRY_{i\varepsilon}$ are country dummies, again with one country excluded. Country dummy variables are not shown in the regression tables to conserve space.[5] u_{it} is the error term in the regression, with $u \sim N(0, \sigma^2)$.The model is identical for technical, cost, revenue, and allocative cross-to-own efficiency ratios, the only difference being that the dependent variable is D_{Cit} or D_{Rit} or D_{Ait} instead of D_{Tit}.

We analyze technical, cost, revenue and allocative efficiency using CFA for insurers from 13 countries, two organizational forms (conventional, takaful), three branches (life, non-life, multi-line), and three company sizes (large, medium, small). We consider total assets as a measure of insurer size (see, e.g., Cummins and Zi, 1998; Diacon et al., 2002) and thus subdivide all companies by their total assets into large, medium, and small insurers. For DEA, We calculate technical, cost, revenue, and allocative efficiency values assuming input orientation and constant returns to scale. We perform a cross-frontier analysis as presented in Cummins, Weiss and Zi (1999), and Cummins, Rubio-Misas, and Zi (2004).

6. Data, Inputs and Outputs

Data on organization type for conventional and takaful insurers, and lines of business are taken from the Capital IQ database for the period 2007-2011. Company-specific information on domiciliary country is taken from the World Bank and Bloomberg. Specifically, interest rate, consumer price index (CPI), gross domestic product (GDP), and GDP per capita are obtained from the World Bank. The rate of return on the stock market index for each country is obtained from Bloomberg. Information on the legal systems is taken from the University of Ottawa's JuriGlobe World Legal Systems database (see http://www.juriglobe.ca/eng/index.php). Corruption is proxied using the Corruption Perceptions Index provided by Transparency International (available at http://www.transparency.org).

[5] There is a tradeoff between selecting a homogenous sample of countries and the number of observations. That is, by selecting a homogenous sample we will reduce the sample size which would make the subsequent results have low statistical power. we control for the heterogeneity between countries by including country dummies. The downside of the country dummies is that they might not fully capture the heterogeneity across countries.

Our calculations include all companies from 13 countries that have positive values for the inputs and outputs described in Table 2. I have an unbalanced panel, i.e., we do not require that each company has data for all years. Our dataset consists of 666 firm-years from 13 countries.

There is widespread agreement in the literature with regard to the choice of inputs (Eling and Luhnen, 2010a; Cummins and Weiss, 2013). Following this literature, we use labor, business services and materials, and equity capital as inputs. A major fraction of operating expenses in insurance is labor related.[6] We thus concentrate on labor to determine the price of the operating-expenses-related input factor. For the price of labor we use selling, general, and administrative expenses divided by the number of employees. As our proxy for business services, we use policy acquisition costs, which include commissions and other underwriting expenses. The price for policy acquisition is proxied by the CPI, which we obtained from the World Bank for each country. The price of equity capital is proxied by a long-term average of the yearly rates of total return of the country-specific stock market indices, obtained from Bloomberg. After converting all numbers into U.S. dollars, we deflate each year's value by the country-specific CPI to the base year 2005 so that all monetary values are directly comparable (Cummins and Zi, 1998). The exchange rates were obtained from the World Bank.

[**Insert Table 2 Here**]

Consistent with the recent insurance efficiency literature, we use the value-added approach to define the outputs (see Berger and Humphrey, 1992; Cummins and Weiss, 2013). We distinguish between three main services provided by insurance companies – risk-pooling/risk-bearing, real financial services, and intermediation. A proxy for the amount of risk-pooling/risk-bearing and real financial services frequently used in the literature is the value of real incurred losses. We proxied the incurred losses by the policy benefits paid. The

[6] Cummins and Weiss (2013) show that the largest share of operating expenses in the U.S. insurance industry are employee salaries and commissions.

intermediation function is proxied by the real value of investments. All monetary values are deflated by the CPI to obtain constant 2005 values.[7]

Panel A of Table 2 summarizes the inputs, outputs, and prices (all in U.S. dollars), and panel B presents summary statistics. As shown in panel B, the total number of observations for the pooled sample is 666 firm-years. The total number of observations for the conventional insurers is 485 firm-years. This is almost 3 times the total number of the observations for takaful insurers, which is 181 firm-years. Most of the firms (conventional and takaful) write both life and non-life insurance. A total of 81 firm-years write only life and health insurance, 364 firm-years write insurance for multi-lines, and 221 firm-years write only non-life insurance. We exclude reinsurers and insurance brokers in our study.

7. Empirical Results

In this section, we present results based on the assumption that conventional and takaful insurers from the 13 countries compete with each other, i.e., they operate on the same efficient frontier. In Table 3, we provide aggregate results for the different organizational forms—conventional and takaful—in the 13 countries' insurance markets. We conduct two different statistical tests. In Table 3, the insurers are grouped by type of organizational form (conventional and takaful) in order to test whether conventional and takaful insurers operate on the same pooled frontier or on separate frontiers.

