Extending the Ramsey Equation further: Discounting under Mutually Utility Independent and Recursive Preferences*

Svenja Hector

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Abstract

I revisit the consumption discount rate for a novel combination of standard assumptions. To disentangle risk and time preferences, I consider a decision maker with recursive preferences à la Kreps and Porteus (1978). Moreover I assume that preferences are mutually utility independent in the sense of Koopmans (1960). In a multiperiod setting with independent growth risk and constant elasticity of substitution, the instantaneous consumption discount rate is diminished by a previously unrecognized horizon effect. This horizon effect may reduce the discount rate to a significant extend, which, in the context of social cost-benefit analysis, may provide a rationale for discounting at a rate that lies substantially below observable market rates of return.

Keywords: discounting, intertemporal decision making, uncertain growth, risk aversion, recursive utility, Kreps-Porteus preferences, Risk-Sensitive preferences, utility independence, horizon, cost-benefit analysis, climate change

JEL codes: H43, D81, D90, Q54

Correspondence:
ETH-Zurich
Department of Management, Technology and Economics
Chair of Integrative Risk Management and Economics
Zurichbergstrasse 18
8032 Zurich, Switzerland
Email: shector@ethz.ch

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1 Introduction

Standard approaches to consumption discounting under certainty originate from the most popular model of intertemporal choice, namely the discounted utility model as introduced by Samuelson (1937) and axiomatized by Koopmans (1960).[1] This model yields the well known Ramsey Equation which aggregates the determinants of the consumption discount rate, impatience and a wealth effect, in an intuitive manner. The predominance of the Ramsey Equation as an organizing principle for discounting sure monetary values in social cost-benefit analysis, most prominently in the assessment of optimal climate policy, was recently confirmed by a panel of leading experts on intergenerational discounting (Arrow et al. 2012).

A crucial assumption which is built into the discounted utility model, and thus into the standard approach to consumption discounting under certainty, is preference independence. Preferences over the consumption of one generation are (mutually) preference independent if they are independent of the consumption levels of generations living in the past and in the future. This assumption largely simplifies the preference representation. In a deterministic setting, Koopmans (1960) showed that preference independence constitutes the key axiom for the existence of an additively separable intertemporal utility function.

Recent contributions in the discounting literature emphasize the role of risk and risk aversion. Gollier (2002a, 2002b) motivates an Extended Ramsey Equation which incorporates discounting for reasons of precaution in the presence of growth risk. The additional effect on a one period (instantaneous) discount rate is marginal, however. This insignificance of the growth risk is partly due to an immanent drawback of the additive expected utility framework from which the Extended Ramsey Equation originates. In this framework it is not possible to disentangle risk aversion from the intertemporal elasticity of substitution (IES). In the additive expected utility model, a meaningful degree of risk aversion goes along with an unrealistically small IES. Gollier (2002a), Hector (2013), Traeger (2011, 2014) and others approach this deficit by employing recursive preferences of the Kreps-Porteus type (Kreps and Porteus 1978) to identify the consumption discount rate and its determinants. Since the degree of risk aversion can be varied independently of the IES

[1] See Frederick et al. (2002) for a discussion of the discounted utility model's historical origins.

in the Kreps-Porteus framework, it is possible to account for higher degrees of risk aversion. Higher degrees of risk aversion then imply a pronounced effect of growth risk on the consumption discount rate.

Utility independence–the equivalent of preference independence in a risky world–is either implicitly assumed or dismissed without discussion in the cited literature on discounting under growth risk. In Gollier's (2002a, 2002b) Extended Ramsey Equation, utility independence goes along with the time additive structure of the expected utility function. Kreps-Porteus recursive preferences, in contrast, are not utility independent by default. In particular, the constant relative risk aversion or 'Epstein-Zin' (Epstein and Zin 1989) parameterization of Kreps-Porteus recursive preferences, which has recently been advocated in context of discounting in social cost-benefit analysis (e.g. Traeger 2011, 2014), does not represent utility independent preferences.

The focus of the present paper is on the instantaneous consumption discount rate of a decision maker whose preferences are Kreps-Porteus recursive and satisfy (mutual) utility independence. The Kreps-Porteus framework is chosen for its flexibility with respect to the disentanglement of risk aversion and the IES. Utility independence is postulated as it is a broadly accepted assumption for intertemporal social welfare considerations, such as those underlying the Ramsey Equation and the Extended Ramsey Equation. In a first instance, I show that utility independence restricts a Kreps-Porteus recursive decision maker's preferences to a very specific parametric form, namely to the constant absolute risk aversion form of Hansen and Sargent's (1995) Risk-Sensitive preferences. If the risk on consumption *levels* is independently distributed, then the utility function of a decision maker with Risk-Sensitive preferences can be written in a temporally additive manner, which is a direct consequence of mutual utility independence. The additive separability of the utility function then implies that the instantaneous consumption discount rate–the rate at which consumption in period 2 is discounted relative to consumption in period 1–is a function of circumstances in periods 1 and 2 only.

