Insurance market equilibrium for correlated risks

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Abstract

We analyze the design of contracts when individual risks are correlated between risk-averse agents, such as in natural disaster risks. If the insurer has access to fair contingent capital, it supplies standard contracts. Yet, contingent capital supplied by reinsurers or investors is costly in practice. In this case, the insurer supplies mutual contracts which are contingent on the state of nature. Without transaction costs between insurer and insured, risk-averse agents fully insure against their individual risk and share collective risk by getting some premium pay-back in normal states. With transaction costs, they only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and get some premium pay-back in normal states.

Keywords: individual risk, collective risk, insurance contracts, default risk, participating insurance.

JEL classification: D86, G22, G28, Q54

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1 Introduction

A single natural or man-made disaster can generate tremendous losses for communities. In the recent years, the International Disaster Database has recorded 125 billion dollars of losses for hurricane Katrina which hit New Orleans in 2005 and 50 billion for hurricane Sandy on the American east coast in 2012. On the other side of the globe, the Wenchuan earthquake destroyed 85 billion dollars of assets in China in 2008 and tsunami Honshu that impacted the Japanese Fukushima nuclear power plant generated a daunting 210 billion loss in 2011. These large scale disasters, that impact a huge number of people at the same time, are characterized by correlated individual risks, leading to a high uncertainty on collective losses. This constrains insurers to secure contingent capital in the case where high collective losses would occur, as explained especially by Jaffee & Russell (1997), Froot (2001), Cummins et al. (2002) and Kunreuther & Michel-Kerjan (2009). The high cost of securing contingent capital through reinsurers or investors (with Cat-Bonds for instance) leads insurers to market costly standard contracts for correlated risks, which opens the door to the emergence of alternative contracts.

In the present paper, we question the types of insurance contracts that emerge in the context of collective loss uncertainty in a community, such as in the case of natural disaster risks. We consider a community of identical risk-averse agents. Each agent faces two individual states: she can either suffer a loss or not. At the collective level, there are two states of nature, the normal one and the catastrophic one, respectively characterized by low and high fraction of the population impacted. We postulate the existence of a representative insurer characterizing either a benevolent public insurer or private insurers in competition.[1] We analyze different types of contracts. In any case, the insurer charges an ex-ante premium and pays an ex-post indemnity to the insured if impacted. We first consider standard contracts in which the indemnity is identical in the two states of nature. We then consider contracts with an indemnity level that is contingent on the state of nature.[2] We finally consider contracts with participation, in the sense that a premium pay-back can be transfered ex-post to the insured if the state of nature is normal. Besides, in any case, we assume that the insurer has access to contingent capital through reinsurers

[1] In the two cases the insurer profit is null, so the analysis and the results are similar.

[2] The contingency is contractual in our framework. However, this framework also models an economy in which the contingency is not contractual (i.e. there is a "default risk" on the indemnity) and insureds have the right perception of "default risk". In Doherty & Schlesinger (1990); Cummins & Mahul (2004); Charpentier & Le Maux (2014), "default risk" is used in this sense.

or investors outside the community.[3]

This framework details how the cost of contingent capital for the insurer is translated into the premium paid by the final insureds. We show that, with standard contracts and costly contingent capital (i.e. above fair prices), the higher the correlation between individual risks, the higher the cost of insurance and the lower the purchased coverage. When fully flexible contracts are supplied (i.e. possibility of contingent indemnity and participation), the purchased contracts are standard only if the insurer has access to contingent capital at a fair price. In this case, the classic results of Mossin (1968) apply: risk-averse agents partially insure if there are transaction costs between insurer and insured and fully insure if not. With costly contingent capital, the insurer sells mutual contracts which are contingent on the state of nature. Compared to the standard contracts, the mutual contracts have either an indemnity contingent on the state of nature or a premium pay-back transfered to the insured in the normal state, or both. Without transaction costs between insurer and insured, risk-averse agents fully insure against their individual risk and share, through premium payback, the part of collective risks which is not covered. With transaction costs between insurer and insured, they partially insure against their individual risk in any collective states. In this case, they get either an indemnity lower in the catastrophic state than in the normal state or a premium pay-back in the normal state, or both. The availability of premium pay-back at a low cost relatively to reinsurance cost enables to lower the coverage difference between the catastrophic state and the normal state.

The literature on individual and collective risks has focused on insurance contracts with indemnity contingent on the state of nature. This contingency can be seen as contractual or as a "default risk" with right perception by insureds.[2] Without focusing on insurer capital issues, Doherty & Schlesinger (1990) and Cummins & Mahul (2004) consider the case where the indemnity level in normal states increases with the premium level but the indemnity level in catastrophic states remains low whatever the premium level. Agents purchase contracts at fair prices that do not fully cover the individual risk in both catastrophic and normal states in order to preserve their welfare level in catastrophic states. Charpentier & Le Maux (2014) consider the case where the indemnity level increases with the premium level in any states and the insurer has an exogenously given level of capital besides premiums. Agents purchase contracts at fair prices that fully cover the individual

[3]Catastrophes may affect a community without affecting the others. These types of risks can be at least partly reinsured thanks to international reinsurance or financial markets.

risk in normal states but not in catastrophic states because of insurer reserve issues. The literature has also focused on alternative contracts that could eliminate individual risks at fair prices. As explained by Borch (1962) and his mutuality principle, in a community where agents are exposed to individual risks with collective components, it is Pareto optimal to eliminate individual risks and to share collective risks, which should be reached with participating insurance contracts (Marshall, 1974). Doherty & Dionne (1993) and Doherty & Schlesinger (2002) consider the case where individual risks can be separated in two components, a collective one and an idiosyncratic one, and where one standard contract is supplied for each component. They prove that this setting leads at fair prices to the full elimination of individual risks in each state of nature. This can also be reached without the separability condition by one contract with premium pay-back, as exhibited in the present paper.