[**Insert Table 3 Here**]

The first test is to determine whether conventional and takaful insurers are operating on the same frontier. Therefore, we compare the efficiency results for conventional and takaful insurers on the pooled frontier ($T_P(y_c; x_c)$ and $T_P(y_k; x_k)$) with the efficiency results for the group specific frontiers ($T_C(y_c; x_c)$ and $T_K(y_k; x_k)$) in Table 3. We use both a parametric t-test

[7] CPI is used as the price for the input variable business services. Thus, business services was not deflated by CPI

as well as a non-parametric Wilcoxon test as significance tests. Table 3 presents the results of the Wilcoxon test. Both tests clearly reject the hypothesis that the takaful production frontier is identical to the pooled frontier for life, non-life, and multi-line insurers at the 1% level (T_P : T_K ***). The same results are obtained for cost (panel B), revenue (panel C), and allocative efficiency (panel D). Also, both tests reject the hypothesis that the conventional production frontier is identical to the pooled frontier for life, non-life, and multi-line insurers at the 1% level (T_P : T_C ***) (with one exception for life technical conventional efficiency rejected at which is the 10% level). This finding is in line with Cummins, Weiss and Zi (1999), who also reject the hypothesis of identical frontiers for mutual and stocks. However, those authors find no significant differences between the stock own and pooled frontiers, which is in contrast to our empirical result. That is, we also reject the hypothesis of identical frontiers for conventional (T_P: T_C ***). We follow Cummins, Weiss and Zi (1999) in concluding that the rejection of identical frontiers implies that efficiency analysis of conventional and takaful insurers should be based on separate frontiers rather than on the pooled frontier. The economic interpretation of this finding is that takaful and conventional insurers use different technologies for producing their respective outputs.

[**Insert Table 4 Here**]

In the second test, we consider the results shown in Table 4 and analyze the efficiency scores based on separate takaful and conventional frontiers in the rows $T_K(y_k; x_k)$ *and* $T_C(y_c; x_c)$. Takaful insurers are significantly more efficient with respect to their own Takaful frontier compared with the efficiency of conventional insurers relative to their own conventional frontier (e.g., 0.96 vs. 0.82 for technical efficiency in multi-line). This result may indicate that there are more degrees of freedom for conventional firms to make mistakes that degrade efficiency when operating in more complex lines of business (Cummins, Rubio-Misas, and Zi; 2004).

The third test is to consider the cross-frontier efficiencies in Table 4, i.e., the efficiency of takaful (conventional) relative to the conventional (takaful) frontier ($T_C(y_k; x_k)$ and $T_K(y_c; x_c)$) and the resulting cross-to-own efficiency ratios. We find that, on average, conventional insurers are more efficient relative to the takaful frontier in technical efficiency for life firms, cost efficiency for life and multi-line firms, revenue and allocative for multi-line insurance. That is the cross-to own efficiency ratios for conventional ($D_{T\{C:K\}(yc;xc)}$) are consistently larger than cross-to-own efficiency ratios for takaful ($D_{T\{K:C\}(yk;xk)}$). The economic interpretation of this result is that conventional firms are dominant for producing conventional output vectors. These differences in technical, cost, revenue, and allocative efficiency are important since they might be an indication of expense preference behavior by life and multi-line takaful insurers.

To provide evidence on whether the differences in cross-to-own frontier efficiencies are robust when controlling for firm and country characteristics, we conduct a frontier distance regression following with cross-to-own frontier ratios (distances between frontiers) as dependent variables and selected characteristics as independent variables (equation 5).[8] The idea is to regress various company-specific and country factors on the distance of takaful to conventional production (and the distance of conventional to takaful), cost, revenue, and allocative frontiers. I run the regression separately for the three main branches: non-life (panel A of Table 5), life (panel B of Table 5) and multi-line insurers (panel C of Table 5). We apply a Tobit regression model with omitted intercept, and the dependent variable is censored at 0. Omitting the intercept allows us to interpret the regression coefficients of the interaction terms as an intercept for insurers in that specific group.

[**Insert Table 5 Here**]

[8] A similar analysis was conducted in Cummins, Rubio-Misas, and Zi (2004).

In the production frontier regression, all size-ownership interaction terms are significantly larger than 1 (z_2 is significant at 1% level) in non-life, life, and multi-line insurance (with one exception for life, medium-size takafuls), implying that they are dominant for producing their specific output vectors. Coefficients of all conventional insurer interaction terms are significantly greater than the corresponding coefficient of the takaful interaction terms for each size quartile in non-life and life. Thus, the production frontier regression supports the efficient structure hypothesis for non-life and life as well as for multi-line conventional and takaful insurers.

Concerning the cost frontier regression, all size-ownership interaction terms are significantly larger than 1 in life and multi-line insurance. The regression results for the cost frontier thus provide additional support for the efficient structure hypothesis for life and multi-line insurance. The coefficient of conventional insurers is larger than the coefficient of takaful insurers in multi-line insurance. That is, the dominance of conventional insurers is higher than the dominance of takaful insurers in multi-line insurance. With respect to the expense preference hypothesis, we find evidence of cost deficiencies for non-life insurers in all size quantiles since the test statistic z_2 is insignificant for takaful insurers. This finding implies that takaful insurers waste more resources than conventional insurers in non-life insurance, which is in line with the expense preference hypothesis.

In the revenue frontier regression, the interaction terms of all size-ownership variables are significantly larger than 1 in life and multi-line insurance. The regression results for the revenue frontier confirm the efficient structure hypothesis for life and multi-line insurance. The coefficient of the interaction term of conventional insurers for medium-size in life, and in all size quantiles in multi-line insurance is higher than the coefficient of the interaction term for the takaful insurers. Thus, the dominance of conventional is higher than the dominance of takaful only for medium-size in life, and all size quantiles in multi-line insurance. For the expense preference hypothesis, we find evidence of revenue deficiencies for non-life insurers in medium-size quantile since the test statistic z_2 is insignificant for takaful insurers. This finding implies that takaful insurers waste more resources than conventional insurers in non-life insurance for only medium-size, which is in line with the expense preference hypothesis.

