The Risk of Variable Annuity Guarantees and Life Insurer Capital

Dr. Etti G. Baranoff, FLMI
Associate Professor of Insurance and Finance
Department of Finance, Insurance and Real Estate
School of Business
Virginia Commonwealth University
Snead Hall,
301 West Main Str., Suite B4167
Richmond VA 23284-4000
Tel: 804-828-3187, Fax: 804-828-3972
Cell: 512-750-6782
www.professorofinsurance.com
ebaranof@vcu.edu or ettib@earthlink.net

Thomas W. Sager
Professor of Statistics
Department of Information, Risk, and Operations Management
The University of Texas at Austin, CBA 5.202
Austin, Texas 78712-1175
(512) 471-3322

Bo Shi
Assistant Professor of Finance
Department of Accounting, Economics and Finance
School of Business Administration
Morehead State University, 316 Combs Bldg.
Morehead, Kentucky 40351
(606) 783-2475

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ABSTRACT

Recently, concerns have been raised about the systemic financial threat potentially posed by life insurers. In part, these concerns have arisen because of life insurer involvement in the sales of variable annuity products with put-like performance guarantees. Guarantees expose life insurers to the market risks of mutual funds that are directed by their policyholders. In 2007, U.S. policyholders held about $500 billion in variable annuity accounts subject to guarantees issued by insurers. In this study we examine life insurer management of the risks of these guarantees, with emphasis on management of capital buffers. We introduce actuarial/regulatory and exposure-based proxies for these risks. Surprisingly, we find a robust and paradoxical result. In the years immediately prior to the Great 2008 Recession, the assumption of additional guarantee risk was associated with reductions in capital (contradicting the finite risk hypothesis), *ceteris paribus*. The result survives the attempts of two regression models designed to uncover explanatory confounders. The result can inform the debate on systemic risk and capital adequacy for life insurers.

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Section I. Introduction

Following the global financial crisis that began in 2008, the G-20 charged the Financial Stability Board (FSB) with the task of creating a system that ensures global financial stability. The first major action was the designation of 29 banks as Globally Systemically Important (G-SIB) in 2012. On July 18, 2013, the FSB also designated nine insurers as Globally Systemically Important (G-SII) using the International Association of Insurance Supervisors (IAIS, May 31, 2012) methodology. Among the factors that are cited in identifying insurers as systemically important is their involvement with variable annuities.[1] In variable annuities, policyholders invest their premiums in mutual funds within an insurance contract. In standard variable annuities, policyholders' portfolio values fluctuate with the market and without guarantee by the insurer. The concern with variable annuities arises because a significant and growing proportion of variable annuities have been sold *with* guarantees that expose insurers to the risks of financial markets.[2] In addition to the potential for global systemic risk, variable annuity guarantees also create idiosyncratic risk for the individual insurer, even if there might be no contribution to systemic risk.

In this paper, we focus on the risks that life insurers incur from variable annuities with guarantees. We use the term *variable annuity guarantee benefits risk* (VAGB risk) to refer to the risks to life insurers that arise from guarantees attached to variable annuities. These risks may be divided into two main categories: risks that arise from living benefit guarantees (LBG) that inure to the policyholder, and risks that arise from death benefit guarantees (DB) that inure to the beneficiaries of the policyholder. We give more emphasis to the LBG category since these guarantees typically expose insurers more directly to the uncertainties of the market. Although there are many variations, an LBG guarantee commonly creates a parallel portfolio that exists only on paper and that runs alongside a policyholder's actual portfolio over time, but with contractually guaranteed minimum returns. Upon exercise of the guarantee, the policyholder is guaranteed the benefit that results from the maximum of his actual portfolio value or the guaranteed value. The policyholder thereby is assured against loss, but retains some upside potential. The insurer assumes the risk that the guaranteed value (the liability) may exceed the actual portfolio value upon exercise. The insurer collects an additional premium in return for the guarantee like a fee for a rider. To assess LBG risk to the insurer, we develop a new risk metric for the potential shortfall between the asset represented by the actual portfolio value and the liability represented by the guaranteed account value. Our LBG risk metric is based upon actuarial calculations that are endorsed by the American Academy of Actuaries for this purpose.[3]

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[1] The FSB empowered the IAIS to create the methodology which includes Variable annuities. See "International Association of Insurance Supervisors, Global Systemically Important Insurers: Proposed Assessment Methodology, May 31, 2012.

[2] In citing variable annuities with guarantees as an important factor in the determination of G-SIIs, the IAIS stated: "Variable annuities most often include some type of guaranteed levels of payment to policyholders: attempting to pay guaranteed amounts could accelerate asset sales by an insurer and exacerbate already distressed market conditions. There is also the possibility that hedging strategies for guarantees could adversely affect markets in times of wider market stress." See: IAIS, May 31, 2012, p. 17.

[3] This is used in the computations for Life Risk Based Capital C-3 phase II

It is essentially a value-at-risk measure that uses simulations of the potential shortfalls in policyholders' variable annuity portfolios. See Section 3 and Appendix 1 for a detailed exposition. In addition to our actuarial metric of LBG risk, we also employ simple exposure metrics for the categories of guarantee risks. We use a death benefit exposure metric based on the value of policyholder portfolios covered by death benefit guarantees, an exposure metric for each type of the LBG,[4] and an overall VAGB exposure metric based on the value of policyholder portfolios covered by any of these types of variable annuity guarantee. See Section 4 for a detailed exposition.

The main purpose of our study is to analyze how U.S. life insurers managed their capital in the light of variable-annuity-with-guarantee risks during the immediate run-up to the financial crisis of 2008. The time period of 2006-2007 provides a natural laboratory for testing insurers' deployment of capital in relation to the creative but relatively unseasoned VAGB products in bullish times before crisis-induced reactions set in.

Our motivation is to test the finite risk hypothesis for these creative products. Under the finite risk hypothesis, there exists a positive relationship between capital and the various enterprise risks, *ceteris paribus* (see Baranoff and Sager, 2002 and 2003 and Baranoff, Papadopoulos and Sager, 2007, Baranoff, Sager, Shi, 2014). Under this hypothesis, insurers tend to balance an increase in one type of risk with a reduction in another type of risk. Thus, if the addition of guarantees to variable annuities contracts increases insurer product risk, we may expect an insurer to raise capital, *ceteris paribus*. An opposing hypothesis, the *excessive risk hypothesis* maintains that under certain conditions, the assumption of additional risk in one area can trigger still further risk taking in another, such as the reduction of capital (see Baranoff and Sager, 2002 and 2003 and Baranoff, Papadopoulos and Sager, 2007, Baranoff, Sager, Shi, 2014.) Nonetheless, most empirical studies have found evidence in favor of finite risk.

Therefore, the main finding of this paper – the inverse relationship between both types of guarantee risk and capital – is unexpected. We looked closely for possible explanations in two different econometric models with various control variables and key explanatory determinants. These included possible confounding with increasing guarantee complexity, with other product risks, with asset risk, with insurer size, modeling artifacts, and offsetting risk mitigation techniques such as use of derivatives. But after careful analysis, the inverse relationship remains. Details are provided in section 5.

Our approach utilizes two main models – a change model and a levels model, which address different confounding effects. First, we analyze the effect of short-term 2006-2007 changes in VAGB risks on changes in capital. Results of this econometric model are presented in Table 5. The purpose of the this model is to address concerns that as the complexity of VAGB guarantees has evolved over time, an insurer with a large volume of accumulated VAGB obligations may have mostly older and lower-risk guarantees, whereas an insurer with a small volume of VAGB obligations may have mostly newer and higher-risk guarantees. Relatively less capital therefore may be necessary for the larger and older than for the smaller and newer obligations. The possible temporal evolution of VAGB risk can be controlled for by analyzing the effect of additional VAGB risks assumed in 2007 on changes in capital in that year. Within-insurer

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[4] These are not presented in the models in the paper, but, available upon request

changes in VAGB risks over one year should be more risk-homogeneous than accumulated VAGB levels over several years. We also analyze the relationship between levels of capital and levels of VAGB risks in the context of a variety of other insurer risks and controls. These model results are shown in Table 6.

Unexpectedly, we find a robust result that VAGB risks are inversely related to capital – as though life insurers were risk-seeking in these guaranteed policy lines. These results hold for our new actuarial LBG risk metric, as well as for our exposure metrics – in both econometric models, whether short-term changes or levels. Our results are paradoxical because the models control for a number of major insurer risks and risk-mitigation variables, such as use of derivatives. Model results for other risks are much as expected.

Although the inverse relationship of VAGB risks to capital is real, the magnitude of its threat to individual insurers or to the financial system may not be urgent. In 2007, the U.S. life insurance industry had issued LBGs on some $508 billion of customer portfolio market value. By the actuarial/regulatory guarantee risk proxy we created, the LBG risk of these guarantees amounted to about $74 billion for the assumed model portfolios of policyholders we constructed. These risks are spread among many insurers and many policyholders. Consistent with other research on systemic risk such as Gallanis (2013) who found that the history of U.S. life insurer insolvencies does not suggest potential for destabilization of the financial system, we would not suggest the guarantees may have de-stabilizing systemic risk characteristics. In a study of standalone Dutch reinsurance companies, Lelyveld, Liedorp and Kampman (2011) found no evidence of systemic risk in the Netherlands, even if multiple reinsurers were to fail simultaneously.

This paper has three major contributions. First, it provides and discusses the role of a new product risk factor, the risk of variable annuity guarantees, in the enterprise risk relationships of life insurers, especially relative to their capital. Second, based on the interest of the FSB and the IAIS, this paper can inform the debate on capital adequacy for variable annuity products with guarantees.[5] Third, it introduces a new actuarial/regulatory proxy for the living benefits guarantee risk of the variable annuity products of life insurers.

The paper is structured as follows: We begin with background information on variable annuities and the new guarantees in section 2. Section 3 explains the new actuarial/regulatory guarantee risk proxy (with a detailed example in Appendix 1). Section 4 provides summary statistics on the VAGB products. The analyses, models and results are provided in Section 5. The paper concludes with a summary in Section 6.

