Risk Aversion and Risk Premiums with Dependent Risks

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Abstract

By using a general bivariate utility function, this paper provides the conditions under which agents would like to remove primary risk in the presence of other dependent risk. For small risks, the conditions for retaining primary risk along with other dependent risk are also provided. The results of this paper indicate that the risk attitude to primary risk depends not only on the dependence relation between the risks, but also on the sign of the second-order cross derivatives of the utility function. In addition, agents also estimate the relative magnitude between the covariance of the risks and variance of the primary risk when they consider retaining the primary risk. Moreover, this paper examines the relation between risk premium for removing all risk simultaneously and those for removing risk sequentially. Rey's (2003a) method to compare the total risk premium with the sum of the partial risk premiums is generalized to the case where there exists dependence relation between risks.

Key words: risk aversion, risk premium, dependent risk, bivariate utility function

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1 Introduction

Pratt (1964) demonstrates the measure of local risk aversion and risk premium for a single risk. However, in the real world, people usually face multiple sources of risks. For instance, agents consider how to invest between two risky assets. In addition, there often exist some non-hedgeable background risks, such as health risk or environmental risk, when agents make decisions for a risky economic activity. An agent may change her or his risk-taking behavior in the presence of other risks.

Many economists try to characterize risk aversion under multiple sources of risks. Kihlstrom and Mirman (1974) extend the Arrow-Pratt results of risk aversion to the multivariate case. Kihlstrom, Romer, and Williams (1981) extend the Arrow-Pratt risk aversion to the case where there exists an independent background risk. Nachman (1982) provides the conditions that guarantee that the derived utility function, which is defined by the integral over variables that denotes other risks of the utility function, preserves the comparative risk aversion relations of the original utility function. Ross (1981) develops a stronger measure of risk aversion in the presence of an additive and mean independent background risk.

Richard (1975) introduces the concept of correlation aversion but named it "multivariate risk aversion" at first. Correlation aversion is a preference for combining good outcome in one dimension with bad outcome in other dimension to combining good outcome with good and bad outcome with bad. He discusses the necessary and sufficient condition for correlation aversion. Epstein and Tanny (1980) make further efforts to illustrate the effect of an increase in correlation of asset returns on optimal portfolio choices under correlation aversion. Eeckhoudt, Rey, and Schlesinger (2007) extend the definition of correlation aversion to cross-prudence and cross-temperance. They illustrate these definitions by "intertemporal consumption choices" model. Tsetlin and Winkler (2009) find the appropriate preference functional, which could represent correlation aversion, cross-prudence and cross-temperance.

Agents may change their risk attitudes in the presence of other risks. Gollier and Pratt (1996) argue that an undesirable lottery could become desirable with an independent background risk. Thus, it is more difficult and complicated to analyze the risk-taking behavior with multiple sources of risks. Most research in this field supposes that the primary risk and other risks are independent. Gollier and Pratt (1996) define "risk vulnerability" to guarantee that risk-averse agents will dislike the primary risk more if the background risk is additive, independent and, expectedly non-positive. They argue that "proper risk aversion" (Pratt and Zeckhauser, 1987) and "standard risk aversion" (Kimball, 1993) are also risk vulnerability. Although they provide necessary and sufficient conditions for this new concept, the conditions are not easy to use. Eeckhoudt, Gollier, and Schlesinger (1996) examine the effects of both first and second degree stochastic dominance changes in an additive, independent background risk on risk-taking behavior. They provide necessary and sufficient conditions to ensure that agents will take more risk-averse behavior under stochastic changes in background risk.

Many researchers try to provide conditions to guarantee that agent would not change their risk attitudes for the primary risk in the presence of other risks. However, Pratt (1990) provides a natural explanation of the reversal phenomenon, i.e. risk attitudes change. He argues that we should not rule out this phenomenon summarily as Ross (1981) did. Besides, he points out that the condition needed to eliminate the reversal phenomenon is very restrictive and inconvenient.

Some classical single-risk results could hold in the presence of other independent risks. When the random initial wealth and losses are independent, Doherty and Schlesinger (1983a) show that partial insurance is optimal given a positive loading factor, i.e. Mossins (1968) result still holds. However, the results may be violated without the assumption of independence. Doherty and Schlesinger (1983a) show that we need to assume the random initial wealth and losses have a bivariate normal distribution to ensure that full insurance is optimal given an actuarially fair premium when there exists negative correlation between the random initial wealth and losses. Thus, we may need more restrictive conditions to obtain unambiguous results when the primary risk and other risks are dependent. Finkelshtain, Kella, and Scarsini (1999) propose the conditions on the utility function for risk aversion becomes more restrictive if there exists dependence relation between risks. Li (2011) uses the concept of expectation dependence introduced by Wright (1987) and analyzes the demand for a risky asset if there exists expectation dependence between the financial risk and the background risk. Denuit, Eeckhoudt, and Menegatti (2011) analyze the effect of the correlation of a financial risk and a non-financial background risk on optimal choices. Dionne and Li (2014) show that a dependent background risk may guarantee first-order risk aversion.

Most of related literature focuses on an additive relation between risks. Few attentions have been given to the effects of multiplicative relation.Turnbull (1983) illustrates that Kihlstrom, Romer, and Williams's (1981) conditions and Ross risk aversion are not sufficient to obtain unambiguous results (a more risk-averse agent has a higher risk premium) with multiplicative risks by using two counterexamples. Franke, Schlesinger, and Stapleton (2006) extend the concept of (additive) risk vulnerability (Gollier and Pratt, 1996) to multiplicative case. They provide some simple sufficient conditions for the new concept of "multiplicative risk vulnerability". However, most of the commonly-used utility functions don't satisfy these sufficient conditions. Later, Franke, Schlesinger, and Stapleton (2011) examine the interaction effects between independent additive risk and multiplicative risk on risk-taking behavior. They argue that the combined effect of these two types of risk could be rather different from their separate effects. In addition, Tsetlin and Winkler (2005) use a model that includes "both additive and multiplicative background risk". They analyze the conditions where investing in the risky project is undesirable if the "project risk" and the background risk are related.

Moreover, the results obtained from univariate utility functions may be violated in the case of multivariate utility functions. Pratt (1988) analyzes the effect of multiple risks on comparative statics properties of risk aversion and develops conditions for univariate utility functions with additive risks. However, these results do not hold for multivariate utility functions. Rey (2003b) extends the results of Doherty and Schlesinger (1983b) to multivariate case. Specifically, she uses a two-argument utility function instead of the univariate utility function. She shows that not only the correlation between risks but also the sign of the partial derivative of second order of utility function are important to decide the optimal insurance contract.

Last but not least, Few studies have been done to analyze the relation between the risk premium for removing all risk simultaneously and those for the sequential removal of risks. Doherty, Louberge, and Schlesinger (1987) discuss this relation for additive risks and multiplicative risks. However, they only consider correlation, which they admit is not a very good statistic for measuring interdependencies. In a mean-variance setting, Courbage (2001) presents the conditions under which the risk premium for removing one risk while retaining the other, called partial risk premium, would be negative for small risks. He also analyzes the relation between total risk premium and partial risk premiums given small risks. Rey (2003a) develops a comparison method to examine the relation between the total risk premium and the sum of partial risk premiums without the assumption of small risks. However, she focuses on the case where there is no interaction effect between risks by using a simple discrete model.