Regarding the allocative cross-to-own efficiency regressions, we find values for all size-ownership interaction terms to be significantly larger than 1 in non-life and life insurance, indicating that conventional and takaful insurers of all sizes are dominant in choosing the cost minimizing combination of inputs for producing their own outputs. However, conventional insurers are significantly more successful in choosing the cost minimizing input combination compared to takaful insurers in multi-line insurance. That is, the takaful size-ownership interaction terms are not significantly greater than 1 (z_2 is not significant). The allocative frontier distance regression thus supports the efficient structure hypothesis and provides evidence for the expense preference hypothesis in multi-line insurance since the coefficient terms for takaful insurers are not significantly greater than 1 for all sizes. In conclusion, the results show that the efficient structure and expense preference hypotheses hold for our international dataset.

In inferring the residual regression variables, recall that the dependent variable is larger than 1 in case of dominance of the own group-specific frontier for producing their own output. Coefficients of the remaining variables greater (less) than 0 are thus associated with wider (smaller) distances between the own-group frontier and the opposing group's frontier. Smaller distances between the conventional and takaful frontiers are likely associated with a higher degree of competition between conventional and takaful insurers, since the comparative advantage of producing the own-group outputs will be smaller (see Cummins, Rubio-Misas, and Zi, 2004). For the sake of brevity, we restrict our discussion to the most relevant results.

The coefficients for market density for multi-line insurance are significantly less than 0 for the cost frontier and allocative frontier. It thus seems that the higher the market density, the lower the distances between the conventional and takaful frontiers. Thus, there is likely to be a higher degree of competition between takaful and conventional insurers in high-density multi-line insurance markets. The coefficients for insurance market penetration are opposite to the density coefficients for multi-line insurers.

We find that the Islamic legal system coefficient is negative and significant for revenue frontiers in life insurance, implying that in countries with legislation based on Islamic law the differences between conventional and takaful frontiers are reduced; i.e., there likely exists a higher degree of competition between conventional and takaful in life insurers. The corruption coefficient is negative and significant for the non-life production frontier, the life cost and allocative frontiers, and the multi-line revenue frontier. This might be due to the fact that some of countries which have an Islamic legal system tend to have a developing insurance market. Thus, many of the international conventional insurers see those markets as an opportunity to expand. This implies that there is strong competition between conventional and takaful insurers in countries that have a higher degree of corruption. The coefficient of HHI_Country is positive and significant for life cost and allocative frontiers, and multi-line revenue and allocative efficiency.[9]

8. Conclusion

An economically interesting feature of emerging/developing insurance markets is the coexistence of two organizational forms: conventional insurers and takaful insurers. We apply cross-frontier analysis to study the efficiency of these two organizational forms and derive conclusions regarding the competition between them. A few authors assess the performance

[9] Tests also were conducted including the age variable, but it was never statistically significant.

difference between these two organizational forms in Malaysia only, but we are the first to analyze this issue in a cross country study. We consider a pooled sample of 13 countries. Our research design has the advantage of allowing us to separate the effects of different countries and organization types on efficiency, thus providing insight into the performance of different organizational forms. Thus, our results are valid not only for one country, but can also aid in comparing conventional and takaful insurers under different market environments. The efficient structure hypothesis holds for the most frontiers (i.e., conventional and takaful insurers are dominant in producing their own outputs). We also find indication of the expense preference hypothesis (takaful insurers are less cost efficient than conventional insurers).

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Table 1

Calculation of pooled, own, and cross-frontiers

Firm i	Type	Input	output	Output/input	Technical efficiency			
					Pooled	Own	Cross	Cross/own
Panel A: Example 1								
1	Conventional	1.00	1.00	1.00	1.00	1.00	1.17	1.17
2	Conventional	2.00	2.00	1.00	1.00	1.00	1.17	1.17
3	Conventional	3.00	3.00	1.00	1.00	1.00	1.17	1.17
4	Conventional	4.00	4.00	1.00	1.00	1.00	1.17	1.17
5	Conventional	5.00	5.00	1.00	1.00	1.00	1.17	1.17
6	Takaful	3.00	2.00	0.67	0.67	0.78	0.67	0.86
7	Takaful	4.00	3.00	0.75	0.75	0.88	0.75	0.86
8	Takaful	5.00	4.00	0.80	0.80	0.93	0.80	0.86
9	Takaful	6.00	5.00	0.83	0.83	0.97	0.83	0.86
10	Takaful	7.00	6.00	0.86	0.86	1.00	0.86	0.86
Panel B: Example 2								
1	Conventional	1.00	1.00	1.00	0.83	1.00	0.83	0.83
2	Conventional	2.00	2.00	1.00	0.83	1.00	0.83	0.83
3	Conventional	3.00	3.00	1.00	0.83	1.00	0.83	0.83
4	Conventional	4.00	4.00	1.00	0.83	1.00	0.83	0.83
5	Conventional	5.00	5.00	1.00	0.83	1.00	0.83	0.83
6	Takaful	3.00	2.00	0.67	0.56	0.56	0.67	1.20
7	Takaful	4.00	3.00	0.75	0.63	0.63	0.75	1.20
8	Takaful	5.00	4.00	0.80	0.67	0.67	0.80	1.20
9	Takaful	6.00	5.00	0.83	0.69	0.69	0.83	1.20
10	Takaful	5.00	6.00	1.20	1.00	1.00	1.20	1.20

Adapted from Biener and Eling (2012).