Section 2. Variable Annuities with Guarantees

Life insurers have long marketed private solutions for old age financial security in the form of annuities. An annuity is a contract between an individual and an insurance company that assures a future flow of income to the individual in return for accumulated premium payments. In the

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[5] For example, see "Variable Annuities – An Analysis of Financial stability" by the Geneva Association 2013 at https://www.genevaassociation.org/media/618236/ga2013-variable_annuities.pdf.

basic annuity, the accumulation amount and the income stream are fixed. In recent decades, life insurers have augmented the basic annuity with additional innovative features:

- The variable annuity – replaces the fixed accumulation amount with an amount that is prospectively uncertain because it depends upon the investment performance of the policyholders' accumulated premium payments. Upon annuitization, the accumulation is converted into an income stream that may be fixed, but may also be variable. The policyholder exercises a degree of control over allocations of his or her premiums and accumulation among a limited number of sponsored equity and fixed income mutual funds. In the standard variable annuity, the risk of poor investment performance lies upon the individual policyholder, not upon the insurer. The insurer administers the account and charges fees but bears no liability for poor performance.
- Guarantees – have recently been added to variable annuities to reduce downside investment risks to policyholders, who pay additional fees (premiums). In guaranteed products, insurers assume the risk that investment performance of the assets in the variable annuity account will be insufficient to fund the guarantees, which are the liability of the insurer. Insurers may limit their own downside risks in these guarantees by risk management techniques, including adding to capital, dynamic hedging and use of returns.

Over the past two decades, sales of variable annuities with guaranteed benefits (VAGB) have grown dramatically. A VAGB contract consists of a regular variable annuity bundled with additional contractual provisions (riders) that offer various assurances to policyholders. For example, guarantees may bind the insurer to credit the policyholder with a minimum annual rate of return on his portfolio accumulation, even if the market value of the portfolio declines.[6] Other forms of guarantees may protect the annuitization income or withdrawal benefits of the policyholder. A popular type is a death benefit guarantee that pays beneficiaries a guaranteed amount even when the policyholder dies during down markets. In each case, *the guarantee generates risk to the insurer*. This is the risk that the policyholder's account value will be insufficient to fund the guaranteed liability of the insurer.

There were four common types of guarantees in variable annuities in 2006 and 2007. These guarantees protect annuitant income, accumulation value, withdrawal rights, and beneficiary benefits in case of policyholder death. The first three of these guarantee types are called living benefits (LBG) because they insure a benefit to the annuitant while living; the fourth is called a death benefit (DB) because it guarantees a minimum payment to the beneficiary if the policyholder should die. When necessary, we refer to all four types collectively as VAGB. The main types of LBG – living benefit guarantees – are as follows:

- *Guaranteed Minimum Income Benefits* (GMIB) protects the annuitization income of the portfolio by guaranteeing a formulaically computed minimum periodic income upon annuitization.
- *Guaranteed Minimum Accumulation Benefits* (GMAB) protects the accumulated value of the annuity from market fluctuations by guaranteeing the greater of the actual assets in the account value or a formulaically computed minimum that increases over the accumulation period.

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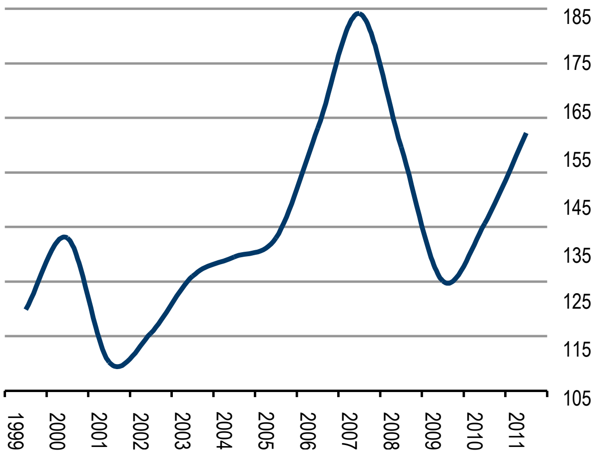
[6] The popular media have covered these products. See for example the Wall Street Journal article from 7/24/2009 "Long Derided, This Investment Now Looks Wise Thanks to Guarantees, Variable Annuities Paid even when Stocks Didn't" by Leslie Scism.

- *Guaranteed Minimum Withdrawal Benefits* (GMWB) protect against illiquidity during the deferment (accumulation) period by guaranteeing the policyholder the right to withdraw a contractual percentage of his account value, without further fee, during and perhaps after the deferment period. GMWB is often combined with features of GMAB and GMIB to provide a product of appealing flexibility.

Each VAGB contract has a portfolio of assets in the account. This account belongs to the policyholder and is required to be separate from, and not commingled with, the life insurer's own accounts. Each life insurer reports the combined market value of its VAGB policyholders' portfolio accounts in its annual statement filed with the National Association of Insurance Commissioners (NAIC). This total asset value for all variable annuities with guarantees is called the *total related account value*. The annual statement also breaks down the insurer's total related account value by the type of guaranteed benefit.

The market for variable annuities boomed from 2002 until the crisis in 2008. Most of the growth occurred in VAGB. Sales were impacted negatively by the Great Recession, but started to rebound in 2009 (see Figure 1). In 2007, the total related VAGB account value was about $1.5 trillion, and the total related LBG account value exceeded $500 billion. From the end of 2006 to the end of 2007, there was a surge in the total related LBG account value of US life insurers, which grew from $383 billion to $508 billion. At the same time, the number of US life insurers offering LBG increased from 79 to 85. Approximately 150 US life insurers offered variable annuities with some kind of guarantee (including death benefit guarantees). See Table 1 for further industry statistics as of 2007.

Figure 1. Annual U.S. Variable Annuity Sales in $ billions, 1999 – 2011



Graph Source: US Variable Annuities by Credit Suisse
Data Source: U.S. Individual Annuities Sales Survey by LIMRA

Literature Review

The literature focuses upon two aspects of this new product risk: the risk of mispricing and the risk of adverse selection. Both of these are fundamental aspects of the guarantee risk.

Milevsky and Posner (2001) value guaranteed death benefits using risk-neutral option pricing theory and estimate the fair mortality and expense risk charge under certain assumptions on volatility and interest rates. Their estimate is much lower than the actual median mortality and expense risk charge of the US insurance industry, per Morningstar Variable Annuity & Life Performance Report of 1999. Although not explicitly stated in their paper, the inference may be drawn that overpricing of death benefit guarantees probably reduces sales of VA with guaranteed death benefits.

Guaranteed living benefits (LBG) are new and the contractual guarantee riders are complex. Guaranteed withdrawal rights are among these living benefits. Milevsky and Salisbury (2006) develop several methods to value guaranteed withdrawal benefits. They find that guaranteed minimal withdrawal benefits were underpriced for 12 major insurers as of mid-2004. Piscopo (2010) estimated the actuarially fair cost of withdrawal benefits at 79-145 basis points, whereas the prevailing charge was actually 50-70 basis points, which implied underpricing. Bauer, Kling and Russ (2008) priced both the guaranteed death benefit and various guaranteed living benefits under a general framework and discovered that guaranteed minimum income benefits were underpriced as well.

In summary, the valuation/pricing literature suggests that insurers overcharged for death benefit guarantees but undercharged for living benefit guarantees. Underpricing of LBG may be expected to contribute to the rapid growth of the LBG market. On the other hand, such underpricing exposes insurers to greater financial risk.

The existence of guarantees against poor investment performance may be expected to create a moral hazard that encourages policyholders to allocate their funds to greater investment risk than they would in the absence of guarantees. One may expect to find that LBG policyholders assume more investment risk than policyholders of variable annuities without guarantees. Using 660,336 individual variable annuity contracts provided by Life Insurance Marketing Research Association (LIMRA) between January 2000 to June 2004, Milevsky and Kyrychenko (2007) compared the asset allocations of policyholders with guaranteed minimum income benefits to the allocations of a control group without guaranteed minimum income benefits. They found that policyholders with the guarantees assume more equity exposure than policyholders without the guarantees. In Milevsky and Kyrychenko (2008), the authors interpret the more aggressive risk-taking of guaranteed policyholders as exercise of a longevity put option embedded in guaranteed benefits. Guarantees permit policyholders to increase the variance of their returns, thus increasing the probability of capturing more upside gains while avoiding the usual concomitant increase in probability of losses on the downside. The protection that insurers are obligated to provide may squeeze insurers’ capital during a market downturn, while insurers’ own asset portfolios suffer as well.

Section 3. The LBG Actuarial/Regulatory Risk Proxy

The risks that variable annuity guarantees pose to life insurers are central to this paper. We are especially interested in the risks of living benefit guarantees. In order to analyze consequences of those risks, it is necessary to measure them. One of the contributions of this paper is the introduction of a new metric for the LBG risk of an insurer. To measure overall VAGB risk generically and DB (death benefit) risk specifically, we use a simple exposure metric to be explained in Section 4. We also introduce an actuarial metric that is explained in this section. Our LBG risk proxy is essentially a value-at-risk (VaR) measure, based on the work of the Variable Annuity Reserve Working Group of the American Academy of Actuaries.[7] Via simulations, the method estimates the potential shortfall in insurer liabilities that are necessary to protect against a perilous tail event of a given probability originating from the insurer's LBG obligations. We provide details on the simulation later in this paper.[8]

Living benefit guarantees vary from contract to contract. To simplify the welter of contractual terms for LBG, we propose an archetype contract for each of the three main classes of LBG. Each LBG archetype that we propose has standard contractual terms:

- *GMIB (income) archetype contract*. There is a single initial investment. The deferment (accumulation) period is ten years. At the end of the deferment period, the annuitant receives a lifetime annual income equal to 5% of the greater of the actual account value or the single initial investment compounded at 5% per annum throughout the deferment period.
- *GMAB (accumulation) archetype contract*. There is a single initial investment. The deferment period is ten years. At the end of the deferment period, the policyholder has a choice: (i) receive as a lump-sum the greater of actual account value or 1.2 times the single initial investment, or (ii) annuitize and receive a lifetime annual income of 5% of the greater of the actual account value or 1.2 times the single initial investment. We suppose that 50 percent of the policyholders choose the lump-sum cash-out, and 50 percent choose to annuitize.
- *GMWB (withdrawal) archetype contract*. There is a single initial investment. The policyholder may withdraw 7% of the single initial investment annually, without penalty fee, until the initial investment is exhausted in 14.2 years. However, each such withdrawal reduces the actual account value. If the actual account value reaches zero on account of poor investment performance before the single initial investment has been recovered, withdrawals may nonetheless continue. After 14.2 years, all of the single

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[7] VARWG proposed a variable annuity reserve valuation method that was presented to the Life and Health Actuarial Task Force and passed by the NAIC in September, 2008. The variable annuity reserve valuation method is also called the Actuarial Guideline XXXXIII (AG 43), corresponding to the previous AG 34 for products with guaranteed death benefits (GMDB) and AG 39 for products with guaranteed living benefits (GMLB). AG 43 has been in effect since December 31, 2009. The scope covers all variable annuity products sold after 1981. The new reserve valuation method is highly consistent with the calculation of risk based capital C3 Phase II, effective in December 2006. In fact, both the reserve valuation calculation and the C3 calculation use the same simulation methodology.