The main contribution of the present paper is to endogenize the choice of capital secured by the insurer to face catastrophic states and to analyze its consequences in terms of purchased insurance whether with standard contracts or mutual contracts (with contingent indemnity or/and premium pay-back). To be realistic, we consider contingent capital supplied to insurer at above fair prices, as well as transaction costs between insurer and insured. The paper is organized as follows. In section 2, we present the model with correlated individual risks and the insurance mechanism. In section 3, we analyze one after another the purchased contract at equilibrium with standard insurance, insurance with contingent indemnity and insurance with contingent indemnity and premium pay-back. In section 4, we illustrate the emergence of the latter contracts with the Caribbean Catastrophe Reinsurance Insurance Facility (CCRIF). In the last section, we conclude.

2 The model

2.1 Correlated individual risks and collective risks

We consider a community populated with N agents identical in terms of preferences, initial wealth and exposure to risk and facing correlated individual risks.[4] We assume that the representative agent maximizes preferences satisfying the Von Neumann-Morgenstern axioms and we denote by $u(.)$ the corresponding utility function which is strictly increasing, globally concave and twice continuously differentiable. The wealth of the representative agent before a potential loss is denoted by w.

[4]Heterogeneity of individuals raises questions related to asymmetric information that are out of the scope of our analysis.

To model correlated individual risks, we consider two states of nature, one catastrophic and one normal. Ex-ante, the representative agent knows that with a probability p (such that $0 < p < 1$), a catastrophe occurs and a fraction q_c of the population endures individually a loss of size l. In the normal state, a smaller fraction $q_n < q_c$ of the population endures the same individual loss l.[5] Conditionally on the state of nature, the individual risk of being impacted is independent across agents. This conditional independence assumption enables the law of large numbers to apply within each state of nature. As we assume that N is large, the individual probability of enduring the loss when the catastrophe occurs and when it does not occur are well approximated by q_c and q_n respectively. The representative agent's wealth profile without risk-sharing scheme is characterized in table 1.

collective state	normal		catastrophe	
probability	$1-p$		p	
individual state	no loss	loss	no loss	loss
probability	$(1-p)(1-q_n)$	$(1-p)q_n$	$p(1-q_c)$	pq_c
agent wealth	w	$w-l$	w	$w-l$

Table 1: agent wealth profile without risk-sharing

By denoting μ the individual probability of experiencing a loss l and δ the coefficient of correlation between individual risks, μ and δ are given by (proof in appendix A.1):

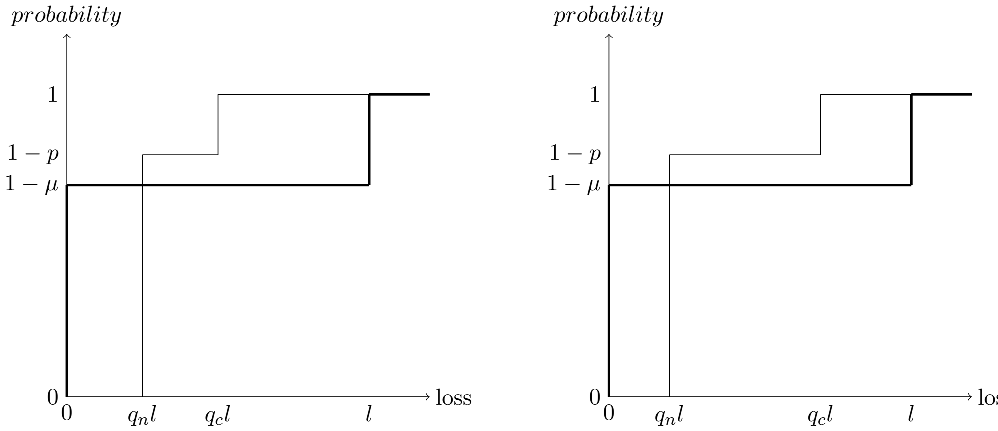
$$\mu = (1-p)q_n + pq_c$$

$$\delta = \frac{p(1-p)}{\mu(1-\mu)}(q_c - q_n)^2$$

In this template, average individual loss depends on the state of nature, its value is $q_n l$ in the normal state and $q_c l$ in the catastrophic state. Thus, its expected value is $\nu = \mu l$ and its variance is $\sigma^2 = \mu(1-\mu)\delta l^2$ (proof in appendix A.2). The higher the individual probability of being affected, the higher the expected average loss. The more correlated the individual risks, the more volatile the average loss.[6]

[5] As pointed out by Malinvaud (1973) and Cass et al. (1996), considering two different loss levels in the two collective states could be considered as two different risks.

[6] The fully correlated case ($\delta = 1$) is characterized by $q_n = 0$, $q_c = 1$ and $0 < p < 1$, in which everyone endures a loss or no one. The no-correlated case ($\delta = 0$), which is not our focus, would correspond to $q_n = q_c$, $p = 1$ or $p = 0$, in which there is only one collective state.



(a) $p = 0.20$, $q_n = 0.25$, $q_c = 0.50$, $\mu = 0.3$, $\delta = 0.048$

(b) $p = 0.20$, $q_n = 0.20$, $q_c = 0.70$, $\mu = 0.3$, $\delta = 0.190$

Figure 1: Cumulative distribution functions for individual loss (thick bars) and average individual loss (thin bars) for two different sets of parameters in 1a and 1b.

Figure 1 illustrates for two different sets of parameters the cumulative distribution functions for the individual loss (thick bars) and the average individual loss (thin bars). The spread between q_n and q_c is smaller in 1a than in 1b, but $p = 0.2$ and $\mu = 0.3$ in both cases. The individual probability of being affected μ is similar for the two sets of parameters, whereas the correlation across individual risks δ is smaller in 1a than in 1b, which makes a difference for risk-sharing mechanism as shown in the paper.

2.2 Insurance mechanism

We consider that the community is equipped with a representative insurer characterizing either a benevolent public insurer or private insurers in competition.[7] These two cases are equivalent because, in both cases, the insurer makes zero profit in both states of nature.[8] In the case of public insurer, it does because it is benevolent. In the case of private insurer, it does because of competition (even if it aims at maximizing its profit).

[7]In the case of private insurers, there should be enough insurers to have perfect competition but not too much relatively to N for the law of large numbers to apply for each insurer in each state of nature.

[8]The appropriate constraints are zero profit in both states of nature and not zero expected profit. Indeed, in the catastrophic state, the insurer cannot pay with some money it does not have, which would be the case with the only condition of zero expected profit. The "zero expected profit" condition is actually a shortcut when transferring money from one state to another can be made at a fair price. Yet, in a world with costly capital or diversification issues, this shortcut cannot be made because agents such as reinsurers or investors bearing this transfer require some payment.