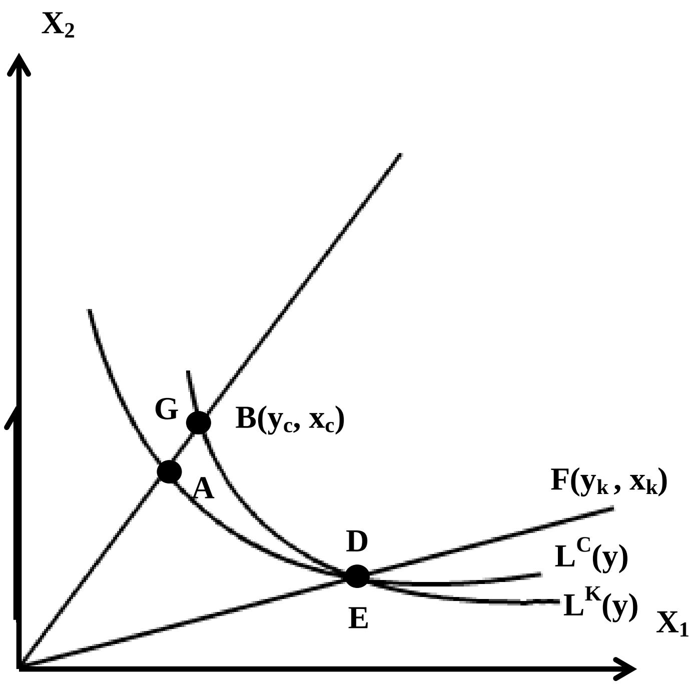

Fig. 1. Own efficiency isoquants for conventional and Takaful insurers. Note: The isoquants $L^C(y)$ and $L^K(y)$ constitute the own production frontier for a fixed output level y with respect to different levels of inputs x_1 and x_2 of conventional (C) and Takaful (K) insurers.
Adapted from Biener and Eling (2012).

Table 2

Inputs and outputs

Panel A: Definition of input, input prices, outputs, and output prices	
Inputs	Proxy
Labor	Number of employees
Business services	Policy acquisition cost
Equity capital	Capital & surplus
Input prices	
Price of labor	Selling, general, & admin. expenses/number of employees
Price of business services	CPI
Price of equity capital	Long-term average stock market return indices
Outputs	
Benefits Paid	Policy benefits paid
Investments	Total investment
Output prices	
Price of benefits paid	(Net premiums earned - policy benefits paid)/policy benefits paid
Price of investment	[(Expected return on stock * investment in equity) + (total investment – investment in equity) * rate of return on debt] / total investment

Panel B: Summary statistics for variables used

variable	Unit	Pooled		Conventional		Takaful	
		Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
Number of employees	Quantity	379.265	449.402	416.101	504.417	282.911	228.953
Acquisition cost	Million $	33.825	13.928	34.017	14.275	33.324	13.002
Capital & surplus	Million $	97.895	100.182	96.830	100.525	100.679	99.487
Selling & Admin expenses/number of employees	Million $	0.125	0.138	0.122	0.126	0.132	0.166
Long-term average stock market return indices	%	0.126	0.509	0.126	0.509	0.126	0.509
Policy benefits paid	Million $	49.814	84.807	53.864	88.236	39.218	74.279
Total investment	Million $	84.807	192.264	139.301	195.379	141.824	184.340
(Net premiums earned - policy benefits paid)/policy benefits paid	%	0.318	0.275	0.314	0.277	0.329	0.271
(Expected return on stock * investment in equity) + (total investment - investment in equity) * rate of return on debt / total investment	%	0.138	0.113	0.146	0.108	0.117	0.124
CPI	%	1.187	0.222	1.187	0.222	1.187	0.222

Note: The values are winsorized at the 5% and 95% level.

Table 3
Results of the cross-frontier analysis (2007-2011, full sample).
Comparison of different types of frontiers for conventional and Takaful insurers

Panel A:	**Technical efficiency**	Non-life		Life		Multi	
		Mean	Wilcoxon	Mean	Wilcoxon	Mean	Wilcoxon
Conventional	$T_P(y_c;X_c)$	0.8618	T_p:T_c***	0.9120	T_p:T_c*	0.8186	T_p:T_c***
	$T_C(y_c;X_c)$	0.8802	T_c:T_k***	0.9137	T_c:T_k***	0.8206	T_c:T_k***
	$T_K(y_c;X_c)$	1.0264	T_p:T_k***	1.1065	T_p:T_k***	0.9909	T_p:T_k***
	$D_{T(C:K)}(y_c;X_c)$	1.1652		1.2118		1.2196	
Takaful	$T_P(y_k;X_k)$	0.8731	T_p:T_k***	0.8172	T_p:T_k***	0.7892	T_p:T_k***
	$T_K(y_k;X_k)$	0.9027	T_k:T_c***	0.9245	T_k:T_c***	0.9587	T_k:T_c***
	$T_C(y_k;X_k)$	1.0473	T_p:T_c***	1.0215	T_p:T_c***	1.1698	T_p:T_c***
	$D_{T(K;C)}(y_k;X_k)$	1.1592		1.1048		1.2077	