[8] Appendix 1 provides examples. We also have the actual computations for the shortfall of the assets against the liabilities of the guarantees, available upon request.

initial investment will have been recovered. At that time, if the actual account value is positive, the policyholder receives the residual account balance as a lump sum payment.

The archetypes seem reasonably representative of their corresponding guarantee types, based upon our review of some available LBG prospectuses. Some that we reviewed were more generous than the archetypes, others less.

Many policies combine two or more of the four main VAGB guarantees, the annual report data from the NAIC break out total related account value by combinations of the four VAGB types. To simplify matters, we assign VAGB contracts with multiple guarantees to one and only one of the LBG archetypes – the withdrawal type (GMWB), which has the least severe risk consequences to the insurers among the three single-benefit types in our simulations (see Appendix 1-Table 1).[9] However, by essentially dropping all but one guarantee from multiple-benefit contracts, our assignment rule is therefore conservative in the computation of LBG risk: GMWB is less risky than GMWB combined with GMIB or GMAB.

Table 1: VAGB Exposure Totals for LBG Insurers and for All VAGB Insurers in 2007

Aggregate variables	**VAGB Insurers (N=143)**	**LBG Insurers (N=83)**
Total assets	$4,061,201	$3,051,759
Total Related Account Value	$1,415,250	$1,340,551
GMIB Account Value	$184,202	$184,202
GMAB Account Value	$58,000	$58,000
GMWB Account Value	$228,638	$228,638
GMDB Account Value	$944,409	$869,711

Figures in the table are in millions

To assess the guarantee risk of LBG to an insurer, we introduce a new metric for LBG risk based upon the actuarial guideline of Variable Annuity Reserve Valuation Method (VARVM), developed by the American Academy of Actuaries for the NAIC.[10] The financial risk to which an LBG insurer is exposed is the possibility that the insurer will have to fund the shortfall between the guarantee value (its liability) and the value of the assets in the policyholder's account. Therefore, VARVM attempts to estimate potential future shortfalls by simulation throughout the duration of the contractual guarantees. In any given iteration of the simulation, VARVM generates two 30-year future projections for a policyholder account: one projection for the growth of the guarantee value over time, and another projection (by simulation) for the changing value of the investment portfolios in the account. The future projection of the guarantee value is

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[9] There are 6 companies in 2006 and 7 companies in 2007 that sell some contracts with multiple guaranteed benefits. Almost all multiple guarantee contracts that we found had a withdrawal guarantee. The most common combination of guarantees is GMWB packaged with GMIB. GMAB is rarely combined with the other living benefits. In the NAIC database, we find only one insurer selling multiple guarantee contracts with GMAB and GMIB in 2007.

[10] Launched by the Variable Annuity Reserve Work Group (VARWG) of the American Academy of Actuaries and passed by the National Association of Insurance Commissioners (NAIC) in September, 2008, as part of Life Risk-Based Capital, C3, Phase II.

governed by the terms of the guarantee specified in the insurance contract. The future projection of the account value is prospectively unknown, but is determined by simulating asset returns based on historical data. At any future time, the potential shortfall equals the difference between the future guarantee (the liability) and the future simulated account value (the asset) at that time. At a given time, there is no potential shortfall if the future guarantee less future account value is negative at that time. Over a 30-year span in a single iteration, potential shortfalls can develop and disappear several times, depending upon the simulated asset returns. For a given iteration of the simulation, the maximum shortfall over the duration of the contract is taken to be the insurer's potential guarantee risk exposure. 10,000 iterations are run, yielding a maximum shortfall for each iteration. The VARVM risk measure is the mean of the largest 3,000 of these 10,000 maximum shortfalls. Thus, the VARVM measure is a Value-at-Risk metric, the 30-percent conditional tail expectation (30% CTE) of the maximum shortfall distribution. After some further adaptations, we adopt the VARVM measure as our LBG risk metric and we call it actuarial/regulatory guaranty risk proxy. Our assumption is that the larger the LBG risk, the more an LBG insurer may need to increase capital to buffer possible future shortfalls.

Among the LBG risks, only withdrawal benefits (GMWB) have a double trigger: the simultaneous occurrence of a substantial market decline with a withdrawal request. By contrast, both the income (GMIB) and accumulation (GMAB) risks have but a single trigger.

Clearly, the adequacy of our guarantee risk proxy depends critically upon having a reasonable model of future market returns for the asset classes for which the guarantees are made. The modeling of future market returns is explained in Appendix 1.

We employ three model portfolios for the LBG accounts that policyholders hold with life insurers: 50% equity and 50% bond, 60% equity and 40% bond, and 80% equity and 20% bond, but show only the 60% equity and 40% bond computation for each of the 3 guaranteed benefits contracts in Table 2. To be sure, actual LBG portfolios display a wide range of investment options. But the three model portfolios represent commonly occurring asset allocations in LBG accounts. For each model portfolio, we calculate the LBG risk as of the end of 2006 and 2007 for each contract with a living benefits guarantee.

Table 2. LBG Risk by subtypes for Insurers having LBG Exposure. 60% equity / 40% bond portfolios. Figures in million USD.

Year	Guaranteed Benefit Products	N	Industry Sum	Mean	Std Dev
2006	Guaranteed Minimum Income Benefit (GMIB)	61	33,660	552	1,001
	Guaranteed Minimum Accumulation Benefit (GMAB)	33	5,916	179	375
	Guaranteed Minimum Withdrawal Benefit (GMWB)	52	15,243	293	558
	LBG risk = GMIB + GMAB + GMWB	71	54,818		
2007	Guaranteed Minimum Income Benefit (GMIB)	62	41,597	671	1,732
	Guaranteed Minimum Accumulation Benefit (GMAB)	38	10,857	286	593
	Guaranteed Minimum Withdrawal Benefit (GMWB)	65	21,710	334	624
	LBG risk = GMIB + GMAB + GMWB	79	74,163		

Section 4. Data

In this paper, we wish to investigate the relationship between capital and the risks of variable annuity guarantees, in the context of other key risks and controls, for 2006 and 2007. We regard the 2006-2007 timeframe of the study as an appropriate period for testing insurer attitudes toward the risks of the relatively new VAGB products. This period is a stable time of consistency and growth in VAGB markets and products. With the coming of the crisis, consistency could no longer be assured.

Our key variables are capital, LBG and other product risks, and asset risk in 2006 and 2007, and changes in these variables between 2006 to 2007. Capital is taken as the ratio of book value of insurer capital to total insurer assets. The calculation of LBG guarantee risk is explained in Section 3. In the analysis, LBG risk is scaled by dividing by the total related account value (total actual asset values in the policyholders' related accounts) to help remove the effects of insurer size. For other product risks and guarantee death benefits or living benefits by contract, we also use exposure measures, as discussed below. For asset risk, we use a volatility-of-returns measure, called *opportunity asset risk* (OAR), as in Baranoff, Papadopoulos and Sager (2007). Like capital and all product and guaranteed contract risks, the asset risk measure is also scaled to adjust for insurer size. For some analyses, capital, LBG and asset risks are used in logarithm form to adjust for skewness. "Log" is prepended to the name of the variable when used in log form. Changes in these variables between 2006 to 2007 are then calculated for each insurer for use in the capital change models.

VAGB risks are only one category of the product risks to which life insurers are exposed. Life insurers typically sell a mix of insurance products that include life insurance, annuities, health and accident insurance, and reinsurance. The particular balance of an insurer's product mix reflects the insurer's *business strategy*, a phrase introduced by Baranoff and Sager (2003). Each product type exposes an insurer to different risks. Baranoff and Sager (2003) argue that the major categories of life insurance products (annuities, life insurance, health and accident insurance, reinsurance) carry distinctive risk characteristics that differentially inform insurer behavior. They find that the greater the uncertainty regarding the outcome of an insurance product, the greater the impetus for accumulating capital. Following their lead, we use the proportion of premiums written in each line (pAnnuity, pLife, pHealth) to capture business strategy effects.[11] Since the sum of all proportions is 100%, we omit pHealth in order to avoid a near multicollinearity in the models. These business strategy product risks are present in our models as exposure measures, unlike our LBG risk measure, which is an actuarially based calculation, as explained in Section 3. Furthermore, VAGB risks do not overlap with the risks represented by pAnnuity, since VAGB accounts are not commingled with insurer accounts.