In this paper, beyond independence assumption for risks, we assume that there exists dependence relation between risks. we focus on the effects of the dependence between risks on agents' risk taking behavior. Specifically, we consider three kinds of dependence relations. We present sufficient conditions to guarantee that agents' risk attitudes for primary risk are preserved in the presence of other risks. We also analyze the case where agents would like to retain the primary risk in the presence of other risks, although they are risk averse to primary risk if there is no uncertainty in other dimensions. In addition, we provide the sufficient conditions where agents prefer retaining primary risk with other dependent risks for "small risks", i.e. the variances of risks are very small.

This paper focuses on the two-dimensional case. Instead of assuming specific specification for the utility function, we use a general bivariate utility function. The additive and multiplicative relations between risks are special cases of the general bivariate utility function. The results obtained from the bivariate utility function are easier to apply in the univariate case. However, as some of literature pointed out, the results obtained from univariate utility functions are not necessary to hold in the case of bivariate utility functions.

In addition, we examine the relation between risk premiums for removing all risk simultaneously and those for removing risk sequentially. Specifically, we consider two ways to pay for the risk premiums. One is paying the risk premiums in both dimensions. The other is paying the risk premiums only in one dimension. In the latter way, we generalize Rey's (2003a) method to compare the total risk premium with the sum of the partial risk premiums in the case where there exists dependence relation between risks. Moreover, we illustrate how to use the generalized comparison method to analyze the relation between risk premiums under two kinds of quadrant dependence assumptions.

The paper proceeds as follows. Section 2 provides the conditions to guarantee that agents would like to remove primary risk in the presence of other dependent risk. Moreover, the conditions for retaining primary risk along with other dependent risk are also provided for small risks. Section 3 devotes to examining the properties of risk premiums. Conclusions are provided in section 4.

2 Risk Attitude under Two Sources of Risk

The agent's preference is represented by a bivariate utility function $u(x, y)$. The utility function is assumed to be concave in the first argument and twice differentiable in both arguments. Without loss of generality, we assume that $u(x,y) > 0$. In addition, suppose that $\lim_{x\to\infty} \frac{\partial u(x,y)}{\partial y}$ and $\lim_{y\to\infty} \frac{\partial u(x,y)}{\partial x}$ exist. ($\tilde{x}$,$\tilde{y}$) is a random pair, where $\tilde{x}$ represents the primary risk while $\tilde{y}$ denotes the other risk. The supports of the distributions of these two random variables are contained in (a,b) and (c,d), respectively. $F(x,y)$ denotes the joint cumulative distribution function (CDF) for $(\tilde{x},\tilde{y})$. In addtion, $F_X(x)$ and $F_Y(y)$ are the marginal CDFs of the random variables $\tilde{x}$ and $\tilde{y}$, respectively. We assume all corresponding expectations exist. We use the same definition for risk aversion to one risk in a model with multiple sources of risk as Finkelshtain, Kella, and Scarsini (1999).

Definition 2.1 *(Finkelshtain, Kella, and Scarsini, 1999) An agent is risk averse to the primary risk, $\tilde{x}$, if the following inequality holds:*

$$Eu(\tilde{x},\tilde{y}) \leq Eu(E\tilde{x},\tilde{y}) \tag{1}$$

Inequality (1) indicates that removing only $\tilde{x}$, in the presence of $\tilde{y}$, will improve the agent's welfare. Inequality (1) will not always be true without some conditions on the preference and the relation between the risks. For instance, we consider $u(x,y) = u(x+y)$. We assume both $\tilde{x}$ and $\tilde{y}$ are zero mean risks and $\tilde{x} = -\tilde{y}$. Thus, there exists perfect negative correlation between $\tilde{x}$ and $\tilde{y}$, i.e. $Cov(\tilde{x},\tilde{y}) = -1$. Now, $Eu(\tilde{x},\tilde{y}) = Eu(\tilde{x}+\tilde{y}) = u(0)$ while $Eu(E\tilde{x},\tilde{y}) = Eu(\tilde{y})$. If u is concave, we know that $u(0) \geq Eu(\tilde{y})$. Thus, inequality (1) doesn't hold. In this simple example, the two risks cancel each other out. Thus, there is no uncertainty if agents retain the risks. However, agents have to face the uncertainty from one risk if they get rid of the other. Therefore, risk averse agents' welfare will decrease by eliminating one risk.

We denote the risk premium for removing the risk $\tilde{x}$ while retaining the other risk $\tilde{y}$ as π_x. π_x is defined by

$$Eu(\tilde{x},\tilde{y}) = Eu(E\tilde{x} - \pi_x,\tilde{y}). \tag{2}$$

Given $u^{(1,0)} \geq 0$, inequality (1) implies that $\pi_x \geq 0$.

We recall the following two kinds of dependence relations.

Definition 2.2 *(Lehmann, 1966) The random pair $(\tilde{x},\tilde{y})$ is positively quadrant dependent, if*

$$F(x,y) \geq F_X(x)F_Y(y),\ \forall x,y. \tag{3}$$

and

Definition 2.3 *(Wright, 1987)* $\tilde{x}$ *is positive first-degree expectation dependent (FED) on* $\tilde{y}$ *if* $FED(\tilde{x}|y) = E\tilde{x} - E(\tilde{x}|\tilde{y} \leq y) \geq 0,\ \forall y.$

Negative quadrant dependence and negative first-degree expectation dependence are defined by reversing the above inequalities.

First-degree expectation dependence is a weaker condition than the quadrant dependence. First, we know the following results by integrating by parts.

$$\int_a^b [F(x|\tilde{y} \leq y) - F_X(x)]dx = E\tilde{x} - E(\tilde{x}|\tilde{y} \leq y) = FED(\tilde{x}|y) \tag{4}$$

We can rewrite (3) as $F(x|\tilde{y} \leq y) \geq F_X(x)\,\forall x, y$. Thus, positive quadrant dependence implies $FED(\tilde{x}|y) \geq 0,\ \forall y$. Likewise, negative quadrant dependence implies negative first-degree expectation dependence.

Next, we examine the conditions to guarantee that agents prefer removing primary risk $\tilde{x}$ in the presence of risk $\tilde{y}$. Since quadrant dependence implies first-degree expectation dependence, the conditions for expectation dependent risks will also hold in the quadrant dependent case. Thus, to simplify the following explanation, we only present the results for expectation dependent risks.

2.1 Conditions for Risk Aversion

The following proposition provides sufficient conditions to ensure that inequality (1) holds.

Proposition 2.4 *Given* $u^{(1,0)} \geq 0$ *and* $u^{(2,0)} \leq 0$, *if either of the following conditions is satisfied*

(1) $u^{(1,1)}(x, y) \leq 0$ *and* $FED(\tilde{x}|y) \geq 0,\ \forall y,$

(2) $u^{(1,1)}(x, y) \geq 0$ *and* $FED(\tilde{x}|y) \leq 0,\ \forall y,$

then inequality (1) holds and $\pi_x \geq 0$.