Panel B:	**Cost efficiency**	Non-life		Life		Multi	
		Mean	Wilcoxon	Mean	Wilcoxon	Mean	Wilcoxon
Conventional	$C_P(y_c;X_c)$	0.5381	C_p:C_c***	0.6980	C_p:C_c ***	0.4392	C_p:C_c***
	$C_C(y_c;X_c)$	0.5724	C_c:C_k***	0.6338	C_c:C_k***	0.4923	C_c:C_k***
	$C_K(y_c;X_c)$	2.0426	C_p:C_k***	0.9930	C_p:C_k***	1.6855	C_p:C_k***
	$D_{C(C:K)}(y_c;X_c)$	3.4051		1.5712		3.7400	
Takaful	$C_P(y_k;X_k)$	0.6058	C_p:C_k***	0.6503	C_p:C_k***	0.3674	C_p:C_k***
	$C_K(y_k;X_k)$	0.7028	C_k:C_c***	0.8305	C_k:C_c***	0.6422	C_k:C_c***
	$C_C(y_k;X_k)$	1.9332	C_p:C_c***	1.2528	C_p:C_c***	1.9497	C_p:C_c***
	$D_{C(K;C)}(y_k;X_k)$	2.9578		1.4955		2.9669	

Panel C: Revenue efficiency

		Non-life		Life		Multi	
		Mean	Wilcoxon	Mean	Wilcoxon	Mean	Wilcoxon
Conventional	$R_P(y_c;X_c)$	0.3734	R_p:R_c***	0.6423	R_p:R_c***	0.4468	R_p:R_c***
	$R_C(y_c;X_c)$	0.4380	R_c:R_k***	0.6486	R_c:R_k***	0.4523	R_c:R_k***
	$R_K(y_c;X_c)$	0.9774	R_p:R_k***	1.0431	R_p:R_k***	1.3800	R_p:R_k***
	$D_{R(C:K)}(y_c;X_c)$	2.2499		1.6172		3.3644	
Takaful	$R_P(y_c;X_c)$	0.4446	R_p:R_k***	0.6271	R_p:R_k***	0.3570	R_p:R_k***
	$R_C(y_c;X_c)$	0.7107	R_k:R_c***	0.8914	R_k:R_c***	0.7852	R_k:R_c***
	$R_K(y_c;X_c)$	1.9718	C_p:R_c***	1.4078	C_p:R_c***	1.8388	C_p:R_c***
	$D_{R(C:K)}(y_c;X_c)$	2.7631		1.5881		2.3113	

Panel D: Allocative efficiency

		Non-life		Life		Multi	
		Mean	Wilcoxon	Mean	Wilcoxon	Mean	Wilcoxon
Conventional	$A_P(y_c;X_c)$	0.6485	A_p:A_c***	0.7671	A_p:A_c***	0.5758	A_p:A_c***
	$A_C(y_c;X_c)$	0.6670	A_c:A_k***	0.6945	A_c:A_k***	0.6403	A_c:A_k***
	$A_K(y_c;X_c)$	1.0081	A_p:A_k***	0.7365	A_p:A_k**	2.3638	A_p:A_k***
	$D_{A(C:K)}(y_c;X_c)$	1.5058		1.0564		3.9743	
Takaful	$A_P(y_c;X_c)$	0.7027	A_p:A_k***	0.7994	A_p:A_k***	0.5342	A_p:A_k***
	$A_C(y_c;X_c)$	0.7752	A_k:A_c***	0.9003	A_k:A_c***	0.6775	A_k:A_c***
	$A_K(y_c;X_c)$	1.2552	A_p:A_c***	1.0591	A_p:A_c***	2.2044	A_p:A_c***
	$D_{A(C:K)}(y_c;X_c)$	1.5946		1.1724		3.1824	

Note: *** indicates a significance level at 1%. The test is defined by the abbreviations T_i, C_i, R_i, and A_i with i= P, C, K. T_P:T_C with conventional indicates, for example, a Wilcoxon test for the difference in mean technical efficiency between the conventional pooled and own frontier. Takaful, T_P:T_K would indicate a Wilcoxon test for the difference in mean technical efficiency between the Takaful pooled and cross frontier.

Table 4
Results of the cross-frontier analysis (2007-2011, full sample).
Comparison of conventional and Takaful on different types of frontiers

Panel A: Technical efficiency

		Non-life				**Life**				**Multi**			
		Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox
Pooled	$T_P(y_c;X_c)$	0.8618				0.9120	**	**	***	0.8186			
	$T_P(y_k;X_k)$	0.8731				0.8172				0.7892			
Own	$T_C(y_c;X_c)$	0.8802				0.9137				0.8206	***	***	***
	$T_K(y_k;X_k)$	0.9027				0.9245				0.9587			
Cross	$T_K(y_c;X_c)$	1.0264				1.1065		*	**	0.9909	***	***	***
	$T_C(y_k;X_k)$	1.0473				1.0215				1.1698			
Cross/Own	$D_{T(C:K)}(y_c;X_c)$	1.1652				1.2118	***	***	***	1.2077			
	$D_{T(K;C)}(y_k;X_k)$	1.1592				1.1048				1.2196			