Although death benefits (DB) are among the four major VAGB risks and could be proxied in a manner similar to our actuarial proxy for the three LBG types, we have chosen to represent DB in our models by an exposure proxy, in the manner of a business strategy variable. There are several reasons for different treatment. First, the literature suggests that DB riders may be

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[11] Reinsurance business is relatively small and concentrated in relatively few insurers.

overpriced (e.g., Milevsky and Posner, 2001), whereas LBG riders may be underpriced (Milevsky and Salisbury, 2006; Piscopo, 2010; Bauer, Kling and Russ, 2008). Second, the literature also suggests that the existence of LBG guarantees may create a greater investment risk taking by policyholders (Milevsky and Kyrychenko, 2007, 2008). (See Section 2 for more detail.) Third, life insurers have long experience in managing the financial consequences of perils with a mortality trigger, but only a short and (during the time of our study) untested experience with LBG perils. Moreover, a DB shortfall in the actuarial computation requires both triggers – a DB shortfall cannot occur without simultaneously a sufficiently severe market decline and a coinciding policyholder death. By contrast, both income (IB) and accumulation (AB) shortfalls require only one trigger: a sufficiently severe market decline. A withdrawal (WB) shortfall also requires two simultaneous triggers: a sufficiently severe market decline and a coinciding withdrawal request. However, whereas it is likely that there is essentially no correlation between mortality and market declines, it is likely that there may be substantial correlation between withdrawal requests and market declines. All of these considerations suggest a distinctiveness to DB risk. Although there is clearly insurer risk to being involved with DB, the 30 percent conditional tail expectation for DB shortfalls that is endorsed by the AAA may be too conservative in its computation to record insurer risk meaningfully. Instead, we choose to represent DB risk by exposure, following the argument of Baranoff and Sager (2003) that an insurer's product mix differentially informs its capital/risk relationships. We measure DB exposure (and LBG exposures, as well) by the total related account value of each type of guarantee, scaled in proportion to the sum of all types. Since the sum of DB and LBG proportions equals 1, if we wish to include DB in a model, we drop LBG exposures in order to avoid a collinearity.

To provide a business strategy proxy for overall VAGB product risks, we use pVAGB which is equal to total account value of variable annuities with guaranteed benefits, divided by life insurers' total assets. Note that the numerator of pVAGB tallies the related account value for all types of guarantees, both living benefits and death benefits. By including death benefits as well as living benefits, this business strategy variable therefore looks to the offering of any type of variable annuity guarantee as the essential strategic and behavioral differentiator among insurers, rather than the type of the guarantee.

We rationalize the inclusion in our study of both VAGB risk and LBG risk variables by the fact that they maintain separate functions in the analysis. The VAGB risk proxies (pVAGB and pDB) are used as controls for business strategy rather than as actuarial/regulatory measures of guarantee risk like the LBG risk proxy. pVAGB is an exposure measure of product risk. To the extent that pVAGB does overlap LBG risk, the presence of pVAGB as a predictor in our models may attenuate the effect of LBG risk, therefore conservatively reducing the significance that we find for LBG risk.

Asset risk (investment risk) is a major risk factor for life insurers, which have large asset portfolios reflecting the long-term nature of life insurer obligations. For asset risk, we use a volatility-of-returns measure known as opportunity asset risk (OAR), the calculation of which follows Baranoff, Papadopoulos, and Sager (2007). Briefly, the calculation for a given insurer for a given year begins with the values of 16 asset classes as reported on the insurer's annual statement. Generally, the values are adjusted book values. To each of the 16 asset class values is applied the monthly percentage changes in a corresponding market index. We use a market index because actual returns of individual insurers' portfolios are not available to us. For example, for the stock portfolio we multiply the value of insurer stocks by monthly percentage changes in the S&P 500 stock index. The product of asset class value and monthly percentage changes in the corresponding index yields 12 monthly returns for the asset class. We then sum the returns of the 16 asset classes, by month, to yield 12 monthly returns for the insurer for the given year. OAR is then the standard deviation of the 12 monthly returns, scaled by dividing by total invested assets to adjust for insurer size. OAR provides a measure of the asset risk life insurers could have experienced if they had invested their asset portfolio in corresponding market indices.

Three of our control variables represent measures of firm size (total assets, total premiums, and total liabilities). These are strongly correlated with each other. A principal components analysis suggests that the first principal component of the three (log) variables explains well over 90% of their variation and weights each variable about equally, whether the components are extracted in standardized or unstandardized mode. To reduce the possibility of multicollinearity, these three variables are combined by taking the logarithm of their geometric mean (logSize). Applying the logarithm also reduces the skewness of the distribution.

Several other predictors are suggested by studies of life insurer capital. The life risk-based capital ratio (RBCratio) is included as an indication of regulatory forbearance. As a performance and/or earnings indicator, return on capital (RetOnCap) is included (e.g., Berger, 1995; and Berger and Patti, 2006 – both studies for banks). This is important to indicate the success of the markets in 2006 and 2007 which may explain some of the results as will be shown in our level and change models. Insurers that are members of an affiliated group of firms may have superior access to investment opportunities and may have different mechanisms for monitoring and/or controlling managerial performance and structuring their capital and asset risk. To capture this possibility, we include a 0-1 indicator for whether the insurer is a member of a group (NGROUP). Stock insurers and mutual insurers are faced with different financing costs when raising capital (Laux and Muermann, 2010). Therefore, stock insurers and mutual insurers may manage their capital differently. So we include a 0-1 indicator for whether an insurer is a stock firm (NTYPE).

Underwriting VAGB products exposes life insurers to additional market risk. To manage the additional market risk of VAGB products, some insurers rely not only on capital, but also on derivative hedging. Life insurers' use of derivatives to hedge financial risks caused by their asset and product portfolios is not new. Hoyt (1989), Colquitt and Hoyt (1997), and Cummins, Phillips and Smith (2001), among others, have studied the use of derivatives to hedge life insurer financial risks. These authors do not separate derivative use by intended risk to be mitigated. Nor do we. The specific purpose of derivative use cannot be ascertained from annual statement data. Presumably at least some use is targeted for VAGB risks. VAGB insurer white papers indicate that dynamic hedging is an integral part of these products.[12] An insurer's posture toward capital

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[12] See: Prudential White Paper "The Importance of Financial Risk Management in Today's Variable Annuity Market" (2010) at: http://www.prudential.com/media/managed/Financial_Risk_White_Paper_Client.pdf . Also see for example, "Variable Annuities with Guarantees and Use of Hedging" (2011) at http://www.genevaassociation.org/PDF/Insurance_And_Finance/GA2011-I&FSC10.pdf.

accumulation is affected by its willingness to manage risk by hedging. Therefore we include a 0-1 indicator for whether an insurer is active in derivative use (IndDeriv). Worthy of note is the fact that in the methodology for designation of Globally Systemically Important Insurers (G-SII), the use of derivatives is included as a contributor to potentially being systemically risky. Table 3 lists definitions of all variables used in our analysis.

Table 3. List of variables

Variable Name	**Definition**
CAP	Book capital / Total assets
pLBG	Living Benefits Guarantee Risk / Total related account value (†)
pVAGB	Variable annuities with guaranteed benefits total account value / Total assets
pDB	Guaranteed minimum death benefit account value / Total related account value
AssetRisk	Opportunity asset risk / Total invested assets
Size	log(Total assets*Total premiums*Total liabilities)/3
pLife	Life premiums / Total premiums
pAnnuity	Annuity premiums / Total premiums
RBCratio	100*Book capital / (2*Authorized capital)
RetOnCap	Income / Book capital
Ntype	Organizational type (1=Stock)
Ngroup	Indicator for member of affiliated group (1=Yes)
IndDeriv	Indicator of derivative activity (1=Yes)

Transformation in the level model	**Changes in the change model**
LogCAP = log(CAP)	Δ CAP = CAP07 - CAP06
LogpLBG = log(0.01+pLBG)	Δ pLBG = pLBG07 - pLBG06
LogpVAGB = log(pVAGB)	Δ pVAGB = pVAGB07 - pVAGB06
LogpDB = log(0.01+pDB)	
	Δ pOppARisk=pOppARisk07 - pOppARisk06
	Δ pLife=pLife07 - pLife06
	Δ pAnnuity=pAnnuity07-pAnnuity06
LogRBCratio=log(RBCratio)	Δ RBCratio = RBCratio07 - RBCratio06
	Δ RetOnCap=RetOnCap07 - RetOnCap06

†Numerator is the raw LBG risk metric discussed in Section 3 – the mean of worst 30% of simulation shortfalls.

In 2006, 152 life insurers were active in VAGB, with 79 of these active in LBG. In 2007, 151 life insurers were active in VAGB, with 85 of these active in LBG. From this total, we dropped 13 LBG insurer-years on account of outliers or anomalous values, and one on account of undue statistical influence on coefficients. Ultimately, we could use 144 VAGB insurers in 2006 (71 LBG), and 143 VAGB insurers in 2007 (79 LBG) in our analyses. Table 4 shows summary statistics on these insurers for 2007. All data are from NAIC annual statements filed for life insurers in 2006 and 2007.