Proof See Appendix. Q.E.D.

If we replace the expectation dependence relations by corresponding quadrant dependence relations, it is straightforward to show that Proposition 2.4 still holds.

Richard (1975) proved that $u^{(1,1)}(x, y) \leq 0$ implies that the agent is correlation averse, while $u^{(1,1)}(x, y) \geq 0$ implies that the agent is correlation loving. Eeckhoudt, Rey, and Schlesinger (2007) provide intuitive interpretation for correlation aversion and correlation loving as following.

Definition 2.5 *(Eeckhoudt, Rey, and Schlesinger, 2007) An agent is correlation averse if she or he prefers the lottery L_1 offering $(x-k, y)$ and $(x, y-c)$ with equal probabilities to the lottery L_2 offering (x, y) and $(x-k, y-c)$ with equal probabilities for all $(x, y) \in R_+^2$, where $k > 0$ and $c > 0$.*

An agent is correlation loving if she or he always prefers the lottery L_2 to L_1. Thus, correlation aversion implies that agents prefer combining good outcome with bad outcome to combining good outcome with good and bad outcome with bad. Positive FED implies that risk $\tilde{x}$ is likely to vary with risk $\tilde{y}$ in the same direction. Given positive FED, Proposition 2.4 indicates that a correlation averse agent would like to remove risk $\tilde{x}$. This results is consistent with the economic interpretation for correlation aversion. Likewise, a correlation loving agent also prefer getting rid of risk $\tilde{x}$, given negative FED.

According to Theorem 1 of Finkelshtain, Kella, and Scarsini (1999), we only need $u^{(2,0)} \leq 0$ to guarantee that inequality (1) holds if risk $\tilde{x}$ is independent of risk $\tilde{y}$. However, according to Proposition 2.4, we find that whether inequality (1) holds depends not only on the dependence relation between the risks, but also on the sign of the second-order mixed derivatives of the utility function, i.e. the risk attitudes for dependence relation.

2.2 Conditions for Retaining Primary Risk

If inequality (1) is reversed, the agent is not willing to remove the primary risk in the presence of the other risk. Thus, the risk premium π_x will be negative, given $u^{(1,0)} > 0$. Inequality (5) implies that the welfare of an agent will diminish by removing the risk $\tilde{x}$.

$$Eu(\tilde{x}, \tilde{y}) \geq Eu(E\tilde{x}, \tilde{y}) \tag{5}$$

It is not easy to provide sufficient conditions for inequality (5) in the general case. However, we could obtain sufficient conditions for small risks. First, we provide the following lemma.

Lemma 2.6 *If $FED(\tilde{x}|y) \geq 0$, $\forall y$, then $Cov(\tilde{x}, \tilde{y}) \geq 0$.*

If $FED(\tilde{x}|y) \leq 0$, $\forall y$, then $Cov(\tilde{x}, \tilde{y}) \leq 0$.

Proof See Appendix. Q.E.D.

Compared to correlation, Lemma 2.6 shows that the FED is a stronger condition. The following proposition shows conditions for "small risks" under which agents prefer retaining the primary risk in the presence of another expectation dependent risk.

Proposition 2.7 *Given $u^{(1,0)} \geq 0$ and $u^{(2,0)} \leq 0$, assume that the variances of both risks are very small. If either of the following conditions is satisfied, then inequality (5) holds and $\pi_x \leq 0$.*

$$\begin{aligned}
&(1)\, u^{(1,1)}(x,y) \geq 0,\ FED(\tilde{x}|y) \geq 0,\ \forall y, \\
&\quad and - \frac{u^{(2,0)}(E\tilde{x}, E\tilde{y})}{u^{(1,1)}(E\tilde{x}, E\tilde{y})} \leq \frac{Cov(\tilde{x},\tilde{y})}{Var(\tilde{x})} \qquad (6) \\
&(2)\, u^{(1,1)}(x,y) \leq 0,\ FED(\tilde{x}|y) \leq 0,\ \forall y, \\
&\quad and - \frac{u^{(2,0)}(E\tilde{x}, E\tilde{y})}{u^{(1,1)}(E\tilde{x}, E\tilde{y})} \geq \frac{Cov(\tilde{x},\tilde{y})}{Var(\tilde{x})}. \qquad (7)
\end{aligned}$$

Proof See Appendix. Q.E.D.

If the expectation dependence relations are changed to the corresponding quadrant dependence relations, Proposition 2.7 still holds. Unlike Proposition 2.4, the information about the dependence relation between the risks and the sign of the second-order mixed derivatives of utility function is not enough to obtain unambiguous results. The left-hand sides of (6) and (7) are very similar to the measure of absolute risk aversion for univariate utility function evaluated at $(E\tilde{x}, E\tilde{y})$. However, the right-hand sides of (6) and (7) are the ratio of covariance to the variance of primary risk. If the absolute value of $Cov(\tilde{x},\tilde{y})$ is small while $Var(\tilde{x})$ is large, conditions (6) and (7) may be violated. Intuitively, negative dependence relation could hedge against uncertainty. Thus, there may exist a motive for retaining the primary risk, which is negative dependent on the other risk. However, given $FED(\tilde{x}|y) \leq 0$ for all y, Proposition 2.7 indicates that agents may be willing to remove the primary risk if the variance of primary risk is too large or the covariance between these risks is too small. Nevertheless, according to Proposition 2.7, agents could prefer retaining the primary risk even though $FED(\tilde{x}|y) \geq 0$ for all y. If $u^{(1,1)}(x,y) \geq 0$, the marginal utility of x is increasing with y. Moreover, $FED(\tilde{x}|y) \geq 0$ for all y implies that $Cov(\tilde{x},\tilde{y}) \geq 0$. Thus, when y is large, the value of x tends to be large while the marginal benefit of x is also large. However, when y is small, the value of x is likely to be small while the marginal cost of x is also small. Therefore, retaining the primary risk may result in a higher level of expected welfare than removing it.

3 The Nature of Risk Premiums

In this section, we examine the relation between risk premiums for removing all risk simultaneously and those for removing risk sequentially. Specifically, we consider two ways to pay for the risk premiums. One is paying the risk premiums in both dimensions. The other is paying the risk premiums only in one dimension.