Panel B: Cost efficiency

		Non-life				**Life**				**Multi**			
		Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox
Pooled	$T_P(y_c;X_c)$	0.5381	*	**	*	0.6980				0.4392	*	***	*
	$T_P(y_k;X_k)$	0.6058				0.6503				0.3674			
Own	$T_C(y_c;X_c)$	0.5724	***	***	***	0.6338	***	***	***	0.4923	***	***	***
	$T_K(y_k;X_k)$	0.7028				0.8305				0.6422			
Cross	$T_K(y_c;X_c)$	2.0426			**	0.9930	**	**	**	1.6855			
	$T_C(y_k;X_k)$	1.9332				1.2528				1.9497			
Cross/Own	$D_{T(C:K)}(y_c;X_c)$	3.4051				1.5712			**	3.7400	***	***	***
	$D_{T(K;C)}(y_k;X_k)$	2.9578				1.4955				2.9669			

Panel C: Revenue efficiency

		Non-life				Life				Multi			
		Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox
Pooled	$T_P(y_c;X_c)$	0.3734		*	*	0.6423				0.4468	***	***	***
	$T_P(y_k;X_k)$	0.4446				0.6271				0.3570			
Own	$T_C(y_c;X_c)$	0.4380	***	***	***	0.6486	***	***	***	0.4523	***	***	***
	$T_K(y_k;X_k)$	0.7107				0.8914				0.7852			
Cross	$T_K(y_c;X_c)$	0.9774	***	***	***	1.0431	***	***	***	1.3800	***	***	***
	$T_C(y_k;X_k)$	1.9718				1.4078				1.8388			
Cross/Own	$D_{T(C:K)}(y_c;X_c)$	2.2499	***	***	***	1.6172				3.3644	***	***	***
	$D_{T(K;C)}(y_k;X_k)$	1.9718				1.5881				2.3113			

Panel D: Allocative efficiency

		Non-life				Life				Multi			
		Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox	Mean	t_a	t_b	Wilcox
Pooled	$T_P(y_c;X_c)$	0.6485		*		0.7671				0.5758			
	$T_P(y_k;X_k)$	0.7027				0.7994				0.5342			
Own	$T_C(y_c;X_c)$	0.6670	*	**	***	0.6945	***	***	***	0.6403			
	$T_K(y_k;X_k)$	0.7752				0.9003				0.6775			
Cross	$T_K(y_c;X_c)$	1.0081	***	***	***	0.8287	***	***	***	2.3638			**
	$T_C(y_k;X_k)$	1.2552				1.2552				2.2044			
Cross/Own	$D_{T(C:K)}(y_c;X_c)$	1.5058	**	***	**	1.1861				3.9743	*	**	***
	$D_{T(K;C)}(y_k;X_k)$	1.5946				1.1724				3.1824			

Note:*, **, *** indicate significance levels of 10%, 5%, and 1%, respectively, of t-test and Wilcoxon test for difference between conventional and Takaful insurer means for the pooled, and cross frontiers and the cross/own ratios. t_a-Test for difference between conventional and Takaful insurers. t_b-Test for whether the difference between conventional and Takaful are > or <0. Wilcox: Wilcoxon test.

Table 5
Frontier distance regression
Panel A

Independent variables	Non-Life											
	Production frontier			Cost frontier			Revenue frontier			Allocative frontier		
	Coeff.	z1	z2	Coeff.	z1	z2	Coeff.	z1	z2	Coeff.	z1	z2
Q1*Conventional	1.223	77.11***	***	1.146	15.14***	**	2.999	2.31**	**	1.316	11.49***	***
Q2*Conventional	1.228	90.25***	***	1.154	12.76***	**	3.106	2.20**	**	1.310	11.41***	***
Q3*Conventional	1.240	58.10***	***	1.182	8.76***	**	2.695	1.92*		1.261	9.73***	**
Q1*Takaful	1.217	56.42***	***	0.830	6.20***		3.495	2.37**	**	1.406	10.25***	***
Q2*Takaful	1.253	39.77***	***	0.920	10.14***		2.892	1.94*		1.285	7.92***	**
Q3*Takaful	1.225	57.57***	***	0.809	6.95***		2.922	1.63		1.353	8.64***	**
Islamic	-0.0111	-0.66		-0.0491	-0.69		0.146	0.27		0.0337	0.48	
Density	-0.000	-1.14		-0.000	-0.57		0.000	0.57		0.000	0.79	
Penetration	7.275	0.93		26.00	1.85*		-177.6	-0.61		-27.28	-0.79	
Corruption	-0.005*	-1.92		-0.0178	-1.18		0.0980	1.06		0.0180	1.54	
HHI_Country	0.000	0.77		-0.000	-0.11		0.0005	0.61		0.000	0.70	
2007	-0.0276	-2.10**		-0.0032	-0.39		9.869	6.50***		0.0722	0.77	
2008	-0.0925	-4.51***		-0.0283	-1.22		-2.469	-2.03**		-0.007	(-0.07)	
2009	-0.0625	-4.30***		-0.0156	-0.74		9.569	6.67***		0.355	2.58**	
2010	0.0173	10.88***		10.83	65.41***		9.948	6.94***		0.126	1.03	
N	221			221			221			221		