Table 4. Summary statistics for life insurers that underwrite and for the subset that underwrite LBG in 2007

Levels Summary Stats in 2007

	VAGBInsurers (N=143)		**LBG Insurers (N=83)**	
Ratio Variables	**Mean**	**Std Dev**	**Mean**	**Std Dev**
CAP	0.0931	0.1141	0.0600	0.0371
pLBG	0.0499	0.0502	0.0499	0.0502
pVAGB	0.3600	0.3791	0.5108	0.3867
pGMDB	0.8256	0.2403	0.6981	0.2480
Asset Risk	0.0022	0.0009	0.0021	0.0008
pLife	0.2951	0.2944	0.2308	0.2448
pAnnuity	0.5277	0.3610	0.6267	0.3294
RBCratio	853.5726	1710.2100	680.5861	727.6594
RetOnCap	0.0407	0.2690	0.0406	0.2564
Indicator Variables	**N**		**N**	
Stock insurers	130		75	
Group members	138		82	
Derivative users	87		61	

Changes Summary Stats

Variable	**Mean**	**Std Dev**
Δ CAP	0.0002	0.0234
Δ pLBG	0.0073	0.0173
Δ pVAGB	-0.0059	0.1340
Δ Asset Risk	0.0002	0.0010
Δ pLife	0.0019	0.1200
Δ pAnnuity	-0.0120	0.1528
Δ RBCratio	-71.2618	593.4396
Δ RetOnCap	-0.0338	0.2879

Section 5. Methodology, Models and Results

In preliminary examination of our raw data, we observed the fact that LBG insurers maintain *lower* capital ratios than other VAGB insurers (mean CAP = 0.0600 for LBG insurers vs mean CAP = 0.1389 for other VAGB insurers in 2007). Further preliminary investigation showed that the capital ratios of LBG insurers remain lower than the capital ratios of other VAGB insurers among large insurers, among small insurers, among insurers that use derivatives, among insurers that do not use derivatives, among insurers with high VAGB exposure, among insurers with low VAGB exposure – or among combinations of these attributes. Moreover, VAGB insurer capital ratios in 2007 were also much lower than the capital ratios of insurers that write no VA guarantees (means of 0.0933 and 0.3750, respectively). Unless accounted for by other factors, this would be a paradoxical inversion of expectation, since assumption of LBG risk could be expected to be balanced by offsetting *additions* to capital, *ceteris paribus*. The simple preliminary analysis therefore challenges the finite risk hypothesis for life insurers active in VAGB.

Seeking a rigorous testing of the finite risk hypothesis, we essayed a regression analysis to control for major factors that could be expected to affect the capital-risk relationship among VAGB insurers. This regression analysis took the form of two models, displayed below as Eqn 1 and Eqn 2, with empirical results for each model shown in Tables 5 and 6, respectively. Both models test the finite risk hypothesis by searching for confounding variables among a variety of relevant enterprise risks and other controls – one model in *change* form (Eqn 1) and the other model in *level* form (Eqn 2). The two models are complementary, rather than competing. Neither provides a sufficient test in itself. Selection of variables (Section 4) for the models is motivated by conventional capital structure modeling, combined with the special concerns of the current study (see Baranoff and Sager, 2002 and 2003 and Baranoff, Papadopoulos and Sager, 2007, Baranoff, Sager, Shi, 2014).

Table 5 shows the empirical results for the following *change* model (Eqn 1), in which all variables are entered as insurer-specific changes between 2006 and 2007, except for size, which is present in level form (as of 2007) to enhance the effect of scaling as a control:

$$\Delta CAP_j = \beta_0 + \beta_{LBG} \Delta LBGrisk_j + \boldsymbol{\beta}' \cdot \Delta \mathbf{x}_j + \varepsilon_j \quad \text{[Eqn 1]}$$

The subscript *j* denotes the insurer. Because of the importance of the $\Delta LBGrisk$ predictor and its coefficient β_{LBG}, its term has been separated out from the other predictors in the statement of the model equation. The other predictors are represented by $\Delta \mathbf{x}$ (the vector of the changes in the continuous variables) and their vector $\boldsymbol{\beta}$ of coefficients. The major variables are scaled to help control for insurer size, but it is not necessary to correct for skew in the changes. There is only one observation on each insurer, yielding a sample of 143 observations in all. Therefore, the model for the Eqn 1 regression is a simple OLS.

Table 6 shows the results for the following *level* model (Eqn 2), in which all variables are entered as their actual values in 2006 or 2007:

$$\log CAP_{ij} = \beta_0 + \beta_{LBG} \log LBGrisk_{ij} + \boldsymbol{\beta}' \cdot \mathbf{x}_{ij} + \varepsilon_{ij} \qquad \text{[Eqn 2]}$$

The subscript *i* denotes the year (2006 or 2007) and the subscript *j* denotes the insurer. Because of the importance of the $\log LBGrisk$ predictor and its coefficient β_{GR}, its term has been separated out from the other predictors in the statement of the model equation. The other predictors are represented by the vector $\mathbf{x}$ and their vector $\boldsymbol{\beta}$ of coefficients. No lags of *CAP* are used as predictors. This is a panel model, with 144 insurers, all but one of which have values in both 2006 and 2007. Thus, there are 287 observations. Since the data are a panel dataset, the error term ε has a covariance structure that makes OLS regression inefficient. In particular, the duplicate appearance of nearly all insurers in the dataset, once in 2006 and once in 2007, makes it likely that the errors are autocorrelated within insurers. The possibility of heteroscedastic errors (different error variance among insurers) also exists. Although the OLS coefficient estimates remain unbiased under such conditions, they will be inefficient – meaning that larger sample sizes than usual are required to achieve satisfactory statistical power in hypothesis tests. We address these issues first by using Generalized Estimating Equation (GEE) methodology (Liang and Zeger, 1986), as implemented in SAS (PROC GENMOD). Within the GEE framework, we model the error autocorrelation as autoregressive order 1. Second, scaling and logging the major variables strongly mitigate potential heteroscedasticity, as well as correcting skew, so that our key predictors end up statistically significant.

The purpose of the change model (Eqn 1 – Table 5) is to investigate an important possible confounding effect that otherwise cannot easily be accounted for – in particular, by the level model of Eqn 2. Suppose that the complexity of VA guarantees has been increasing over time and that greater complexity correlates with greater risk. Then past LBG sales may have required less risk mitigation (including capital buffering) by insurers than recent LBG sales. If that is so, then a large current volume of LBG obligations may represent a long-term accumulation of LBG sales, much of which may have been written under older, less complex, and less risky contractual terms. By contrast, a small current volume of LBG obligations likely represents a more recent accumulation – hence more complex and riskier. The large accumulation therefore might pose a lower actual level of LBG risk relative to its size than the small accumulation. For historical reasons, insurers with large LBG obligations might therefore have buffered their LBG risks with relatively less capital, *ceteris paribus*, than similarly positioned insurers with small LBG obligations. This effect could account for the lower capital ratios that we observed among insurers with large LBG risk in our simple preliminary analysis. The effect is not controlled for in the level model of Eqn 2.

One way to control for this potential confounding effect and to negate possible long-term trends in LBG complexity is to analyze the effect of short-term change in LBG obligations. Presumably, LBG obligations acquired in calendar 2007 will be more nearly homogeneous in their complexity and risk than the level of LBG obligations acquired over a period of several years. Therefore, in a modeling environment of appropriate controls, changes in capital ratios that occur in 2007 might be interpreted as responses to corresponding changes in LBG risk that occur in 2007. One would expect a positive relationship, *ceteris paribus*. The change model of Eqn 1 is designed to test this hypothesis.

Table 5. Regression model results for VAGB insurers, N = 143 insurers, changes from 2006 to 2007. Response variable = ΔCAP. See Table 3 or the text for variable definitions.

	N=143	
Predictor	Estimates	P-Val
Intercept	-0.0235	0.3209
ΔpLBG	-0.2336	0.0975
ΔpVAGB	-0.0510	0.0004
ΔAssetRisk	0.3952	0.8484
ΔpAnnuity	-0.0145	0.2414
ΔpLife	0.0292	0.0636
ΔRBCratio	0.0000	0.0403
ΔRetOnCap	0.0034	0.6104
Size	0.0011	0.3171
	$Adj.\ R^2 = 0.1277$	

A quick scan of Table 5 shows that the hypothesis of *finite risk* is not confirmed: Increases in LBG risk from 2006 to 2007 are associated with *decreases* in capital ratios, even in the context of other relevant risks and controls, although the effect is marginal in statistical significance (p = 0.0488 one-tail, p = 0.0975 two-tail). However, the effect of change in overall VAGB exposure on capital ratios is strongly negative (p = 0.0004). In fact, of the predictors and controls examined, change in VAGB exposure has the strongest effect on capital ratio adjustments among all 2007 change variables. This finding emerges in spite of the effective loss of half of the sample due to reducing the two annual data points for each insurer to one by differencing, and in spite of the generally inherent difficulty of estimating short-term fluctuations in a time series. The apparent violation of finite risk remains.

However, there are important potential explanations that cannot be tested by the change model of Eqn 1. As an example relevant to this paper, suppose that there is a gradient among insurers in their appetites for VA guarantee risk, with some insurers preferring to buffer a given VAGB risk with less capital than other insurers. One possible reason that an insurer may prefer a smaller capital buffer could be a desire to expand rapidly and build market share in the popular VAGB area – perhaps coupled with a lower evaluation of the risk of such products. Suppose further that as an insurer acquires additional VAGB risk, it always correspondingly sets aside a constant matching proportion of capital – but that in keeping with the first supposition, these matching capital proportions differ among insurers, per the gradient. Then changes in VAGB risk will relate positively to changes in capital for every insurer. However, the largest levels of VAGB risk will likely accrue to insurers with the *lowest* capital ratios. The reason is that, *ceteris paribus*, insurers that impose the fewest internal constraints (e.g., capital buffer additions) on their sales may be expected to grow their business most rapidly, especially in an historical time period pre-dating heightened market discipline in the form of either close regulatory scrutiny or accelerated guarantee exercise triggered by policyholder portfolio losses. The result is that low capital ratio insurers tend to migrate over time toward the high end of VAGB levels. In this scenario, the risk gradient will become negative, with the largest levels of VAGB associated with the smallest capital ratios. The change model will record a positive relationship between VAGB risk and capital and fail to detect the negative gradient. But the level model can detect it. Moreover, the scenario clearly is consistent with an hypothesized increasing trend in risk/complexity of VAGB products, even as insurers maintain a negative capital gradient in their appetites for VAGB risk. Although we do not necessarily either endorse or hypothesize the preceding as an explanation for our empirical results, we observe that it is consistent with them.

We now turn to the level model of Eqn 2 for further possible explanations. Table 6, which reports empirical results for the level model, repeats the same set of predictors and controls as Table 5, except for the addition of indicators, which do not often change value in the short run. There are four versions of the Table 6 model. Model A is the baseline version. In two of the other models (B, D), we have added interactions to show the effect of derivative use on major predictors. In two of the models, we have broken out guaranteed death benefit exposure from other VAGB risks (C, D).