3.1 Pay the Risk Premiums in Both Dimensions

First, we assume that agents could pay the risk premiums in both dimensions. Sequentially, the risk premium for getting rid of $\tilde{x}$ first is π_x, defined by (2). Proposition 2.4 and Proposition 2.7 provide the conditions for $\pi_x \geq 0$ and $\pi_x \leq 0$, respectively. After $\tilde{x}$ is removed, the conditional risk premium for getting rid of $\tilde{y}$ is $\pi_{y|C_x}$, defined by

$$Eu(E\tilde{x} - \pi_x, \tilde{y}) = u(E\tilde{x} - \pi_x, E\tilde{y} - \pi_{y|C_x}) \tag{8}$$

If agents get rid of $\tilde{y}$ first, we denote the risk premium for removing $\tilde{y}$ is π_y. In addition, the conditional risk premium for removing $\tilde{x}$ after getting rid of $\tilde{y}$ is denoted by $\pi_{x|C_y}$. According to (2) and (8), we have

$$\begin{aligned} Eu(\tilde{x}, \tilde{y}) &= Eu(E\tilde{x} - \pi_x, \tilde{y}) = u(E\tilde{x} - \pi_x, E\tilde{y} - \pi_{y|C_x}) \qquad (9) \\ &= Eu(\tilde{x}, E\tilde{y} - \pi_y) = u(E\tilde{x} - \pi_{x|C_y}, E\tilde{y} - \pi_y) \qquad (10) \end{aligned}$$

If the utility function is also increasing and concave in the second argument, i.e. $u^{(0,1)} \geq 0$ and $u^{(0,2)} \leq 0$, it is straightforward to show that $\pi_{y|C_x} \geq 0$. Given $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$, π_x, $\pi_{y|C_x}$, π_y, and $\pi_{x|C_y}$ are unique.

Now, we consider the case where both risks are removed simultaneously. The risk premiums are denoted by π'_x and π'_y, which are defined by

$$Eu(\tilde{x}, \tilde{y}) = u(E\tilde{x} - \pi'_x, E\tilde{y} - \pi'_y) \tag{11}$$

Suppose that the utility function, $u(x, y)$, is increasing in both arguments. Given π'_x, then π'_y is unique. However, π'_x can be arbitrary since we always can find a solution for π'_y to ensure that (11) is satisfied [1]. Thus, unlike the sequential case, there are infinite solutions for π'_x and π'_y, which satisfy (11). Actually, (π_x, $\pi_{y|C_x}$) and (π_y, $\pi_{x|C_y}$) are two sets of particular solutions for π'_x and π'_y. In addition, π'_x could be negative even though inequality (1) holds.

We summarize the above results as follows.

[1] If the second dimension is bounded, given an extreme value of π'_x, we might not find a solution for π'_y. However, since π'_x and π'_y are continuous variables here, there are still infinite solutions for π'_x and π'_y.

Proposition 3.1 *Assume that $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$, we can obtain the following results for risk premiums if agents can pay the risk premiums in both dimensions.*

(1) There are infinite possible sets of solutions for risk premiums for removing all risks simultaneously.

(2) The set of risk premiums for removing the risks sequentially is unique. Nevertheless, the particular solutions depend on the order of removing the risk. In addition, when $u^{(1,1)}(x,y) \leq 0$, the sequential risk premiums satisfy the following conditions[2]:

$$\pi_x \geq (\leq) \pi_{x|C_y} \quad if \quad \pi_{y|C_x} \leq (\geq) \pi_y. \tag{12}$$

3.2 Pay the Risk Premiums in the First Dimension

Next, we analyze the nature of risk premiums in the case of paying the risk premiums only in the first dimension. We can consider the first dimension represents wealth level while the second dimension denotes a nonpecuniary variable, such as health. Assume that agents can only pay for risk premiums with money for all risk. Thus, all risk premiums are paid in the first dimension of the utility function.

The following two equations characterize the risk premiums for removing the risks sequentially.

$$\begin{aligned} Eu(\tilde{x}, \tilde{y}) &= Eu(E\tilde{x} - \pi_x, \tilde{y}) = u(E\tilde{x} - \pi_x - \pi_{y|C_x}, E\tilde{y}) && (13) \\ &= Eu(\tilde{x} - \pi_y, E\tilde{y}) = u(E\tilde{x} - \pi_y - \pi_{x|C_y}, E\tilde{y}) && (14) \end{aligned}$$

Given $u^{(1,0)} \geq 0$ and $u^{(0,2)} \leq 0$, we can obtain $\pi_{y|C_x} \geq 0$. Moreover, π_x, $\pi_{y|C_x}$, π_y, and $\pi_{x|C_y}$ are still unique in the sequential case as long as the utility function is increasing in the first argument. In addition, according to (13) and (14), we find the following relation.

$$\pi_x + \pi_{y|C_x} = \pi_y + \pi_{x|C_y} \tag{15}$$

If agents are willing to remove all risk simultaneously, they need to pay risk premiums π'_x and π'_y, which are defined by

$$Eu(\tilde{x}, \tilde{y}) = u(E\tilde{x} - \pi'_x - \pi'_y, E\tilde{y}) \tag{16}$$

[2] Define $u(x,y) =$ a constant. We totally differentiate u and obtain

$0 = u^{(1,0)}dx + u^{(0,1)}dy + u^{(1,1)}dxdy$.

So $dx = -\frac{u^{(0,1)}dy}{u^{(1,0)} + u^{(1,1)}dy}$. We need $u^{(1,1)} \leq 0$ to guarantee $dy \leq 0 \Rightarrow dx \geq 0$.

Although there are still infinite solutions for π'_x and π'_y, the sum of π'_x and π'_y is unique, i.e.

$$\pi'_x + \pi'_y = Constant \tag{17}$$

Likewise, (π_x, $\pi_{y|C_x}$) and (π_y, $\pi_{x|C_y}$) are two sets of particular solutions for π'_x and π'_y. The above results are summarized in the following proposition.

Proposition 3.2 *Given $u^{(1,0)} \geq 0$, we have the following results if agents pay the risk premiums only in one dimension.*
(1) There are infinite possible sets of solutions for risk premiums for removing all risks simultaneously. However, the sum of the risk premiums is unique.
(2) The set of risk premiums for removing the risks sequentially is unique. In addition, although the particular solutions depend on the order of removing the risks, the sum of the risk premiums is constant.

Let π denotes the total risk premium for removing both risks simultaneously, i.e. $\pi = \pi'_x + \pi'_y$. Now, we examine the relation between π and the sum of π_x and π_y. According to (15), we know that $\pi = \pi_y + \pi_{x|C_y}$. Thus, we obtain

$$\pi \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \pi_x + \pi_y \iff \pi_x \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \pi_{x|C_y}.$$

The comparison between π_x and $\pi_{x|C_y}$ can help us to determine the relation between π and $\pi_x + \pi_y$. We need three steps to compare π_x with $\pi_{x|C_y}$. In order to simplify notations, we define function f and function v as follows.

$$f(x) = Eu(x, \tilde{y}), \quad v(x) = u(x, E\tilde{y}) \ \ \forall x$$

Given $u^{(1,0)} \geq 0$, it is straightforward to show that $\partial f/\partial x \geq 0$. We can rewrite (13) as

$$Eu(\tilde{x}, \tilde{y}) = Eu(E\tilde{x} - \pi_x, \tilde{y}) = f(E\tilde{x} - \pi_x) \tag{18}$$

Given no uncertainty on the second dimension, the risk premium for getting rid of $\tilde{x}$ is $\hat{\pi}_x$, defined by

$$Eu(\tilde{x}, E\tilde{y}) = u(E\tilde{x} - \hat{\pi}_x, E\tilde{y}) = v(E\tilde{x} - \hat{\pi}_x) \tag{19}$$

First, we compare $\pi_{x|C_y}$ and $\hat{\pi}_x$. According to (14) and (19), both $\pi_{x|C_y}$ and $\hat{\pi}_x$ are risk premiums for getting rid of $\tilde{x}$, which are conditional on there is no uncertainty on the second dimension. However, the initial wealth in the first dimension is different. If $\pi_y > 0$, the initial wealth in the first dimension is less in the former case. However, the initial wealth is greater in the former case given $\pi_y < 0$. Thus, we can consider that the comparison between $\pi_{x|C_y}$ and $\hat{\pi}_x$ depends on the 'wealth effect'. According to Rey (2003a), we use the following definitions to characterize the properties of risk aversion in the first dimension.