The representative insurer supplies insurance contracts to risk-averse agents. The insurance premium charged ex-ante is denoted by $\alpha \geq 0$. The indemnity paid ex-post to the insured if impacted is denoted by $\tau \geq 0$ in the normal state and $\tau - \epsilon \geq 0$ in the catastrophic state. The indemnity flexibility models an insurance scheme in which the coverage is contingent on the state of nature, which is relevant either for the situation in which the contingency is contractual or for the situation in which the contingency is a "default risk" with right perception by insureds. The insurance is potentially participating, in the sense that the insureds can get an ex-post premium pay-back $\beta \geq 0$ in the normal state. The insurance scheme has transaction costs equal to a fraction $\lambda^i \geq 0$ of the indemnity, in which λ^i is called the insurance loading factor. It also has potentially transaction costs for participation equal to a fraction $\lambda^p \geq 0$ of the premium pay-back, in which λ^p is called the participation loading factor. We assume that $\lambda^i \geq \lambda^p$.[9]

The representative insurer can secure contingent capital through reinsurers or investors outside the community, to be able to pay the higher total claims of the catastrophic state. Securing contingent capital $c \geq 0$ for the catastrophic state costs $(1 + \lambda^c)pc$, in which $\lambda^c \geq 0$ is called the contingent capital loading factor.[10] For contingent capital to be relevant, we need to have $(1 + \lambda^c)p < 1$.

With this insurance mechanism, the representative agent's wealth $\tilde{y}$ and the representative insurer's profit $\tilde{\pi}$ are detailed in table 2.[11]

collective state	normal		catastrophe	
probability	$1 - p$		p	
individual state	no loss	loss	no loss	loss
probability	$(1-p)(1-q_n)$	$(1-p)q_n$	$p(1-q_c)$	pq_c
agent wealth	$w - \alpha + \beta$	$w - \alpha - l + \tau + \beta$	$w - \alpha$	$w - \alpha - l + \tau - \epsilon$
insurer profit	$\alpha - (1+\lambda^i)q_n\tau - (1+\lambda^c)pc - (1+\lambda^p)\beta$		$\alpha - (1+\lambda^i)q_c(\tau-\epsilon) - (1+\lambda^c)pc + c$	

Table 2: agent wealth and insurer profit profiles

[9] λ^i and λ^p correspond to frictional costs: λ^i should include in particular the costs of expertise, management and illiquidity and λ^p should include in particular the cost of illiquidity. Note that the costs in λ^i should be more numerous than the costs in λ^p, that is why we assume $\lambda^i \geq \lambda^p$.

[10] λ^c corresponds to frictional costs with reinsurers or investors, as detailed in Froot (2001).

[11] The insurer profit written in table 2 is actually the insurer profit per individual. With one insurer, it should be multiplied by N to get to the insurer profit. With M insurers, it should be multiplied by $\frac{N}{M}$ to get to the representative insurer profit.

3 Insurance contracts at equilibrium

This section introduces progressively insurance contract flexibility in the context of correlated individual risks. We analyze one after another the equilibrium with standard contracts, contracts with contingent indemnity and contracts with contingent indemnity and premium pay-back.

3.1 Standard insurance

If the representative insurer supplies standard insurance contracts, a premium $\alpha \geq 0$ is required in exchange for an indemnity $\tau \geq 0$ in case of individual loss in any state of nature, without any premium pay-back. In this case, the representative agent's wealth $\tilde{y}$ and the representative insurer's profit $\tilde{\pi}$ boil down to table 3.

collective state	normal		catastrophe	
individual state	no loss	loss	no loss	loss
agent wealth	$w - \alpha$	$w - \alpha - l + \tau$	$w - \alpha$	$w - \alpha - l + \tau$
insurer profit	$\alpha - (1 + \lambda^i)q_n\tau - (1 + \lambda^c)pc$		$\alpha - (1 + \lambda^i)q_c\tau - (1 + \lambda^c)pc + c$	

Table 3: agent wealth and insurer profit profiles

As explained in the previous section, the representative insurer makes zero-profit in both states of nature. Thus, it secures the contingent capital c per agent and supplies the contracts (α, τ) such that:

$$c = (1 + \lambda^i)(q_c - q_n)\tau \tag{1}$$

$$\alpha = (1 + \lambda^i)\Big(1 + \frac{p}{\mu}(q_c - q_n)\lambda^c\Big)\mu\tau \tag{2}$$

The representative insurer has to secure contingent capital (1) if risks are correlated ($q_c > q_n$), because its coverage expenses are uncertain. The contract it supplies has a premium (2) higher than the actuarially fair premium $\mu\tau$. The total insurance loading factor is composed by the exogenous insurance loading factor λ^i and the contingent capital loading factor λ^c translated to final insureds. The translated loading factor increases with risk correlation because the higher the correlation, the higher the insurer uncertainty and the larger the necessary contingent capital. We define the total loading factor λ^t such that: $1 + \lambda^t = (1 + \lambda^i)(1 + \frac{p}{\mu}(q_c - q_n)\lambda^c)$.

The representative agent purchases the contract that maximizes her welfare:

$$\begin{aligned} &\max_{\tau} \ \mathbb{E}(u(\tilde{y})) \\ &\text{s.t. } \alpha = (1+\lambda^t)\mu\tau \end{aligned} \tag{3}$$

The purchased indemnity τ is determined by the first order condition of (3), which relates the marginal trade-off between risk-aversion and insurance costs:

$$\frac{u'(w-\alpha-l+\tau)}{u'(w-\alpha)} = \frac{1+\lambda^t}{1-\frac{\mu}{1-\mu}\lambda^t} \tag{4}$$

(4) tells that the representative agent partially insures if λ^t is strictly positive, i.e. if λ^i is strictly positive or if risks are correlated and λ^c is strictly positive. If λ^t is too large, the representative agent does not even purchase insurance.

Proposition 1 *If insurance is not a Giffen good*[12]*, then* $\frac{d\tau}{d\delta} \leq 0$ *and* $\frac{d\tau}{d\lambda^c} \leq 0$.