Table 6. Regression model results for VAGB insurers, N = 287 insurer-years, 2006-2007. Response variable = log CAP. See Table 3 or the text for variable definitions.

	MODEL A		MODEL B		MODEL C		MODEL D	
Predictor	Estimates	P-Val	Estimates	P-Val	Estimates	P-Val	Estimates	P-Val
Intercept	2.2073	<.0001	2.8160	<.0001	1.5309	0.0001	2.2542	<.0001
LogpLBG	-0.0278	<.0001	-0.0570	<.0001	-0.1050	<.0001	-0.0979	<.0001
LogpVAGB	-0.0332	<.0001	-0.0583	<.0001	-0.0353	<.0001	-0.0529	<.0001
LogpDB					-0.1791	<.0001	-0.1102	<.0001
LogAssetRisk	0.0800	<.0001	0.0560	<.0001	0.0899	<.0001	0.0569	<.0001
Size	-0.3303	<.0001	-0.3785	<.0001	-0.3112	<.0001	-0.3580	<.0001
pLife	-0.0101	0.2905	-0.0176	0.0757	-0.0227	0.0161	-0.0219	0.0249
pAnnuity	-0.3993	<.0001	-0.3782	<.0001	-0.3998	<.0001	-0.3755	<.0001
LogRBCratio	0.4297	<.0001	0.4325	<.0001	0.4321	<.0001	0.4311	<.0001
RetOnCap	0.0444	<.0001	0.0390	<.0001	0.0486	<.0001	0.0407	<.0001
Ntype	-0.6531	0.0039	-0.6712	0.0040	-0.6539	0.0032	-0.6683	0.0035
Ngroup	0.8614	0.0104	0.9414	0.0066	0.8403	0.0109	0.9109	0.0074
IndDeriv	-0.1131	<.0001	-1.2545	<.0001	-0.1423	<.0001	-0.9516	<.0001
IndDeriv_by_logpVAGB			0.0283	<.0001			0.0201	<.0001
IndDeriv_by_logAssetRisk			0.0513	<.0001			0.0621	<.0001
IndDeriv_by_size			0.0699	<.0001			0.0574	<.0001
Sample Size = 287	*Adj.* R^2 = 0.3083		*Adj.* R^2 = 0.3135		*Adj.* R^2 = 0.3153		*Adj.* R^2 = 0.3170	

One of the first impressions that emerges even from a casual perusal of Table 6 is the consistency of corresponding coefficient values in all four level models, except for the variables affected by interactions, Therefore, we discuss Model A at length and comment on the differences shown in Models B, C, D.

To test our major focus in this paper, we examine the coefficients of the guarantee risks, logpLBG and logpVAGB. LogpLBG is our calculation of LBG risk by AAA-endorsed actuarial considerations In our panel for Table 6, 83 insurers have LBG risk in 2007 and 78 in 2006. The remaining 60-odd death-benefit-only VAGB insurers receive the same *de minimis* score (log (0.01)) on LBG risk. LogpVAGB a broad measure of exposure to VA guarantees that includes both living and death benefit guarantees, but only measured using exposures, not actuarial considerations. As an exposure, pVAGB parallels our treatment of other product risks (pAnnuity, pLife, pAnnuity) in being calculated as a proportion of total exposure. All of the insurers in our panel for Table 6 are subject to VAGB exposure risk. The two main guarantee risks are negative and strongly significant in all models in spite of the controls. In Models C and D, we include the separate exposure risk measure for death benefits to provide a separate test of death benefit guarantees. It, too, is strongly negatively associated with lower capital ratios. Thus, the basic conundrum of low capital ratios associated with high guarantee risk remains and the finite risk hypothesis is not met.

Since both logpLBG and the dependent variable (logCap) are in log scale, the coefficient of logpLBG is interpreted as an elasticity. A one percent increase in LBG risk is associated with a roughly 0.0278% decrease in the capital ratio in Model A. Likewise, a one percent increase in overall VAGB exposure is associated with a 0.0332% decrease in the capital ratio, *ceteris paribus*. The same applies in Models C and D.[13]

Hedging may substitute for capital accumulation as a buffer against risk. We therefore expect that use of derivatives may be associated with lower capital ratios, given the same profile of other risks and controls. That is what we find. Insurers that use derivatives maintain about exp(-0.1131) = 89.31% (Model A) of the capital ratios of equivalent insurers that do not use derivatives.

Because of the importance of hedging to managing risk, we have included interactions to expose possible complex relationships of capital with derivative use. In Models B and D, we interact derivative use with three key predictors. The effect is to change the coefficients of those predictors between the group of 87 derivative-using and the group of 56 non-derivative-using insurers in our panel. In Model B, the coefficient of logpLBG is -0.0583 for non-derivative using insurers and is (-0.0583 + 0.0283) = -0.0300 among derivative-using insurers. The coefficient remains negative in both groups, but less so among derivative users. The positive sign of the coefficient adjustment for derivative users (+0.0283) is consistent with the usual expectation that increasing risk is offset by higher capital ratios, *ceteris paribus*. However, one cannot hold the uninteracted logpLBG constant if the interacted logpLBG changes. The two must be taken as a combination, resulting in a net negative -0.0300 coefficient for derivative users. Still, it is curious that non-derivative users would have lower capital ratios than otherwise equivalent derivative users for the same given level of LBG risk. Moreover, the data to which we have access do not

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[13] There is a small nuance: Since the sum of death benefit account value and LBG account value equals total VAGB account value, death benefit exposure in Models C and D cannot increase without a corresponding decrease in LBG exposure. A one percent increase in DB exposure coupled with a one percent decrease in LBG exposure net out with respect to the logpVAGB variable in Models C and D. However, in order for *ceteris paribus* to apply to coefficient interpretation under the scenario of a one percent increase in DB exposure, there would be required a corresponding one percent reduction in LBG exposure that would need to leave the actuarial logpLBG unchanged. With a reduction in LBG account value, the average actuarial LBG risk of the remaining 99 percent of LBG account value would need to increase slightly.

reveal the reasons for insurer derivative positions. Insurers may use derivatives for investment purposes as well as to hedge.

Apparently, the interactions have a dramatically large effect on the intercept adjustment for derivative users, dropping the adjustment from -0.1131 in Model A to -1.2545 in Model B. However, the extreme adjustment of Model B comes into play only if the interacted variable values are close to zero, and most derivative users are far from zero in logpLBG, logAssetRisk, and size. For non-derivative users, the overall intercept (2.8160) has risen in partial compensation. Moreover, the capital ratio estimates provided by Model B overall do not differ substantially from those provided by Model A. The mean magnitude of the difference in estimates from Model A to Model B is less than 6 percent for the 287 insurer-year observations.

Model D includes not only interaction terms but also a separate measure of guaranteed death benefits exposure. The empirical results are consistent with the other three Models: Both derivative users and non-derivative users have significantly net negative coefficients for LBG risk, but with derivative users less so. The effect of exposure to death benefit guarantees is stronger than the effect of corresponding exposures to LBG risk. For derivative users, the intercept adjustment drops substantially from Model C to D (from -0.1423 to -0.9516) to accommodate upward adjustment of estimates coming from the positive interaction coefficients, while the overall intercept rises substantially in compensation for non-derivative users. Like the Model A – Model B differences, the differences in capital estimates between Models C and D are small.

Other product risks are also controlled in the models. The proportions of premiums written in life and annuity lines are included as exposure metrics. The other major product activity of life insurers, health and accident insurance (along with the much smaller reinsurance) is omitted to avoid a near multicollinearity, since the sum of all premium proportions equals 1.0. Initially, the negative coefficients of these *other* product risks (pLife and pAnnuity) appear puzzling. Why should an increase in exposure to these risks result in lower capital, *ceteris paribus*? It is because an increase in either of these risks, holding the other fixed, necessarily results in a *decrease* in the riskier lines of the omitted health and reinsurance business exposure.[14]

Per Model A, a one percent increase in asset risk (OAR) is associated with an estimated 0.0800 percent *increase* in the capital ratio, *ceteris paribus*. This positive relationship accords with the usual expectation of risk-balancing behavior under the finite risk hypothesis. In the other

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[14] The proportion of one line cannot increase without another decreasing. Some authors have argued (e.g., Baranoff and Sager, 2002) that health lines are riskier than life lines, which are riskier than annuity lines. So if an increase of 1% in pAnnuity necessarily results in a corresponding 1% decrease in health exposure, then net risk would decrease. Essentially, that would be a substitution of less risky annuities for riskier health policies. Hence, capital as a risk buffer could also be reduced. Thus the negative coefficients of pLife and pAnnuity are consistent with the view that health lines are the riskiest. Moreover, the magnitudes of the coefficients are consistent with the ordering of the riskiness of lines given by Baranoff and Sager (2002). The magnitude of the annuity coefficient (-0.3993 in Model A) is larger than the life coefficient (-0.0101). This suggests that swapping out 1% of total premiums from health lines to annuity lines frees up more capital (greater risk reduction) than swapping out 1% of total premiums from health to life (less risk reduction). Indeed, pLife is statistically indistinguishable here from the omitted health (and reinsurance) proportion.

Models, asset risk also has roughly comparable positive coefficients, which are increased further for derivative users, per the interaction coefficients.

Size has its expected negative relationship with capital ratios. Large size provides a buffer against shocks that could debilitate small insurers. Therefore, large insurers need lower capital ratios than small insurers. The interaction of size with derivative use shows that the strong negative effect of size on capital ratio is reduced somewhat for both derivative-using insurers. However, the combination coefficient remains well on the negative side of zero.

Among other control variables, logRBCratio, RetOnCap, and Ngroup all relate positively to the capital ratio. The larger the RBC ratio, the stronger the financial position of the insurer, so the more likely the insurer has a large capital ratio. The greater the return on capital, the more likely the insurer is doing well financially and the more funds it could use to add to capital. Membership in a related group of insurers may provide access to sources of capital not available to single insurers and may be another indicator of large size. Ntype has a large negative coefficient (-0.6531). *Ceteris paribus*, agency theory predicts stock insurers to be more driven than mutual insurers to maximize value by optimally employing resources, including capital. Therefore, lower capital ratios are expected. Although stock insurers are much smaller than mutuals, as a group, and small insurers hold much higher capital ratios than large insurers, the effects of size are controlled in the Models. This permits the negative relationship predicted for Ntype by agency theory to emerge.