Definition 3.3 *(Rey, 2003a)*

(1) $u(x,y)$ *is DARA utility function in the first dimension if* $\frac{\partial}{\partial x}(-\frac{u^{(2,0)}(x,y)}{u^{(1,0)}(x,y)}) < 0 \ \ \forall y$;

(2) $u(x,y)$ *is CARA utility function in the first dimension if* $\frac{\partial}{\partial x}(-\frac{u^{(2,0)}(x,y)}{u^{(1,0)}(x,y)}) = 0 \ \ \forall y$;

(3) $u(x,y)$ *is IARA utility function in the first dimension if* $\frac{\partial}{\partial x}(-\frac{u^{(2,0)}(x,y)}{u^{(1,0)}(x,y)}) > 0 \ \ \forall y$.

We can use the following lemma to compare $\pi_{x|C_y}$ with $\hat{\pi}_x$.

Lemma 3.4

(1) Given $\pi_y > 0$, $\pi_{x|C_y} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \hat{\pi}_x \quad if \ \ u(x,y) is \left\{ \begin{array}{c} DARA \\ CARA \\ IARA \end{array} \right\}$ *in the first argument;*

(2) Given $\pi_y < 0$, $\pi_{x|C_y} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \hat{\pi}_x \quad if \ \ u(x,y) is \left\{ \begin{array}{c} IARA \\ CARA \\ DARA \end{array} \right\}$ *in the first argument*

According to Definition 3.3 and Pratt's Theorem 1 (1964), the proof for Lemma 3.4 is trivial. Next, we need to find the conditions to determine the sign of π_y. We will consider two kinds of quadrant dependence in the following analysis. Besides quadrant dependence, we propose the following definition:

Definition 3.5 *Consider a pair of random variables* $(\tilde{x}, \tilde{y})$*, we say that there is a positive second-degree quadratic dependence (SQD) between* $\tilde{x}$ *and* $\tilde{y}$ *(denoted by* $SQD(\tilde{y}|x,y) \geq 0$*) if*

$$\int_c^y [F_Y(z|\tilde{x} \leq x) - F_Y(z)]dz \geq 0, \ \forall x, y. \tag{20}$$

Negative second-degree quadratic dependence is defined by reversing the above inequality.

We notice that $F(x,y) - F_X(x)F_Y(y) \geq 0 \Rightarrow SQD(\tilde{y}|x,y) \geq 0 \Rightarrow FED(\tilde{y}|x) \geq 0$.

Following the proof for Proposition 2.4, we can find the sufficient conditions for $\pi_y > 0$ as follows.

Sufficient Condition 1:

Given $u^{(1,0)} > 0$, $u^{(0,2)} < 0$, if one of the following conditions is satisfied, then $\pi_y > 0$.

Quadrant Dependence:

$$(1)\ u^{(1,1)}(x,y) \leq 0 \ and \ F(x,y) \geq F_X(x)F_Y(y) \ \ \forall x,y, \tag{21}$$

$$(2)\ u^{(1,1)}(x,y) \geq 0 \ and \ F(x,y) \leq F_X(x)F_Y(y) \ \ \forall x,y. \tag{22}$$

Second-Degree Quadratic Dependence:

$$(1)\ u^{(1,1)}(x,y) \leq 0 \ and \ SQD(\tilde{y}|x,y) \geq 0 \ \ \forall x, \tag{23}$$

$$(2)\ u^{(1,1)}(x,y) \geq 0 \ and \ SQD(\tilde{y}|x,y) \leq 0 \ \ \forall x. \tag{24}$$

If we assume that $u(x,y)$ is strictly convex in the second argument, i.e. $u^{(0,2)} > 0$, it is straightforward to obtain the sufficient conditions for $\pi_y < 0$ by using the proof for Proposition 2.4.

Sufficient Condition 2:

Given $u^{(1,0)} > 0$, $u^{(0,2)} > 0$, if one of the following conditions is satisfied, then $\pi_y < 0$.

Quadrant Dependence:

$$(1)\ u^{(1,1)}(x,y) \leq 0 \ and \ F(x,y) \leq F_X(x)F_Y(y) \ \ \forall x,y, \tag{25}$$

$$(2)\ u^{(1,1)}(x,y) \geq 0 \ and \ F(x,y) \geq F_X(x)F_Y(y) \ \ \forall x,y. \tag{26}$$

Second-Degree Quadratic Dependence:

$$(1)\ u^{(1,1)}(x,y) \leq 0 \ and \ SQD(\tilde{y}|x,y) \leq 0 \ \ \forall x, \tag{27}$$

$$(2)\ u^{(1,1)}(x,y) \geq 0 \ and \ SQD(\tilde{y}|x,y) \geq 0 \ \ \forall x. \tag{28}$$

Next, we need to analyze the 'risk effect' on the risk premiums. First, we can rewrite the expected utility $Eu(\tilde{x},\tilde{y})$ as following

$$\begin{aligned} & Eu(\tilde{x},\tilde{y}) \\ = & \int_c^d \int_a^b u(x,y)f(x,y)dxdy \\ = & \int_c^d \int_a^b u(x,y)f_X(x)f_Y(y)dxdy + \int_c^d \int_a^b u(x,y)[f(x,y) - f_X(x)f_Y(y)]dxdy \end{aligned} \tag{29}$$

The first term in (29) denotes the 'independent risk effect' while the second term represents the 'dependent risk effect'. To simplify notations, I define the first term in (29) as

$$E_I[u(\tilde{x},\tilde{y})] = \int_c^d \int_a^b u(x,y) f_X(x) f_Y(y) dx dy. \tag{30}$$

Given $E_I[u(\tilde{x},\tilde{y})]$, the risk premium for removing the risk $\tilde{x}$ is π_I, which is defined by

$$E_I[u(\tilde{x},\tilde{y})] = E[u(E\tilde{x} - \pi_I, \tilde{y})] = f(E\tilde{x} - \pi_I) \tag{31}$$

According to (19) and (31), we know

$$Eu(\tilde{x}, E\tilde{y}) = Ev(\tilde{x}) = v(E\tilde{x} - \hat{\pi}_x) \tag{32}$$

$$E_I[u(\tilde{x},\tilde{y})] = Ef(\tilde{x}) = f(E\tilde{x} - \pi_I) \tag{33}$$

According to Pratt's Theorem 1 (1964), we obtain the following lemma.