Panel B

Independent variables	Life											
	Production frontier			Cost frontier			Revenue frontier			Allocative frontier		
	Coeff	z1	z2	Coeff	z1	z2	Coeff	z1	z2	Coeff	z1	z2
Q1*Conventional	1.141	27.22***	***	1.652	188.09***	***	1.766	14.21***	***	1.452***	33.57	***
Q2*Conventional	1.147	45.38***	***	1.652	136.41***	***	1.784	17.40***	***	1.445***	63.97	***
Q3*Conventional	1.142	41.49***	***	1.653	139.62***	***	1.769	17.43***	***	1.452***	53.41	***
Q1*Takaful	1.061	55.47***	***	1.571	120.50***	***	1.806	20.81***	***	1.474***	71.06	***
Q2*Takaful	1.018	19.29***		1.567	134.11***	***	1.711	11.44***	***	1.520***	27.93	***
Q3*Takaful	1.108	56.94***	***	1.562	111.17***	***	1.866	20.18***	***	1.409***	88.65	***
Islamic	-0.110	-1.63		0.000808	0.09		-0.227	-2.02**		0.134	1.50	
Density	0.000	0.41		-0.000	-1.68*		-0.000	-0.73		-0.000	-0.73	
Penetration	-106.6	-4.87***		37.80	5.24***		-83.42	-2.30**		161.6	5.23***	
corruption	0.00758	4.98		-0.003	-4.36***		0.00460	0.84		-0.0118	-7.76***	
HHI_Country	-0.0001	-2.46**		0.000	3.56***		-0.0002	-2.07**		0.000190	2.60**	
2007	0.0678	2.78***		-0.180	-31.17***		-0.0116	-0.15		-0.236	-9.17***	
2008	0.289	4.88***		0.379	41.42***		0.240	1.47		-0.00797	-0.12	
2009	0.0320	3.70***		-0.331	-19.58***		-0.651	-8.43***		-0.333	-34.85***	
2010	-0.0455	-3.39***		-0.308	-11.75***		-0.255	-2.50**		-0.224	-5.62***	
N	81			81			81			81		

Panel C

Independent variables	Multi											
	Production frontier			Cost frontier			Revenue frontier			Allocative frontier		
	Coeff	z1	z2	Coeff	z1	z2	Coeff	z1	z2	Coeff	z1	z2
Q1*Conventional	1.121	94.31***	***	1.973	37.28***	***	2.757	200.09***	***	1.716	27.87***	***
Q2*Conventional	1.127	75.39***	***	1.933	39.52***	***	2.782	181.62***	***	1.661	29.41***	***
Q3*Conventional	1.124	79.37***	***	1.958	38.46***	***	2.771	173.05***	***	1.692	28.98***	***
Q1*Takaful	1.157	70.05***	***	1.270	14.48***	***	2.118	85.34***	***	1.058	8.37***	
Q2*Takaful	1.129	75.09***	***	1.230	18.73***	***	2.170	96.72***	***	0.948	12.37***	
Q3*Takaful	1.137	87.92***	***	1.201	17.79***	***	2.165	133.65***	***	0.932	12.64***	
Islamic	-0.0214	-1.34		0.109	1.56		-0.0388	-1.63		0.149	1.69*	
Density	9.67e-08	1.76*		-0.000	-2.14**		0.000*	1.71		-0.000	-1.79*	
Penetration	-0.0689	-0.30		5.092	1.91*		-1.272	-2.31**		4.380	1.44	
Corruption	-0.00467	-1.26		0.0224	1.64		-0.0114	-2.68***		0.0326	1.95*	
HHI_Country	0.000	1.17		-0.000	-1.58		0.000	1.69*		-0.001	-1.81*	
2007	0.0512	1.84*		0.536	24.70***		4.428	133.38***		-0.0846	-2.46**	
2008	0.192	42.20***		6.418	37.09***		0.806	32.02***		9.588	47.49***	
2009	0.144	14.42***		0.457	25.07***		-0.363	-9.72***		0.253	8.63***	
2010	0.0996	20.05***		0.559	10.33***		0.386	254.12***		0.270	10.62***	
N	364			364			364			364		

Z_1 significantly different from 0, based on a two-tail test. Z_2 significantly greater or less than 1 based on a one-tail test. Note: The dependent variable is the ratio of the efficiency of each conventional (Takaful) firm relative to the Takaful (conventional) frontier to the efficiency of each conventional (Takaful) firm relative to its own frontier. This is a measure of the distance between the conventional and Takaful frontiers for the ith firm's input-output vector.
*, **, *** indicate significance levels of 10%, 5%, and 1% respectively.

Appendix
The DEA Methodology[10]

Distance Functions and Efficiency

To analyze production frontiers, I employ both input and output-oriented distance functions (Fare et al., 1985). Suppose a firm uses input vector $x = (x_1, x_2, \ldots, x_m)^T \in R_+^m$ to produce output vector $y = (y_1, y_2, \ldots, y_n)^T \in R_+^m$, where T denotes the vector transpose. A production technology which transforms inputs into outputs can be modeled by an input correspondence $y \rightarrow V(y) \subseteq R_+^m$, such that for any $y \in R_+^m, V(y)$ denotes the subset of all input vectors for a given decision making unit (DMU) that minimizes input consumption conditional on outputs:

$$D_1(y,x) = \sup\left\{\theta: \frac{x}{y} \in V(y)\right\}, \tag{1}$$

where θ is a scalar, i.e., a radial distance. In the output-oriented case, technology is modeled by an output correspondence $x \rightarrow P(x) \subseteq R_+^n$, such that $P(x)$ denotes the subset of all output vectors obtainable from input vector $x \in R_+^m$.