Proof If insurance is not a Giffen good, then $\frac{d\tau}{d\lambda^t} \leq 0$ by definition. Results are then directly obtained by the fact that λ^t increases with δ or with λ^c.

Proposition 1 tells that the higher the price of contingent capital (due to higher correlation or loading factor), the lower the purchased coverage if insurance is not a Giffen good. Correlation between individual risks generates collective loss volatility, which forces the insurer to secure contingent capital to be able to cover insureds at the same level in any collective state. The insurer has to translate the cost of contingent capital to final insureds, which can crowd-out insurance demand. Lowering indemnity only in the catastrophic state can reduce the necessary contingent capital, that is why we introduce this flexibility in the following section.

3.2 Insurance with contingent indemnity

Relatively to the scheme in section 3.1, we now consider that the representative insurer supplies contracts in which the indemnity is contingent on the state of nature.[13] We denote by $\tau \geq 0$ the indemnity in the normal state and $\tau - \epsilon \geq 0$ the indemnity in the catastrophic state. The representative agent's wealth $\tilde{y}$ and the representative insurer's profit $\tilde{\pi}$ are then represented in table 4.

[12] Insurance is called a Giffen good if there exist an interval on which the coverage demand is increasing in the loading factor. Briys et al. (1989) provide the necessary and sufficient condition for insurance not to be a Giffen good. They show that insurance is not a Giffen good if and only if the Arrow-Pratt coefficient of absolute risk aversion decreases not too rapidly when wealth increases.

[13] Contingent indemnity contracts are relevant only with two strictly different states of nature (i.e. $q_n < q_c$ and $0 < p < 1$).

collective state	normal		catastrophe	
individual state	no loss	loss	no loss	loss
agent wealth	$w - \alpha$	$w - \alpha - l + \tau$	$w - \alpha$	$w - \alpha - l + \tau - \epsilon$
insurer profit	$\alpha - (1 + \lambda^i)q_n\tau - (1 + \lambda^c)pc$		$\alpha - (1 + \lambda^i)q_c(\tau - \epsilon) - (1 + \lambda^c)pc + c$	

Table 4: agent wealth and insurer profit profiles

The representative insurer makes zero-profit in both states of nature. Thus, it secures the contingent capital c per agent and supplies the contracts (α, τ, ϵ) such that:

$$c = (1 + \lambda^i)(q_c - q_n)\tau - (1 + \lambda^i)q_c\epsilon \tag{5}$$

$$\alpha = (1 + \lambda^t)\mu\tau - (1 + \lambda^i)(1 + \lambda^c)pq_c\epsilon \tag{6}$$

with $1+\lambda^t = (1+\lambda^i)(1+\frac{p}{\mu}(q_c-q_n)\lambda^c)$. Allowing for a lower indemnity in the catastrophic state ($\epsilon \geq 0$) enables to lower the necessary contingent capital (5) and the premium (6). The premium is reduced through two channels. The first channel is standard: a lower indemnity implies a lower premium. The second channel is due to the fact that the insurer needs less contingent capital and appears through λ^c in the second term on the right hand-side of (6).

The representative agent purchases the contract that maximizes her welfare:

$$\begin{aligned} &\max_{\tau,\epsilon} \ \mathbb{E}(u(\tilde{y})) \\ &\text{s.t. } \alpha = (1 + \lambda^t)\mu\tau - (1 + \lambda^i)(1 + \lambda^c)pq_c\epsilon \end{aligned} \tag{7}$$

The purchased indemnities τ and $\tau - \epsilon$ are determined by the first order conditions of (7) (proof in appendix B.1):

$$\frac{u'(w - \alpha - l + \tau)}{u'(w - \alpha)} = \frac{(1 + \lambda^i)(1 - \frac{p}{1-p}\lambda^c)}{1 - \frac{\mu}{1-\mu}\lambda^t} \tag{8}$$

$$\frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = \frac{(1 + \lambda^i)(1 + \lambda^c)}{1 - \frac{\mu}{1-\mu}\lambda^t} \tag{9}$$

The coefficient on the right hand-side of (8) (respectively (9)) is lower (respectively greater) than in (4). This suggests that flexibility on indemnity level leads to purchase a contract with a better (respectively worse) coverage in the normal (respectively catastrophic) state than with standard insurance.

Proposition 2 *The purchased contract is such that:*

(i) $\epsilon = 0$ *when* $\lambda^c = 0$*;*

(ii) $\epsilon > 0$ *when* $\lambda^c > 0$.

Proof The combination of (8) and (9) gives:

$$\frac{u'(w-\alpha-l+\tau-\epsilon)}{u'(w-\alpha-l+\tau)} = \frac{1+\lambda^c}{1-\frac{p}{1-p}\lambda^c} \tag{10}$$

which gives directly the result.

Proposition 3 *With a given* $\lambda^c > 0$*, the purchased contract is such that:*

(i) $\tau - \epsilon < l$ *and* $\tau \leq l$ *for high* λ^i*;*

(ii) $\tau - \epsilon < l$ *and* $\tau > l$ *for low* λ^i.

Proof (9) tells that $\tau - \epsilon < l$ for any λ^i if $\lambda^c > 0$ because the coefficient on its right hand-side is strictly larger than one when $\lambda^c > 0$. (8) tells that $\tau \leq l$ if and only if $\lambda^i \geq \lambda^{i*}$ with λ^{i*} such that $\frac{(1+\lambda^{i*})(1-\frac{p}{1-p}\lambda^c)}{1-\frac{\mu}{1-\mu}\lambda^t} = 1$.