Lemma 3.6 $\hat{\pi}_x \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \pi_I$ *if and only if* $-\frac{v''(x)}{v'(x)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} -\frac{f''(x)}{f'(x)} \quad ,\forall x$

Lemma 3.6 illustrates that the comparison between $\hat{\pi}_x$ and π_I depends on whether agents will change their risk attitudes for risk $\tilde{x}$ in the presence of the other risk $\tilde{y}$. If agents become more risk averse to risk $\tilde{x}$ in the presence of risk $\tilde{y}$, i.e. $-\frac{f''(x)}{f'(x)} > -\frac{v''(x)}{v'(x)}$, they are willing to pay more to remove the risk $\tilde{x}$. However, given risk $\tilde{y}$, if agents become less risk averse to risk $\tilde{x}$, i.e. $-\frac{f''(x)}{f'(x)} < -\frac{v''(x)}{v'(x)}$, they would pay less for getting rid of $\tilde{x}$ in the presence of risk $\tilde{y}$.

Finally, we need to analyze the 'dependent risk effect' by comparing π_I with π_x. We can use Corollary 4 of Levy and Paroush (1974) to obtain the following result:

$$\gamma \equiv \int_c^d \int_a^b u(x,y)[f(x,y) - f_X(x) f_Y(y)] dx dy = \int_c^d \int_a^b u^{(1,1)}(x,y)[F(x,y) - F_X(x) F_Y(y)] dx dy \tag{34}$$

According to (18), (29), (31), and (34), we have

$$Eu(\tilde{x},\tilde{y}) = f(E\tilde{x} - \pi_I) + \gamma = f(E\tilde{x} - \pi_x) \tag{35}$$

Given $u^{(1,0)}(x,y) > 0$, $f(x)$ is an increasing function. Thus, using equation (34) and (35), we derive Lemma 3.7 to compare π_I with π_x, given quadrant dependence relation between risks.

Lemma 3.7 *If either of the following conditions is satisfied, then* $\gamma \leq 0$ *and* $\pi_I \leq \pi_x$.

(1) $u^{(1,1)}(x,y) \leq 0$ *and* $F(x,y) \geq F_X(x)F_Y(y)\ \forall\, x, y$,

(2) $u^{(1,1)}(x,y) \geq 0$ *and* $F(x,y) \leq F_X(x)F_Y(y)\ \forall\, x, y$.

If either of the following conditions is satisfied, then $\gamma \geq 0$ *and* $\pi_I \geq \pi_x$.

(3) $u^{(1,1)}(x,y) \leq 0$ *and* $F(x,y) \leq F_X(x)F_Y(y)\ \forall\, x, y$,

(4) $u^{(1,1)}(x,y) \geq 0$ *and* $F(x,y) \geq F_X(x)F_Y(y)\ \forall\, x, y$.

The first part of Lemma 3.7 demonstrates that agents would like to pay more for removing the risk $\tilde{x}$ while retaining the risk $\tilde{y}$ in the dependent case than in the independent case, if the dependence relation is not consistent with their correlation risk attitude. For example, if agents are correlation averse, they are willing to combine good outcome with bad outcome. In other words, they prefer a negative dependence relation between risks. Thus, they dislike risk $\tilde{x}$ more and are willing to pay more to get rid of it, given a positive quadrant dependence relation. However, the second part of Lemma 3.7 shows that the risk premium for removing risk $\tilde{x}$ is less in the dependent case than in the independent case if agents' risk attitude for correlation is consistent with the dependence relation. For instance, given a negative quadrant dependence between $\tilde{x}$ and $\tilde{y}$, there exists a motive for a correlation averse agent to retain the risk $\tilde{x}$. In addition, the sign of γ represents the dependent effect. If γ is greater than zero, the dependent effect is positive. However, if γ is less than zero, the dependence between risks has negative effect on agents' risk attitude for risk $\tilde{x}$.

Given $u^{(1,0)} > 0$ and $u^{(0,2)} < 0$, conditions (1) and (2) in Lemma 3.7 are sufficient for $\pi_y > 0$. In addition, conditions (3) and (4) in Lemma 3.7 are sufficient for $\pi_y < 0$ if we have $u^{(1,0)} > 0$ and $u^{(0,2)} > 0$.

Rey (2003a) illustrates how to compare the total risk premium π with the sum of π_x and π_y by analyzing 'wealth effect' and 'risk effect'. However, by using a simple discrete model, she focuses on the case where there is no interaction effect between risks, i.e. $u^{(1,1)} = 0$ or $Cov(\tilde{x}, \tilde{y}) = 0$. In this paper, we generalize her idea. We need to consider three effects rather than two to compare π with $\pi_x + \pi_y$ if there exist interaction effect between risks. All in all, we summarize the comparison method as:

$$\pi \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \pi_x + \pi_y \;\Leftrightarrow\; \pi_x \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \pi_{x|C_y} \Leftarrow \underbrace{\pi_{x|C_y} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \hat{\pi}_x}_{\textit{wealth effect}}, \quad \underbrace{\hat{\pi}_x \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \pi_I}_{\textit{independent risk effect}} \quad \text{and} \quad \underbrace{\pi_I \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \pi_x}_{\textit{dependent risk effect}}$$

Using this comparison method and Lemma 3.4-3.7, we obtain the following two propositions for quadrant dependent risks.

Proposition 3.8 *Given $u^{(1,0)} > 0$ and $u^{(0,2)} < 0$, $\pi \leq \pi_x + \pi_y$ if all the following conditions hold.*

(1) the utility function is CARA or IARA in the first argument,

(2) $-\frac{v''(x)}{v'(x)} \leq -\frac{f''(x)}{f'(x)}$,

(3) $u^{(1,1)}(x, y) \leq 0$ and $F(x, y) \geq F_X(x)F_Y(y) \;\; \forall x, y$,

or $u^{(1,1)}(x, y) \geq 0$ and $F(x, y) \leq F_X(x)F_Y(y) \;\; \forall x, y$.

and

Proposition 3.9 *Given $u^{(1,0)} > 0$ and $u^{(0,2)} > 0$, $\pi \geq \pi_x + \pi_y$ if all the following conditions hold.*

(1) the utility function is CARA or IARA in the first argument,

(2) $-\frac{v''(x)}{v'(x)} \geq -\frac{f''(x)}{f'(x)}$,

(3) $u^{(1,1)}(x, y) \leq 0$ and $F(x, y) \leq F_X(x)F_Y(y) \;\; \forall x, y$,

or $u^{(1,1)}(x, y) \geq 0$ and $F(x, y) \geq F_X(x)F_Y(y) \;\; \forall x, y$.