The output distance function for a DMU maximizes output conditional on inputs:

$$D_0(y,x) = \inf\left\{\theta: \frac{y}{\theta} \in P(x)\right\}. \tag{2}$$

The input distance function is the reciprocal of the minimum equi-proportional contraction of the input vector x, given output y, i.e., input-oriented technical efficiency $TE_1(y,x) = 1/D_1(y,x)$, and a similar interpretation applies for output-oriented efficiency.

The choice of input versus output orientation is based on the microeconomic theory of the firm. In microeconomic theory, the objective of the firm is to maximize profits by minimizing costs and maximizing revenues. Cost minimization involves choosing the optimal amounts and mix of inputs to produce a given output vector, and revenue maximization involves choosing the optimal amounts and combination of outputs conditional on the input vector. Hence, the input orientation is adopted to estimate technical efficiency in the cost minimization problem, and the output orientation is adopted for the revenue maximization problem.

The minimum cost function or cost frontier is defined using the distance function approach (e.g., Cooper et al., 2004). Let $x_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T \in \Re_+^m$ denote the input vector from firm j, $y_j = (y_{1j}, y_{2j}, \ldots, y_{nj})^T \in \Re_+^m$ denote the output vector from firm j, and $w_j = (w_{1j}, w_{2j}, \ldots, w_{mj})^T \in \Re_+^m$ denote the input price vector from firm j. Then the cost frontier is:

$$C(y_j, w_j) = \mathrm{Min}_{xj}\{w_j^T x_j : x_j \in V(y_j)\}, \tag{3}$$

where $C(y_j, w_j)$ = the cost frontier for firm j with output-input vector (y_j, x_j). The optimal vector x_j^* minimizes the costs of producing y_j given the input prices w_j. Cost efficiency for firm

[10] This is a modified version of the appendix in Cummins et al. (2010).

j is calculated as $\eta_j = w_j^T x_j^* / w_j^T x_j$, where x_j represents actual input usage and $0 < \eta_j \leq 1$.

The maximum revenue function or revenue frontier is defined analogously to the cost function, also using the distance function approach. Let $p_j = (p_{1j}, p_{2j}, \dots, p_{nj})^T \in \Re_{++}^n$ denote the output price vector corresponding to the output vector y_j. Then the revenue frontier is defined as:

$$R(x_j, p_j) = \underset{y_j}{Max} \{p_j^T y_j : y_j \in P(x_j)\}, \quad (4)$$

where $R(x_j, p_j)$ = the revenue function for firm j. The optimal output vector y_j^* maximizes revenue conditional on inputs x_j and output prices p_j. Revenue efficiency is calculated as the ratio $C = p_j^T y_j / p_j^T y_j^*$, where y_j is the actual output vector. Therefore, $0 < y_j \leq 1$.

Estimating Efficiency

DEA efficiency is estimated by solving linear programming problems. For example, the technical efficiency with respect to the pooled frontier is estimated by solving the following problem, for each firm, j= 1, 2, …, Q in each year of the sample period:

$$[D(y_j, x_j)]^{-1} = TE(y_i, x_j) = \min \theta_j \quad (5)$$

Subject to: $Y\lambda_j \geq y_j$,
$X\lambda_j \leq \theta_j x_j$,
$\lambda_j \geq 0$,

where Q=total number of insurers, Y is an n ×Q output matrix for all firms in the sample, and X an m × Q input matrix for all firms in the sample; y_j is an n × 1 output vector and x_j an m × 1 input vector for firm j, and $\lambda_j$is a Q × 1intensity vector.

Constraining the λ_j only to be non-negative imposes constant returns to scale. Imposing the additional constraint, $\sum_{i=1}^{Q} \lambda_{ji} = 1$ allows for VRS, and changing he constraint to $\sum_{i=1}^{Q} \lambda_{ji} \leq 1$ estimates the frontier under NIRS, where λ_{ji} is the ith element of the vector λ_j.

The following problem is solved as the first step to obtain cost efficiency of firm j:

$$\underset{x}{Min} \; w_j^T x_j \quad (6)$$

Subject to: $Y\lambda_j \geq y_j$,
$X\lambda_j \leq \theta_j x_j$,
$\lambda_j \geq 0$,

where $w_j$is an m × 1 vector of input prices, and x_j is an m × 1 vector of input quantities. As in the case of technical efficiency, constraining λ_j only to be non-negative imposes CRS. Imposing the additional constraint $\sum_{i=1}^{Q} \lambda_{ji} = 1$ imposes VRS and imposing the constraint $\sum_{i=1}^{Q} \lambda_{ji} \leq 1$ imposes NIRS. The solution of (7) is the cost-minimizing input vector for firm j(x_j^*). Cost efficiency is then calculated as explained above.

Revenue efficiency is obtained by solving the following problem:

$$\min_{y} \; p_j^T y_j \qquad (7)$$

Subject to: $Y\lambda_j \geq y_j$,
$X\lambda_j \leq \theta_j x_j$,
$\lambda_j \geq 0$

The solution is the revenue maximizing output vector y_j^*.

Appendix References

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