Proposition 2 states that standard contracts ($\epsilon = 0$) are purchased if and only if contingent capital is supplied at fair prices by reinsurers or investors to the insurer ($\lambda^c = 0$). When contingent capital is costly for the insurer ($\lambda^c > 0$), it is valuable for the insured to purchase a contract with a lower indemnity in the catastrophic state than in the normal state for the same loss l ($\tau - \epsilon < \tau$). The higher λ^c, the more valuable the insurance contract with contingent indemnity, because it enables to reduce the contingent capital that has to be secured by the insurer. Proposition 3 expresses in particular that, when contingent capital is costly ($\lambda^c > 0$), the insured purchases a contract with at least partial coverage in the catastrophic state whatever the transaction costs between insured and insurer (λ^i), which means that this type of contracts with contingent indemnity cannot fulfill the mutuality principle when there are no transaction costs between insured and insurer ($\lambda^i = 0$). The result by Charpentier & Le Maux (2014), telling that only partial insurance is purchased for the catastrophic state, is thus confirmed in the context where the insurer is free to choose its level of reserve, only if contingent capital is costly. Proposition 3 also relates that, when contingent capital is costly ($\lambda^c > 0$), the insured would like to purchase a contract with an indemnity larger than the loss in the normal state if the transaction costs with the insurer are low enough. This counter-intuitive result relatively to the literature comes from the risk of catastrophe. Indeed, to get a sufficient coverage in case of catastrophe without paying too much for the reinsurance, risk-averse agents are fine to pay premiums that are collectively higher than necessary for full coverage in the normal state and thus to receive claims higher than their loss in the normal state. This behavior suggests that a better designed contract would redistribute in the normal state the excessive amount equally among the insureds through premium pay-back. That is why we introduce in the following section the possibility for insureds to get premium pay-back, which is equivalent to participate in the capital reserve of the insurer.

3.3 Insurance with contingent indemnity and premium pay-back

Relatively to the scheme in section 3.2, we now consider that the representative insurer supplies participating contracts, in the sense that ex-post premium pay-back can be transferred to insureds in the normal state.[14] We denote by $\beta \geq 0$ the premium pay-back in the normal state. In this scheme, the representative agent's wealth $\tilde{y}$ and the representative insurer's profit $\tilde{\pi}$ are written in table 5.

collective state	normal		catastrophe	
individual state	no loss	loss	no loss	loss
agent wealth	$w - \alpha + \beta$	$w - \alpha - l + \tau + \beta$	$w - \alpha$	$w - \alpha - l + \tau - \epsilon$
insurer profit	$\alpha - (1+\lambda^i)q_n\tau - (1+\lambda^c)pc - (1+\lambda^p)\beta$		$\alpha - (1+\lambda^i)q_c(\tau - \epsilon) - (1+\lambda^c)pc + c$	

Table 5: agent wealth and insurer profit profiles

The representative insurer makes zero-profit in both states of nature. Thus, it secures the contingent capital c per agent and supplies the contracts $(\alpha, \tau, \epsilon, \beta)$ such that:

$$c = (1+\lambda^i)(q_c - q_n)\tau - (1+\lambda^i)q_c\epsilon - (1+\lambda^p)\beta \tag{11}$$

$$\alpha = (1+\lambda^t)\mu\tau - (1+\lambda^i)(1+\lambda^c)pq_c\epsilon + (1+\lambda^p)(1 - \frac{p}{1-p}\lambda^c)(1-p)\beta \tag{12}$$

with $1+\lambda^t = (1+\lambda^i)(1+\frac{p}{\mu}(q_c - q_n)\lambda^c)$. Allowing for a premium pay-back in the normal state ($\beta \geq 0$) enables to lower the necessary contingent capital (11). It impacts the premium (12) through two channels. The first channel is standard: a higher pay-back implies a higher premium. The second channel corresponds to a decrease through λ^c in

[14] Participating contracts are relevant only with two strictly different states of nature (i.e. $q_n < q_c$ and $0 < p < 1$).

the second term on the right hand-side of (12): a higher pay-back implies a higher reserve through the premium and a lower contingent capital through reinsurers or investors.

The representative agent purchases the contract that maximizes her welfare:

$$\begin{aligned} &\max_{\tau,\epsilon,\beta} \mathbb{E}(u(\tilde{y})) \\ &\text{s.t. } \alpha = (1+\lambda^t)\mu\tau - (1+\lambda^i)(1+\lambda^c)pq_c\epsilon + (1+\lambda^p)(1-\frac{p}{1-p}\lambda^c)(1-p)\beta \end{aligned} \tag{13}$$

The purchased indemnities τ and $\tau - \epsilon$ and the premium pay-back β are determined by the first order conditions of (13) (proof in appendix B.2):

$$\frac{u'(w-\alpha-l+\tau+\beta)}{u'(w-\alpha+\beta)} = \frac{1+\lambda^i}{1-\frac{q_n}{1-q_n}\lambda^i+\frac{1}{1-q_n}\lambda^p} \tag{14}$$

$$\frac{u'(w-\alpha-l+\tau-\epsilon)}{u'(w-\alpha)} = \frac{1+\lambda^i}{1-\frac{q_c}{1-q_c}\lambda^i-\frac{1}{1-q_c}(\frac{1}{p(1+\lambda^c)}-1)\lambda^p} \tag{15}$$

$$\frac{u'(w-\alpha)}{u'(w-\alpha+\beta)} = \frac{(1-\frac{q_c}{1-q_c}\lambda^i-\frac{1}{1-q_c}(\frac{1}{p(1+\lambda^c)}-1)\lambda^p)(1+\lambda^c)}{(1-\frac{q_n}{1-q_n}\lambda^i+\frac{1}{1-q_n}\lambda^p)(1-\frac{p}{1-p}\lambda^c)} \tag{16}$$

The coverage for individual risk in the normal state (14) does not depend on λ^c contrary to the previous contracts and the coverage for individual risk in the catastrophic state (15) depends on λ^c only if $\lambda^p > 0$. This means that, with participating contracts, the coverage choice is less dependent on the costs of contingent capital supplied by reinsurers or investors. The choice of participation β which appears in (16) is directed in two opposite directions by λ^c and λ^p. The larger λ^c relatively to λ^p, the larger the participation β.

Proposition 4 *The purchased contract is such that:*

(i) $\epsilon = 0$ *and* $\beta = 0$ *when* $\lambda^c = 0$*;*

(ii) $\epsilon > 0$ *or* $\beta > 0$ *when* $\lambda^c > 0$*.*

Proof The combination of (14), (15) and (16) gives:

$$\frac{u'(w-\alpha-l+\tau-\epsilon)}{u'(w-\alpha-l+\tau+\beta)} = \frac{1+\lambda^c}{1-\frac{p}{1-p}\lambda^c} \tag{17}$$

which gives directly the result.