We can propose another version of Propositions 3.8 and 3.9. For example, in Proposition 3.8, we need to show $\pi < \pi_x + \pi_y$, or say $\pi_x > \pi_{x|C_y}$. From Dionne and Li (2014, Proposition 4.3), we know that π_x is first-order risk aversion and $\pi_{x|C_y}$ is second-order risk aversion. So, for small risks, $\pi_x > \pi_{x|C_y}$. Hence, another version of Proposition 3.8 for small risks is

Proposition 3.10 $\pi < \pi_x + \pi_y$ *for small risks if $u^{(1,1)} \leq 0$ and $FED(\tilde{x}|y) \geq 0$.*

The comparison method generalized in this paper can be used to examine the relation between the total risk premium π and the sum of π_x and π_y under other assumptions about the dependent relation between risks. To illustrate this, we will use this method to analyze the relation between π and $\pi_x + \pi_y$, given second-degree quadratic dependent risks.

The different assumptions about dependent relation mainly affect the 'dependent risk effect'. Thus, we still can use Lemma 3.4 and Lemma 3.6 to analyze the 'wealth effect' and the 'independent risk effect'. However, we need to know what the 'dependent risk effect' would be if there exists second-degree quadratic dependence between risks.

By integrating by parts in (34), we can obtain

$$
\begin{aligned}
&\gamma \\
\equiv\ & \int_c^d \int_a^b u(x,y)[f(x,y) - f_X(x)f_Y(y)]dxdy \\
=\ & \int_c^d \int_a^b u^{(1,1)}(x,y)[F(x,y) - F_X(x)F_Y(y)]dxdy \\
=\ & \int_a^b u^{(1,1)}(x,d)F_X(x)[\int_c^d (F(y|x) - F_Y(y))dy]dx \\
& - \int_a^b \int_c^d u^{(1,2)}(x,y)[\int_c^y (F(x,z) - F_X(x)F_Y(z))dz]dydx \\
=\ & \int_a^b u^{(1,1)}(x,d)F_X(x)FED(\tilde{y}|x)dx - \int_a^b \int_c^d u^{(1,2)}(x,y)[\int_c^y (F(x,z) - F_X(x)F_Y(z))dz]dydx \\
=\ & \int_a^b u^{(1,1)}(x,d)F_X(x)FED(\tilde{y}|x)dx - \int_a^b \int_c^d u^{(1,2)}(x,y)F_X(x)[\int_c^y (F_Y(z|\tilde{x} \le x) - F_Y(z))dz]dydx \\
=\ & \int_a^b u^{(1,1)}(x,d)F_X(x)FED(\tilde{y}|x)dx - \int_a^b \int_c^d u^{(1,2)}(x,y)F_X(x)SQD(\tilde{y}|x,y)dydx. \qquad (36)
\end{aligned}
$$

According to (36), we know that the sign of γ depends on the signs of $u^{(1,1)}$, $u^{(1,2)}$, $FED(\tilde{y}|x)$ and $SQD(\tilde{y}|x,y)$. However, second-degree quadratic dependence implies first-degree expectation dependence. Thus, we have the following lemma.

Lemma 3.11 *If either of the following condition is satisfied, then $\gamma \le 0$, and $\pi_I \le \pi_x$.*

(1) $u^{(1,1)}(x,y) \le 0$, $u^{(1,2)}(x,y) \ge 0$, and $SQD(\tilde{y}|x,y) \ge 0$ $\forall$ x and y.

(2) $u^{(1,1)}(x,y) \ge 0$, $u^{(1,2)}(x,y) \le 0$, and $SQD(\tilde{y}|x,y) \le 0$ $\forall$ x and y.

If either of the following condition is satisfied, then $\gamma \ge 0$, and $\pi_I \ge \pi_x$.

(1) $u^{(1,1)}(x,y) \le 0$, $u^{(1,2)}(x,y) \ge 0$, and $SQD(\tilde{y}|x,y) \le 0$ $\forall$ x and y.

(2) $u^{(1,1)}(x,y) \ge 0$, $u^{(1,2)}(x,y) \le 0$, and $SQD(\tilde{y}|x,y) \ge 0$ $\forall$ x and y.

By using lemma 3.4, 3.6, and 3.11, and the sufficient conditions for $\pi_y > 0$ and $\pi_y < 0$, we have the following two propositions for second-degree quadratic dependent risks.

Proposition 3.12 *Given $u^{(1,0)} > 0$ and $u^{(0,2)} < 0$, $\pi \leq \pi_x + \pi_y$ if all the following conditions hold.*

(1) the utility is CARA or IARA in the first argument,

(2) $-\frac{v''(x)}{v'(x)} \leq -\frac{f''(x)}{f'(x)}$,

(3) $u^{(1,1)}(x,y) \leq 0$, $u^{(1,2)}(x,y) \geq 0$, and $SQD(\tilde{y}|x,y) \geq 0 \ \forall \ x$ and y

or $u^{(1,1)}(x,y) \geq 0$, $u^{(1,2)}(x,y) \leq 0$, and $SQD(\tilde{y}|x,y) \leq 0 \ \forall \ x$ and y.

and

Proposition 3.13 *Given $u^{(1,0)} > 0$ and $u^{(0,2)} > 0$, $\pi \geq \pi_x + \pi_y$ if all the following conditions hold.*

(1) the utility is CARA or IARA in the first argument,

(2) $-\frac{v''(x)}{v'(x)} \geq -\frac{f''(x)}{f'(x)}$,

(3) $u^{(1,1)}(x,y) \leq 0$, $u^{(1,2)}(x,y) \geq 0$, and $SQD(\tilde{y}|x,y) \leq 0 \ \forall \ x$ and y

or $u^{(1,1)}(x,y) \geq 0$, $u^{(1,2)}(x,y) \leq 0$, and $SQD(\tilde{y}|x,y) \geq 0 \ \forall \ x$ and y.

All in all, the comparison method generalized in this paper could be a useful tool to analyze the relation between the total risk premium π and the sum of π_x and π_y under different assumptions of dependence relations between risks.

4 Conclusion

In this paper, we examine the effects of dependence relation between risks on agents' risk taking behavior. Under some conditions, agents would like to remove the primary risk in the presence of the other dependent risk. However, agents may prefer retaining the primary risk in some particular situations. The risk attitude to primary risk in the presence of other dependent risk depends not only on the dependence relation between the risks, but also on the sign of the second-order mixed derivative of the utility function, i.e. the risk attitude for dependence relation. These results are similar to those of Rey's (2003b). In addition, agents also estimate the relative magnitude between the covariance of the risks and variance of the primary risk when they consider retaining the primary risk.

The sign of risk premiums for removing the primary risk in the presence of other dependent risk depends on the change of agents' welfare. If agents' welfare decreases by getting rid of the primary risk, the risk premium will be negative. In other word, agents prefer retaining the primary risk. Given the utility function is increasing in both arguments, the sequential solutions for risk premiums are unique no matter how we would pay the risk premiums (in different dimensions or in only one dimension). However, there are infinite solutions for risk premiums when we remove all risk simultaneously. Nevertheless, the sum of the risk premiums is unique when we pay the risk premiums only in one dimension.

Moreover, the generalized comparison method provided in this paper could be a useful tool to examine the relation between the total risk premium and the sum of partial risk premiums under different assumptions of dependence relations between risks. Given any dependence relation that is stronger than quadrant dependence, we can use exactly the same way to analyze the relation between risk premiums as we do for quadrant dependence.