Section 6. Conclusion

Following the inception of the global Great Recession in 2008, authorities have sought to identify sources of financial systemic risk. The insurance industry has not escaped scrutiny. Some life insurers have been designated as Globally Systemically Important Insurer (G-SII). One of the factors that contribute to the potential for systemic importance for life insurers is their exposure to guarantees in variable annuities (VAGB). There are four major types of such guarantees: death benefits and three living benefits (LBG): income, accumulation, and withdrawal. LBG guarantees assure policyholders that they will be credited with specified benefits, regardless of the actual performance of the equity and bond markets in which policyholders have invested their annuity premiums. In down markets, insurers may need to draw upon their own reserves to make good on the guarantees. In 2007, U.S. life insurers collectively had issued LBG guarantees on more than $500 billion of policyholder account value. Including death benefits, the policyholder account value subject to guarantees exceeded $1.5 trillion.

In previous studies of capital/risk interrelationships in the life insurance industry in the US, the finite risk hypothesis has prevailed. In this study, we have tested this hypothesis for both LBG risk and VAGB risk generally. Preliminary analysis suggested the possible violation of the finite risk hypothesis, even after roughly controlling for relevant confounding factors. This paper provides rigorous models to test the finite risk hypothesis and finds that the preliminary simple analysis holds.

Using a new actuarially-based proxy of LBG risk, we have investigated the relationship between U.S. life insurers' capital and their LBG risks in a context of other enterprise risks, including exposure to other product risks, as well as exposure to the broader and more inclusive class of all VAGB products. The new proxy of the LBG risk is a value-at-risk measure, using the American Academy of Actuaries methodology for regulatory measure of Risk Based capital. Through simulation, the proxy estimates the mean of the upper 30% tail of the maximum shortfall that an insurer may need to cover from its own funds in order to make good on its guarantees (liability). For 2007, we calculated the collective LBG risk of U.S. life insurers to exceed $74 billion by this LBG risk proxy.

Our regression models are designed to control for identifiable confounding variables and other factors that the literature has found to have important effects on capital. We deploy two related models. The first, a change model (Eqn 1), controls for possible trends in the complexity (or other historical factors) that may have led to an increase in variable annuity guarantee risk of all types over time. The change model controls for this effect by relating 2006-2007 additions to guarantee risk to changes in capital in a context of controls. The second model, a level model (Eqn 2), controls for level effects that the change model does not, such as natural gradients in the appetites of insurers for guarantee risks.

Both regression models yield consistent empirical results. The more LBG risk that U.S. life insurers held (or added in 2007), the *less* capital they held (or added) in relation to their assets – even controlling for their size, other important risks, and other factors recognized in the literature to be important determinants of capital structure. Hedging with derivatives is an alternative to the accumulation of capital to buffer risks. But even taking derivative use into account, the paradoxical negative relationship between capital and guarantee risk remains.

We are not ready to offer an explanation for the apparent paradox of violating the finite risk hypothesis for VAGB products. But one interpretation that is consistent with our results is that insurers may have failed to price the LBG correctly in the run-up to the financial crisis of 2008.[15] All of the analytical results, taken together, are consistent with the interpretation that in the run-up to the financial crisis, insurers seemed to treat both LBG and VAG in an excessive risk mode in relationship to capital ratios. By contrast, the treatment of asset risk and other product risks was in accord with the expectation of the finite risk hypothesis.

The period before the 2008 crisis is an ideal time to test insurer risk management of these innovative new VAGB products. Sales were rapidly growing and market reversals had not yet tested insurer VAGB risk/capital relationships under stress. Also, regulatory reaction to the crisis had not yet materialized. Thus, our results may inform global policy deliberations.

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[15] This may also help explain the underpricing of such products as shown in the literature review section.

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Appendix 1: Calculation of Actuarial/Regulatory LBG risk proxy

The calculation of Living Benefits Guaranty (LBG) risk involves separate computations for each of the three types of LBG types of annuities: income, accumulation and withdrawal. We illustrate the computations for each of our three archetype contracts for a hypothetical insurer, Acme Life.

Income guarantees. Let us suppose that Acme Life has just received $100,000 in premiums for a variable annuity contract with income guarantees. We suppose that the contract follows the terms of the GMIB (income guarantee) archetype contract. For this illustration, we assume that the contract funds are invested 60% in equities and 40% in bonds. In its separate accounts, Acme records a total related account value of $100,000. This is the actual account value for the policyholder. Acme sets up a corresponding "shadow" account, "Guaranteed Contract Value", to keep track of the guaranteed amount on which it may become obligated to pay via its income guarantee. Initially, this amount is also $100,000, but will increase by 5% per year according to the terms of the GMIB archetype contract. To calculate LBG risk for this example, the year-to-year change in actual account value is determined by simulation of market returns. The American Academy of Actuaries has simulated 10,000 iterations. For each year of an iteration, Acme calculates a shortfall or surplus = guaranteed contract value - actual account value that the simulations generate, and the greatest annual shortfall for the (at most) 30-year duration of the iteration is recorded

To illustrate the calculations, we randomly selected one simulated return series out of the 10,000 conducted by the Academy of Actuaries for the regulators.[16] At the end of the first year, the total related account value (which is the asset that could be built in the variable annuity) is simulated to be increasing to $108,917 because of favorable market returns (see Appendix Table 1, I). The Guaranteed Contract Value has grown to $105,000, per the contract. So, the asset is is assumed to grow by simulation to 108,917 while the liability should the guarantee be exercised would be $105,000. At this point the actual account value (asset) is more than sufficient to cover the income guarantee in year one.[17] There is no shortfall. In fact, there is a surplus of $3,917. These figures are shown in Table 1, I for year one. For year two, there is also a surplus. But in year three, the simulations created an asset value that declines to $97,687, whereas the steady growth of the income guarantee (the liability) makes the Guaranteed Contract Value – the amount on which Acme may become obligated to pay income – equal to $115,763 (see Table 1, I). There is a shortfall of $18,076 in year 3. If the income guarantee were to be triggered now, Acme would be obligated to an annual payout of 5% on the gap, liability – assets, in an amount of $18,076. This is a deficit representing the guarantee risk in year 3.Using the simulations of the American Actuarial Society, in our example, there are also shortfalls in each of the remaining years 4-10. At the end of year 10, the policyholder annuitizes the larger of the actual account value (asset= $110,896) or the Guaranteed Contract Value (liability=$162,889). Our guarantee risk for this example is the difference. The primary guaranty risk emanates from an asset/liability mismatch in the hedge: the policyholder's simulated annuity assets may be insufficient to fund the insurer's contractual guarantee liability.

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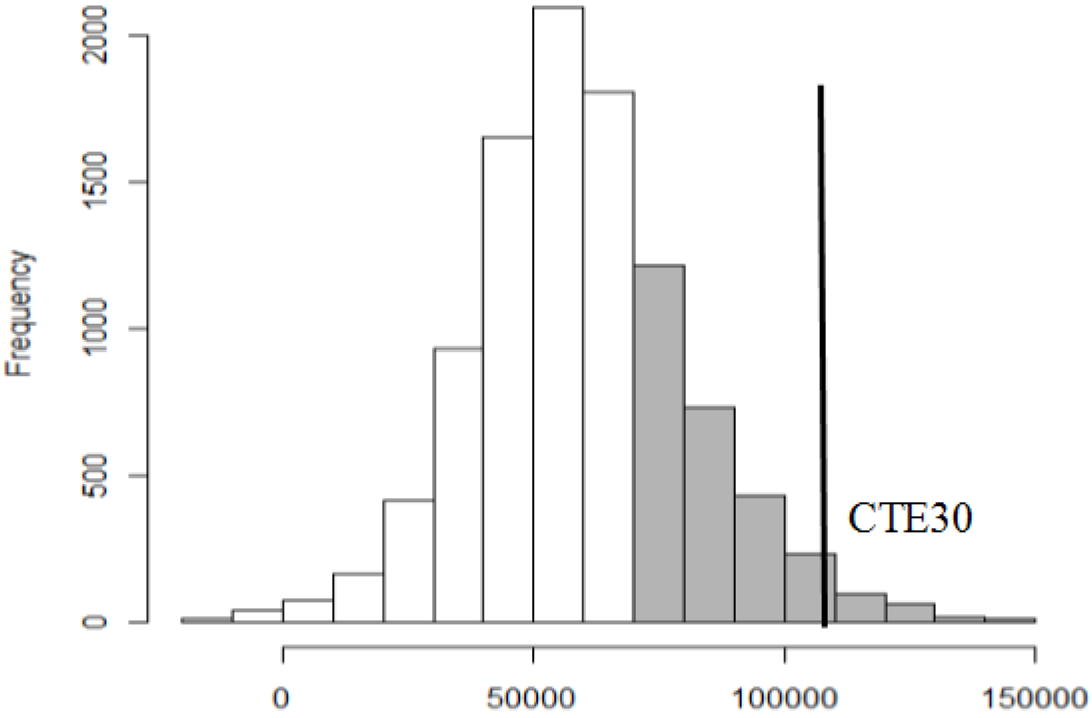
[16] Actually, two simulations – one for equities and one for bonds (they are correlated), since the example hypothesizes 60% investment in equities and 40% in bonds.

[17] There have been no payouts. The GMIB archetype contract provides that the beneficiary can ultimately draw an annual income equal to 5% of the greater of the actual account value or the Guaranteed Contract Value.