Proposition 5 *With a given* $\lambda^c > 0$*, the purchased contract is such that:*

(i) $\tau = l$*,* $\epsilon = 0$ *and* $\beta > 0$ *for* $\lambda^i = \lambda^p = 0$*;*

(ii) $\tau - \epsilon < l$, $\tau < l$ *and* $\beta > 0$ *for* $\lambda^i > \lambda^p > 0$ *and* λ^p *low enough relatively to* $p\lambda^c$*.*

Proof (14), (15) and (16) give directly the result for $\lambda^i = \lambda^p = 0$ because, in this case, the coefficients on the right hand-side of (14) and (15) are equal to one and the coefficient on the right hand-side of (16) is strictly larger than one. For $\lambda^i > \lambda^p > 0$, the coefficients in (14) and (15) are strictly larger than one (remembering that $p(1+\lambda^c) < 1$) and the coefficient in (16) is still strictly larger than one if $\lambda^p < \lambda^{p*}$ with λ^{p*} such that $\frac{(1-\frac{q_c}{1-q_c}\lambda^i - \frac{1}{1-q_c}(\frac{1}{p(1+\lambda^c)}-1)\lambda^{p*})(1+\lambda^c)}{(1-\frac{q_n}{1-q_n}\lambda^i + \frac{1}{1-q_n}\lambda^{p*})(1-\frac{p}{1-p}\lambda^c)} = 1$, which gives the second part of the result.

Proposition 4 states that standard contracts ($\epsilon = 0$ and $\beta = 0$) are purchased if and only if contingent capital is supplied at fair prices by reinsurers or investors to the insurer ($\lambda^c = 0$). When contingent capital is costly for the insurer ($\lambda^c > 0$), the purchased contract by the insured has either a lower indemnity in the catastrophic state than in the normal state ($\tau - \epsilon < \tau$) or a premium pay-back in the normal state ($\beta > 0$), or both. The higher λ^c, the more valuable these flexibilities in the insurance contract are. Proposition 5 expresses that an insurance contract with premium pay-back is preferred by the insured when contingent capital through reinsurers or investors is costly relatively to the participation by insureds. As the coefficient on the right hand-side of (17) is similar to the one of (10), we can say that the possibility of premium pay-back should lead to a better coverage in the catastrophic state relatively to the insurance without participation. Proposition 5 also relates that the insured purchases full insurance for individual risk ($\tau = l$ and $\epsilon = 0$) when there are no transaction costs between insured and insurer ($\lambda^i = 0$ and $\lambda^p = 0$), which means that participating contracts fulfill the mutuality principle. With transaction costs between insured and insurer ($\lambda^i > \lambda^p > 0$), the insured purchases a contract with partial coverage in both collective states ($\tau - \epsilon < l$ and $\tau < l$). In the following section, we illustrate through an example that contracts with contingent indemnity and participation can indeed be implemented when risks are correlated and contingent capital through reinsurers or investors is costly.

4 The case of CCRIF

In this section, we show that the policies sold by the Caribbean Catastrophe Reinsurance Insurance Facility (CCRIF) display the features that we highlighted in the previous section: reinsurance, contingent indemnity and participation. The Caribbean Catastrophe Reinsurance Insurance Facility is a non for profit multi-country insurance pool. Created in 2007, it currently offers disaster-relief insurance policies to sixteen Caribbean countries, protecting them against earthquake, hurricane and excess rainfall losses. Its effectiveness during the five first years of existence has conducted the program to be extended to Central American countries, starting from 2016. The facility aims at pooling the risks faced by its members and reduce the cost the members would individually face if they directly insured on the reinsurance market. The annual reports of the CCRIF are publicly available[15] and provide useful information about the catastrophe insurance contracts proposed to the sixteen members. Relatively to our model, the involved countries represent the risk-averse agents and the CCRIF represent the benevolent public insurer.

Collective risk

CCRIF reports a stable number of 29 or 30 sold policies each year since its inception. The collective risk by the pool has remained rather stable as well. Figure 2 displays the cumulative density function reported by CCRIF in its annual reports since year 2007-2008. The darker lines represent the cumulative distribution of the risk faced by the pool in the earlier periods of its existence. Contrary to a traditional insurance where the collective risk divided by the number of insureds becomes negligible when the number of insureds grows, CCRIF faces a significant collective risk. That is why this community of countries represents a good example for our analysis.

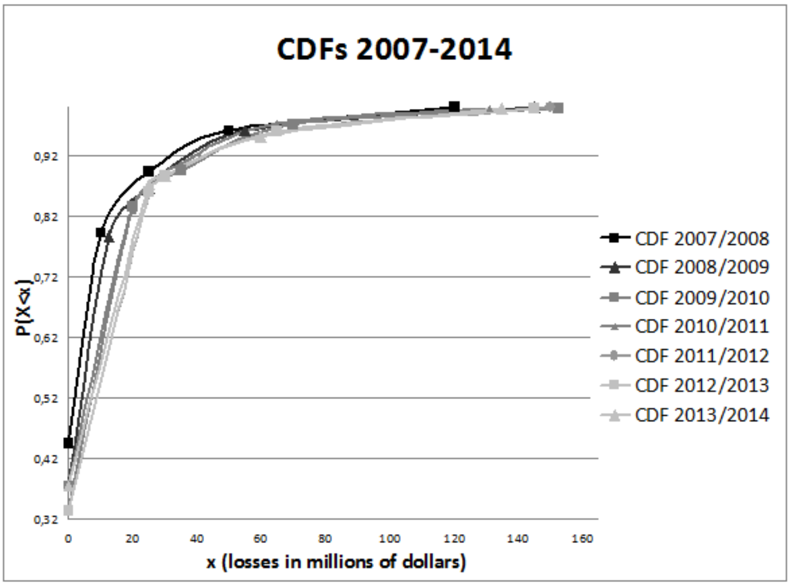


Figure 2: Collective risk

[15] http://www.ccrif.org/content/publications/reports/annual

Reinsurance cost

Using these cumulative distribution functions with the information about the structure of the reinsurance scheme bought by CCRIF, we can compute an estimated loading factor paid by the organization as : $\lambda^c = \frac{\alpha^R}{\mathbb{E}(L)} - 1$, where α^R is the premium paid by CCRIF to reinsurers and $\mathbb{E}(L)$ is the expected loss reinsured. Figure 3 displays its evolution through the years and shows that CCRIF faces a significant loading factor on reinsurance.