5 Appendix

5.1 Proof of Proposition 2.4

Since $u^{(2,0)} \leq 0$, $u^{(1,0)}(E\tilde{x}, y) \geq \frac{u(x,y)-u(E\tilde{x},y)}{x-E\tilde{x}}$ for $x \geq E\tilde{x}$. This inequality implies (37). Likewise, we also can obtain (37) by concavity of the utility function in the first argument when $x \leq E\tilde{x}$. Thus, we have

$$u(x,y) - u(E\tilde{x}, y) \leq u^{(1,0)}(E\tilde{x}, y)(x - E\tilde{x}),\ \forall x \tag{37}$$

Taking expectation on both sides of inequality (37), we obtain

$$Eu(\tilde{x}, \tilde{y}) - Eu(E\tilde{x}, \tilde{y}) \leq E[u^{(1,0)}(E\tilde{x}, \tilde{y})(\tilde{x} - E\tilde{x})] \tag{38}$$

Since $E[u^{(1,0)}(E\tilde{x}, \tilde{y})(\tilde{x} - E\tilde{x})] = Eu^{(1,0)}(E\tilde{x}, \tilde{y})E(\tilde{x} - E\tilde{x}) + Cov(u^{(1,0)}(E\tilde{x}, \tilde{y}), \tilde{x} - E\tilde{x})$,

$$Eu(\tilde{x}, \tilde{y}) - Eu(E\tilde{x}, \tilde{y}) \leq Cov(u^{(1,0)}(E\tilde{x}, \tilde{y}), \tilde{x}) \tag{39}$$

Using Theorem 1 of Cuadras (2002), we obtain

$$\begin{aligned}
Cov(u^{(1,0)}(E\tilde{x}, \tilde{y}), \tilde{x}) &= \int_c^d \int_a^b u^{(1,1)}(E\tilde{x}, y)[F(x,y) - F_X(x)F_Y(y)]dxdy \\
&= \int_c^d \int_a^b u^{(1,1)}(E\tilde{x}, y)F_Y(y)[F(x|\tilde{y} \leq y) - F_X(x)]dxdy \\
&= \int_c^d u^{(1,1)}(E\tilde{x}, y)F_Y(y)[\int_a^b (F(x|\tilde{y} \leq y) - F_X(x))dx]dy \\
&= \int_c^d u^{(1,1)}(E\tilde{x}, y)F_Y(y)FED(\tilde{x}|y)dy
\end{aligned} \tag{40}$$

Since $F_Y(y) \geq 0$, $Cov(u^{(1,0)}(E\tilde{x}, \tilde{y}), \tilde{x}) \leq 0$ if either of the conditions of Proposition 2.4 is met. According to (39), $Cov(u^{(1,0)}(E\tilde{x}, \tilde{y}), \tilde{x}) \leq 0$ implies that inequality (1) holds.

5.2 Proof of Lemma 2.6

Using Lemma 2 of Lehmann (1966), we obtain

$$\begin{aligned} Cov(\tilde{x},\tilde{y}) &= E(\tilde{x}\tilde{y}) - E(\tilde{x})E(\tilde{y}) \\ &= \int_c^d \int_a^b [F(x,y) - F_X(x)F_Y(y)]dxdy \\ &= \int_c^d \int_a^b [F(x|y) - F_X(x)]F_Y(y)dxdy \\ &= \int_c^d F_Y(y)FED(\tilde{x}|y)dy \end{aligned} \tag{41}$$

Since $F_Y(y) \geq 0$, the sign of $Cov(\tilde{x},\tilde{y})$ is the same as the sign of $FED(\tilde{x}|y)$.

5.3 Proof of Proposition 2.7

Concavity of the utility function in x implies that $u^{(1,0)}(x,y) \leq \frac{u(x,y)-u(E\tilde{x},y)}{x-E\tilde{x}}$ for $x \geq E\tilde{x}$. We can obtain (42) by the above inequality. Likewise, (42) still holds when $x \leq E\tilde{x}$. Thus, we have

$$u(x,y) - u(E\tilde{x},y) \geq u^{(1,0)}(x,y)(x - E\tilde{x}),\ \forall x \tag{42}$$

Taking expectation on both sides of inequality (42), we have

$$\begin{aligned} Eu(\tilde{x},\tilde{y}) - Eu(E\tilde{x},\tilde{y}) &\geq E[u^{(1,0)}(\tilde{x},\tilde{y})(\tilde{x} - E\tilde{x})] \\ &= Eu^{(1,0)}(\tilde{x},\tilde{y})E(\tilde{x} - E\tilde{x}) + Cov(u^{(1,0)}(\tilde{x},\tilde{y}), \tilde{x} - E\tilde{x}) \\ &= Cov(u^{(1,0)}(\tilde{x},\tilde{y}), \tilde{x}) \end{aligned} \tag{43}$$

If we assume that the variances of both risks are very small, we can take first-order Taylor expansion of $u^{(1,0)}(x,y)$ around $E\tilde{x}$ and $E\tilde{y}$, and have the following approximation

$$u^{(1,0)}(x,y) = u^{(1,0)}(E\tilde{x},E\tilde{y}) + (x - E\tilde{x})u^{(2,0)}(E\tilde{x},E\tilde{y}) + (y - E\tilde{y})u^{(1,1)}(E\tilde{x},E\tilde{y}) \tag{44}$$

Thus,

$$\begin{aligned} & Cov(u^{(1,0)}(\tilde{x},\tilde{y}),\tilde{x}) \\ = \ & u^{(2,0)}(E\tilde{x},E\tilde{y})Cov(\tilde{x} - E\tilde{x},\tilde{x}) + u^{(1,1)}(E\tilde{x},E\tilde{y})Cov(\tilde{y} - E\tilde{y},\tilde{x}) \\ = \ & u^{(2,0)}(E\tilde{x},E\tilde{y})Var(\tilde{x}) + u^{(1,1)}(E\tilde{x},E\tilde{y})Cov(\tilde{x},\tilde{y}) \end{aligned} \tag{45}$$

Since $u^{(2,0)} \leq 0$, the first term of (45) is non-positive. Thus, the second term of (45) must be non-negative if $Cov(u^{(1,0)}(\tilde{x},\tilde{y}),\tilde{x}) \geq 0$. According to Lemma 2.6, $Cov(\tilde{x},\tilde{y})$ and $FED(\tilde{x}|y)$

have the same sign. Thus, if the sign of $u^{(1,1)}(x,y)$ and $FED(\tilde{x}|y)$ are the same, the second term of (45) is non-negative. However, we also need a third condition, i.e. (6) or (7), to ensure that the absolute value of the second term of (45) is greater than that of the first term of (45) and thus $Cov(u^{(1,0)}(\tilde{x},\tilde{y}),\tilde{x}) \geq 0$. Accoridng to (43), $Cov(u^{(1,0)}(\tilde{x},\tilde{y}),\tilde{x}) \geq 0$ implies that inequality (5) holds.

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