For the selected iteration we demonstrate here, the worst shortfall is in year 9 (asset – liability=$53,560). The GMIB guarantee risk proxy calculation records the worst shortfall figure of $53,560 for this iteration. Since the Academy of Actuaries does 10,000 iteration in the simulation, for each of the remaining 9,999 iterations, the value of the shortfall for the worst year is recorded. After 10,000 iterations of the simulation, there are 10,000 worst shortfalls, of which our illustrative $53,560 is one. Appendix Figure 1 displays Academy of Actuaries' histogram. As the histogram shows, very few of the iterations result in surplus (to the left of zero) which we will regard as no guarantee risk since the mismatch is in favor of the assets ; almost all result in shortfall (right of zero) which provides the guarantee risk. Moreover, the shaded region in the figure shows the worst 30% of the 10,000 iterations. **Our GMIB guarantee risk is defined to be the mean of these 30% worst shortfalls (CTE30)**.

Appendix Figure 1. Worst Shortfalls (Mismatch between assets and liabilities = guarantee risk) in 10,000 Iterations of Income Archetype Contract

GMIB Shortfall Distribution



Accumulation and Withdrawal Guarantees. To illustrate the computation of LBG risk for accumulation and withdrawal guarantees, we proceed as in the case of the income guarantee case just presented by supposing that Acme Life receives $100,000 of premiums for an archetype accumulation contract (GMAB) and $100,000 for an archetype withdrawal contract (GMWB). We again assume that the funds are invested 60% in equities and 40% in bonds. In its separate accounts, Acme begins each example with a total related account value of $100,000 and a corresponding shadow account, "Guaranteed Contract Value". Appendix Table 1 cases II and III show the year-by-year progression of shortfall or surplus for each contract. Each example uses the same simulation data as for the income example, but the shortfalls and surpluses differ due to the different terms of the governing archetype contracts for the liabilities. Thus, the assets values are simulated in the same way as for the example above.

For example, in the accumulation example (Appendix Table 1, II), the Guaranteed Contract Value rises at a compounded rate from the initial $100,000 to the guaranteed $120,000 amount in year 10, when half the insureds take a lump-sum payout and half annuitize. The largest deficiency is $31,272, which occurs in year five.

In the withdrawal benefit example (Appendix Table 1, III), the beneficiary withdraws the permitted 7% x $100,000 = $7,000 annually. The withdrawal is real money, so reduces any market gain or the principal in the actual account. Moreover, the insurer no longer need to continue to guarantee the payment of the withdrawal, once paid. Thus, withdrawals reduce both the actual account value (the asset) and the Guaranteed Contract Value until the initial $100,000 investment has been recovered. At that point, in year 15, there is a remainder of $7,027 in the insured’s account. Per the GMWB archetype contract, the insured then receives the remainder as a lump-sum payout.

Table 1 (App 1)

Computation of Living Benefits Guarantee risk = Asset/Liability mismatch between guaranteed contract value and simulated assets in the account over the life of the guarantee ***

Year End	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
60% Equity 40% Bond Simulated Return Series as of 2006*											
	1.09	1.14	0.98	0.96	0.78	0.84	0.96	1.02	1.02	1.11	
I. With GMIB 5% rollup, 10 years waiting period**											
Simulated asset values in the account	$108,917	$113,630	$97,687	$96,263	$78,272	$83,784	$95,781	$102,414	$101,573	$110,896	
Guaranteed Contract Value (liabilities)	$105,000	$110,250	$115,763	$121,551	$127,628	$134,010	$140,710	$147,746	$155,133	$162,889	
Payout	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	
Deficiency	($3,917)	($3,380)	$18,076	$25,287	$49,356	$50,225	$44,929	$45,332	$53,560	$51,993	
II. With GMAB 1.2 times initial premium return, 10 years of waiting period**											
Simulated asset values in the account	$108,917	$113,630	$97,687	$96,263	$78,272	$83,784	$95,781	$102,414	$101,573	$110,896	
Guaranteed Contract Value (liabilities)	$101,840	$103,714	$105,622	$107,565	$109,545	$111,560	$113,613	$115,703	$117,832	$120,000	
Payout	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	
Deficiency	($7,077)	($9,916)	$7,935	$11,302	$31,272	$27,776	$17,832	$13,289	$16,259	$9,104	
III. With GMWB 7% withdrawal each year**											
Simulated asset values in the account	$101,917	$99,327	$78,391	$70,248	$50,120	$46,649	$46,328	$42,536	$35,187	$31,417	$
Guaranteed Contract Value (liabilities)	$93,000	$86,000	$79,000	$72,000	$65,000	$58,000	$51,000	$44,000	$37,000	$30,000	$
Payout	$7,000	$7,000	$7,000	$7,000	$7,000	$7,000	$7,000	$7,000	$7,000	$7,000	
Deficiency	($8,917)	($13,327)	$609	$1,752	$14,880	$11,351	$4,672	$1,464	$1,813	($1,417)	($

* This portfolio return series is randomly selected from 10,000 simulated return series as of year 2006. Equity returns and bond returns are simulated separately, although with correlation. The method can be found on the **American Academy of Actuaries** website. For this example, we assume 60% of assets allocated to equity and 40% to bond.

** Terms of various guaranteed benefit contracts (GMIB, GMAB and GMWB) can be found in Section 2 in the article.

*** The calculation in this table reflects only Step 1 of the guarantee risk calculation described in Section 2. To get the final LBG Risk, similar calculation for each type of contract will be repeated 10,000 times using 10,000 different simulated portfolio return series. See step 2 or the appendix section for details.

After assigning each of an insurer's LBG contracts to one and only one of the three archetypes, we simulate the potential shortfalls between the guaranteed account and the asset value in the account. The primary risk emanates from the potential asset/liability mismatch of the hedge for each of the three archetypes as shown in Table 1. If the assets (policyholder portfolio value) are insufficient to fund the liability (contractually guaranteed account value) (liability), then the shortfall is a proxy measure of the risk. As noted, our method is an adaptation of the guarantee reserves calculation method developed by the Variable Annuity Reserve Working Group

(VARWG) of the American Academy of Actuaries and approved by NAIC. There are two major steps in the calculation:

Step 1 (Simulation): 10,000 portfolio return series are simulated according to historical data on major market indexes. Each of the 10,000 iterations provides hypothetical, but realistic, monthly stock and bond returns for the next 30 years.[18] For each simulated return series, we project the monthly variable annuity account values for the total of the three archetypes until the end of the deferment period.[19] At the same time, we calculate the guaranteed contract values through the end of the deferment period according to the terms of each archetype contract and sum the three types. This is the liability of the insurer. At each year end during the deferment period, the annual shortfall is the contractually guaranteed value, less the simulated asset values in the account. A positive shortfall is a potential financial obligation which is the guarantee risk for the insurer; a negative shortfall (surplus) is not since it belongs to the policyholder. The actual computation in Excel spreadsheet is available upon request. Each iteration of the simulation thus yields a set of annual shortfalls until the end of the deferment period. For each iteration, the worst of its annual shortfalls is recorded. By this means, we obtain a simulated distribution, consisting of 10,000 worst-in-deferment-period shortfalls. The Appendix Table 1 illustrates the calculation details for one of the 10,000 iterations of the shortfall for the three archetype contracts.

Step 2 (Conditional Tail Expectation): Our raw LBG risk proxy is the mean of the 3,000 largest of the 10,000 worst shortfalls in the simulation.[20] Although many VAR calculations use the 5% tail, the method endorsed by VARWG uses the 30% tail, which we follow for consistency. In the analyses, we scale the raw guarantee risk proxy by dividing it by the insurer's total related account value. This helps to neutralize the effect of insurer size.

Clearly, the adequacy of our guarantee risk proxy depends critically upon having a reasonable model of future market returns for the asset classes for which the guarantees are made. Following is a brief explanation of how the market return series are generated.[21] The Life Capital Adequacy Subcommittee (LCAS) has identified 19 asset classes commonly held in variable annuities accounts. These asset classes include money market funds, U.S. intermediate term government bonds, diversified large capital U.S. equity, etc. LCAS collects historical monthly return data for the 19 asset classes, which start as early as December 1955 and end in December 2003. The 19 asset classes are divided into bond-type assets and equity-type assets. LCAS runs simulation

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[18] Equity returns and bond returns series are simulated separately. The original VARWG simulated return series are posted on the website of the American Academy of Actuaries. The historical data used for simulating the return series on the website extends from 1955 to 2003.

[19] The reserve calculation method proposed by VARWG aims at the entire duration of annuity products, including the payout period. However, our focus is on the risk of benefit guarantees during the deferment period, not on the risk caused by mortality, expenses, interest rates or other sources of annuity risk. Therefore, we terminate the projections at the end of deferment period, which is ten years for our GMIB and GMAB archetype contracts.

[20] Thus the guarantee risk is the 30 percent Conditional Tail Expectation (30 CTE).

[21] Comment on the Exposed AG VACARVM from the American Academy of Actuaries' Variable Annuity Reserve Work Group, presented to the National Association of Insurance Commissioners' Life and Health Actuarial Task Force, September 2007.

models for each type, incorporating intercorrelations among the 19 asset classes.[22] After estimating parameters using historical data, the models mix in normally distributed random perturbations to simulate 360 months of future returns for each asset class. The American Academy of Actuaries website provides 10,000 downloadable iterations of the simulation, where each iteration consists of the 19 simulated series for each of the next 360 months – called "Pre-Packaged Scenarios". In January 2006, LCAS released "interest rate generator" - a set of Microsoft Excel® macros to help actuaries update the simulation for bond-type assets by incorporating historical data that post-date 2003. We have utilized these macros for this paper and updated the data through 2006 or 2007 for our bond simulations.[23]

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[22] Stochastic models for bonds and equities, and parameter descriptions can be found in "Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products" by Life Capital Adequacy Subcommittee (LCAS). American Academy of Actuaries, June 2005.

[23] We also note that as of the time of this writing, the online equity simulations have not been updated to incorporate historical equity returns that post-date 2003; nor have any macros been posted for this purpose. In order to include the most recent equity returns in calculations of our guarantee risk proxy, we therefore replicated the LCAS methodology using the stochastic analysis model for equities recommended by LCAS and historical data on equity returns through the year of calculation of LBG risk (2006 or 2007).