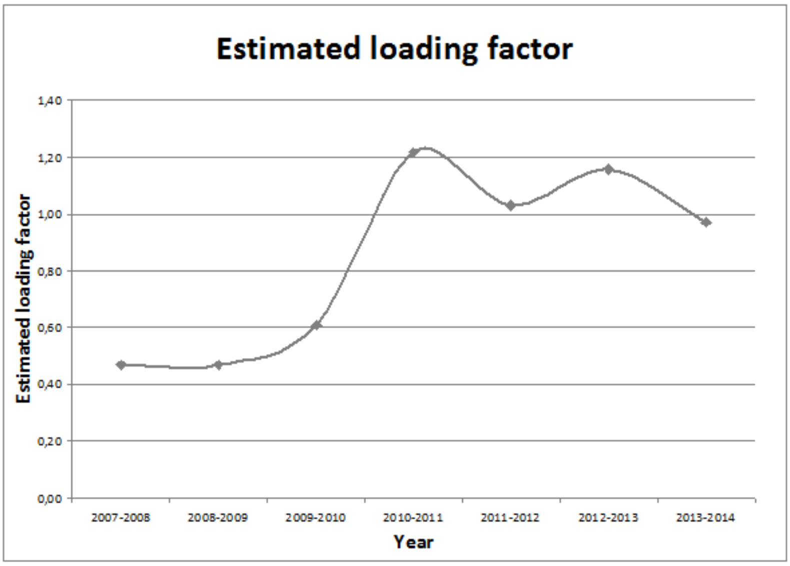


Figure 3: Reinsurance loading factor

Contingent indemnity

The situation faced by CCRIF is much richer than the two collective sates described in our model but the possibility of a default is acknowledged by CCRIF when it states "The CCRIF can currently survive a series of loss events with a less than one in 10,000 chance of occurring in any given year. Due to planned premium reductions, the level of security drops somewhat through the course of our 10-year forward modeling. However, the lowest projected survivability for CCRIF in the 10-year modeled period is about a one in three thousand chance of claims exceeding capacity (and thus defaulting on its obligations) in any one year." Even-though defaults are associated to severe costs in terms of image and trust, which are not captured in our model, CCRIF takes a pragmatic approach by acknowledging the existence of a a trade-off between cost and solvency, which results in a small but positive probability of default. Similarly to our analysis, the indemnity is thus contingent on the collective state with lower indemnities in the most catastrophic states.

Premium policies

In addition to the regular insurance premiums, the facility required its members to pay an up-front participation fee. Audited financial statements report that "it is Managements intent that participation fee deposits are available to fund losses in the event that funds from retained earnings, reinsurers and the Donor Trust are insufficient. If deposits are used to fund losses, it is also Managements intent that any subsequent earnings generated by the Group will be used to reinstate the deposits to their original carrying value". Figure 4 shows that the total amount of premiums was effectively much higher the first year than the following years, because no large catastrophes occurred during these years as shown by the claim payouts. The insurance policy announcing ex-ante lower premiums in case of no catastrophes is equivalent to constant premiums over the years with premium pay-back in case of no catastrophes. The bars in Figure 4 illustrate these premium pay-back ($\beta > 0$), assuming that the premium remains similar to the first year.

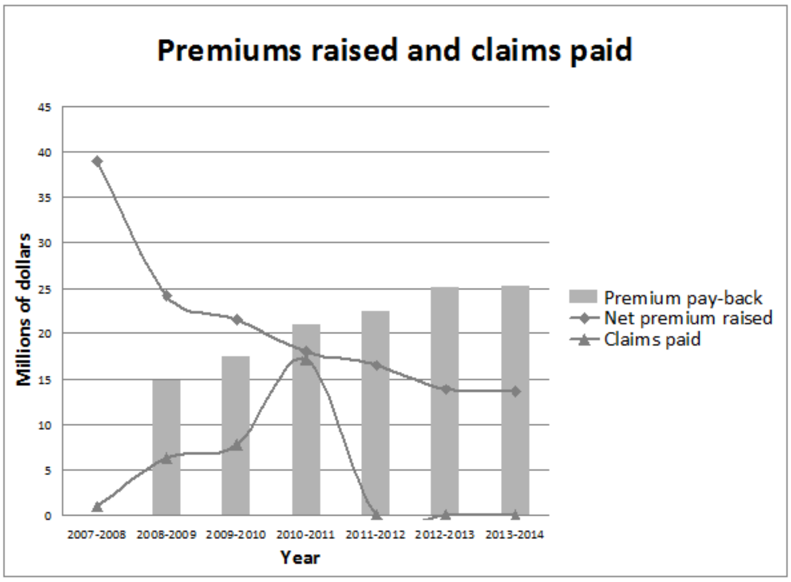


Figure 4: Premium, premium pay-back and payouts

5 Conclusion

In the present paper, we have built a simple model to analyze the type of insurance contract that emerges when risks are correlated between risk-averse agents. For the sake of realism, we have considered a representative insurer choosing the contingent capital it secures to face catastrophic states and we have assumed that the contingent capital is supplied by reinsurers or investors above fair prices. In this scheme, the insurer supplies mutual contracts which are contingent on the state of nature. Without transaction costs between insurer and insured, risk-averse agents fully insure against their individual risk and share collective risk by getting some premium pay-back in normal states. With transaction costs, they only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and get some premium pay-back in normal states. The availability of premium pay-back enables to improve their coverage level in the catastrophic state. This analysis helps to understand the limits that risk correlation and costly contingent capital through reinsurers or investors represent for risk-sharing and how the contracts can be improved through more flexibility. Indeed, contracts with contingent indemnity and premium pay-back enable to share better individual risks and collective risks when diversification outside the community is costly. We illustrate these mechanisms with the example of the Caribbean Catastrophe Reinsurance Insurance Facility (CCRIF) that combines the three characteristics of our proposed contract: reinsurance is bought to hedge against the most likely losses, while participation covers higher collective losses and the most unlikely losses (less than 1-in-10,000 chance of occurring) are left uncovered. Participation and contingent indemnity enable the facility to grant significant discounts and supply better insurance contracts.

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A Appendix for section 2

A.1 Derivation of the coefficient of correlation

With the loss represented by the random variable $\tilde{x}^i$ for individual i, $\tilde{x}^i$ is well approximated when N is large by:

$$\tilde{x}^i = \begin{cases} -l & \text{with probability} \quad \mu \\ 0 & \text{with probability} \quad 1-\mu \end{cases}$$

$$\tilde{x}^i \tilde{x}^j = \begin{cases} l^2 & \text{with probability} \quad (1-p)q_n^2 + pq_c^2 \\ 0 & \text{with probability} \quad 1-(1-p)q_n^2 - pq_c^2 \end{cases}$$

The correlation between individual risks is:

$$\delta = \frac{COV(\tilde{x}^i, \tilde{x}^j)}{(VAR(\tilde{x}^i)VAR(\tilde{x}^j))^{0.5}}$$

We have:

$$\begin{aligned} \mathrm{COV}(\tilde{x}^i, \tilde{x}^j) &= \mathbb{E}(\tilde{x}^i \tilde{x}^j) - \mathbb{E}(\tilde{x}^i)\mathbb{E}(\tilde{x}^j) \\ &= l^2((1-p)q_n^2 + pq_c^2) - (-l\mu)^2 \\ &= l^2((1-p)q_n^2 + pq_c^2 - \mu^2) \end{aligned}$$

$$\begin{aligned} \mathrm{VAR}(\tilde{x}^i) &= \mathbb{E}((\tilde{x}^i)^2) - \mathbb{E}(\tilde{x}^i)^2 \\ &= l^2\mu - (-l\mu)^2 \\ &= l^2\mu(1-\mu) \end{aligned}$$

The coefficient of correlation is then:

$$\begin{aligned} \delta &= \frac{(1-p)q_n^2 + pq_c^2 - \mu^2}{\mu(1-\mu)} \\ &= \frac{p(1-p)}{\mu(1-\mu)}(q_c - q_n)^2 \end{aligned}$$

A.2 Derivation of the variance of the average individual loss

With the average individual loss represented by the random variable $\tilde{X}$, we have:

$$\tilde{X} = \begin{cases} q_c l & \text{with probability} \quad p \\ q_n l & \text{with probability} \quad 1-p \end{cases}$$

This can also be written as $\tilde{X} = \tilde{q}l$ where:

$$\tilde{q} = \begin{cases} q_c & \text{with probability} \quad p \\ q_n & \text{with probability} \quad 1-p \end{cases}$$

Hence, the variance of the average individual loss is: $\sigma^2 = \text{Var}(\tilde{X}) = \text{Var}(\tilde{q})l^2$, with:

$$\tilde{q}^2 = \begin{cases} q_c^2 & \text{with probability} \quad p \\ q_n^2 & \text{with probability} \quad 1-p \end{cases}$$

$$\begin{aligned} \text{Var}(\tilde{q}) &= \mathbb{E}(q^2) - (\mathbb{E}(q))^2 \\ &= (1-p)q_n^2 + pq_c^2 - \mu^2 \end{aligned}$$

The variance of the average individual loss is then:

$$\sigma^2 = \delta\mu(1-\mu)l^2$$

B Appendix for section 3

B.1 First order conditions of (7)

The first order conditions of (7) are:

$$-(1+\lambda^t)\mu\mathbb{E}(u'(\tilde{y})) + (1-p)q_n u'(w-\alpha-l+\tau) + pq_c u'(w-\alpha-l+\tau-\epsilon) = 0 \quad (18)$$

$$(1+\lambda^i)(1+\lambda^c)pq_c\mathbb{E}(u'(\tilde{y})) - pq_c u'(w-\alpha-l+\tau-\epsilon) = 0 \quad (19)$$

With the definition of $\mathbb{E}(u'(\tilde{y}))$, the combinations of the first order conditions of (7) give:

$$u'(w-\alpha-l+\tau-\epsilon) = (1+\lambda^i)(1+\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (20)$$

$$u'(w-\alpha-l+\tau) = (1+\lambda^i)(1-\frac{p}{1-p}\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (21)$$

$$u'(w-\alpha) = (1-\frac{\mu}{1-\mu}\lambda^t)\mathbb{E}(u'(\tilde{y})) \quad (22)$$

The combinations of the latter equations finally give the desired results.

B.2 First order conditions of (13)

The first order conditions of (13) are:

$$-(1+\lambda^t)\mu\mathbb{E}(u'(\tilde{y})) + (1-p)q_n u'(w-\alpha-l+\tau+\beta) + pq_c u'(w-\alpha-l+\tau-\epsilon) = 0 \quad (23)$$

$$(1+\lambda^i)(1+\lambda^c)pq_c\mathbb{E}(u'(\tilde{y})) - pq_c u'(w-\alpha-l+\tau-\epsilon) = 0 \quad (24)$$

$$-(1+\lambda^p)(1-\frac{p}{1-p}\lambda^c)(1-p)\mathbb{E}(u'(\tilde{y}))+(1-p)(1-q_n)u'(w-\alpha+\beta)+(1-p)q_n u'(w-\alpha-l+\tau+\beta)=0 \quad (25)$$

With the definition of $\mathbb{E}(u'(\tilde{y}))$, the combinations of the first order conditions of (13) give:

$$u'(w-\alpha-l+\tau-\epsilon)=(1+\lambda^i)(1+\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (26)$$

$$u'(w-\alpha-l+\tau+\beta)=(1+\lambda^i)(1-\frac{p}{1-p}\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (27)$$

$$u'(w-\alpha+\beta)=(1-\frac{q_n}{1-q_n}\lambda^i+\frac{1}{1-q_n}\lambda^p)(1-\frac{p}{1-p}\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (28)$$

$$u'(w-\alpha)=(1-\frac{q_c}{1-q_c}\lambda^i-\frac{1}{1-q_c}(\frac{1}{p(1+\lambda^c)}-1)\lambda^p)(1+\lambda^c)\mathbb{E}(u'(\tilde{y})) \quad (29)$$

The combinations of the latter equations finally give the desired results.