A more popular assumption on the form of risk in the context of intertemporal decision making is that of independently distributed risk on consumption *growth*. I impose this assumption in my examination of the instantaneous consumption discount rate of a Risk-

Sensitive decision maker, which is conducted in a multiperiod framework with constant elasticity of substitution. I show that this discount rate is subject to an effect that is not present in previous approaches to discounting under risk. This effect, which I denote as the *horizon effect*, is a function of the decision maker's temporal horizon *after* the period to which the discount rate applies. The horizon effect, which may diminish the discount rate to a significant extent, accounts for the fact that the Risk-Sensitive decision maker can be averse towards correlation in consumption levels, which is introduced through the risk on second period consumption growth. The longer is the temporal horizon of the decision maker, the longer is the time-frame over which the risk on second-period growth, and thus the correlation in consumption levels, persists. In essence, a Risk-Sensitive decision maker who assesses the value of second period consumption relative to first period consumption, discriminates between second period risk that persists over the entire horizon, and second period risk that is independent of risk after period 2. The presence of persistent risk (risk on second period growth) amplifies the Risk-Sensitive agents precautionary savings motive, and thus decreases his instantaneous consumption discount rate.

In order to clarify the mechanism through which the horizon effect acts on a Risk-Sensitive agent's valuation of second period consumption, I derive an analytical solution for the instantaneous consumption discount rate. This analytical solution–the *Extended Ramsey Equation for Risk-Sensitive preferences*–allows for a direct comparison to the original Ramsey Equation and its previous extensions, and it discloses the double role which the rate of pure time preference (the utility discount rate) takes on in presence of a horizon effect. Furthermore, the analytical solution for a Risk-Sensitive agent's discounting function facilitates a simple numerical example, which sheds light on the horizon effect's magnitude and thus on its significance. Moreover, the numerical example illustrates that the horizon effect may serve as a rational for discounting below observable market rates of return in social cost-benefit analysis.

The remainder of this paper is structured as follows. In section 2 I describe the notion of preference and utility independence. I present the terminology and formal definitions in the static context of multiattribute utility theory to familiarize the reader with these concepts. In section 3, I introduce the preferences of the decision maker under consideration. I develop a definition of (mutual) utility independence for preferences over temporal lotteries which is then imposed on a Kreps-Porteus recursive decision maker. I show that the specified preferences are of the Risk-Sensitive type. In section 4, I examine the consumption discount rate of a decision maker with Risk-Sensitive preferences and prove the existence and the direction of the horizon effect. In section 5, I take a closer look at the horizon effect. I derive the Extended Ramsey Equation for Risk-Sensitive preferences, discuss its relation to the Ramsey equation and its previous extensions, emphasize the special role of the rate of pure time preference, and illustrate the horizon effect numerically. Section 6 concludes.

2 Background: Utility independence

Assumptions of preference or utility independence are standard in the context of utility functions $U(x_1, x_2, ...x_n)$ that aggregate the felicity from different attributes. The representation of preferences over multiple attributes is largely simplified if preferences over a specific attribute (or over lotteries on an attribute) are independent of common levels of other attributes. If mutual preference or utility independence holds, preferences over deterministic attributes and preferences that satisfy the axioms of expected utility theory can be represented through a utility function that is decomposable into smaller units: $U(x_1, x_2, ...x_n) = f(u_1(x_1), u_2(x_2), ...u_n(x_n))$. In particular, an (expected) utility function over n attributes can be decomposed into an additive or multiplicative form if preferences satisfy mutual preference or utility independence.[2]

In this section, I describe the notion of mutual preference and utility independence in the (mostly) static context of multiattribute utility theory (MAUT). The purpose of this description is to familiarize the reader with the basic idea behind these independence concepts. This familiarity will help the understanding of the next section, in which I adjust the definition of mutual utility independence to the temporal and recursive setting of my analysis.

The main reference for independence concepts in the deterministic or expected utility context of MAUT is Keeney and Raiffa's (1976) volume on Decisions with Multiple Objec-

[2] Keeney and Raiffa (1976).

tives. For comprehensive surveys on various independence assumptions, their implications in MAUT and the relevant literature see Farquhar (1977) and Yilmaz (1978). The definitions of conditional preferences, preference independence and utility independence below are as in Farquhar.

2.1 Terminology and conditional preferences

Consider a decision maker with preferences $\succeq$ on a set of possible outcomes X, which contains n different attribute sets X_i with $i = 1, 2, ...n$. The set of possible outcomes is the Cartesian product of the attribute sets: $X = X_1 \times X_2 \times ... \times X_n$. An element $x_i \in X_i$ is a specific level of attribute i. A specific outcome $x \in X$ is written as the n-tuple $x = (x_1, x_2, ...x_n)$. In risky situations, the decision maker's preferences $\succeq$ are defined over the set P of lotteries on X. An element $p \in P$ is a lottery that assigns probabilities l^ω, with $\omega = 1, ...N$ and $\sum_{\omega=1}^{N} l^\omega = 1$, $l^\omega > 0 \; \forall \; \omega$, to specific outcomes $x^\omega \in X$, such that $p = \sum_{\omega=1}^{N} l^\omega x^\omega$.

For the definitions of preference and utility independence below it will be useful to partition the attribute space and introduce conditional preference relations. The attribute space $i = 1, 2, ...n$ can be partitioned into the nonempty sets I and $\bar{I}$ such that $X = X_1 \times X_2 \times ... \times X_n$ can be expressed as $X = X_I \times X_{\bar{I}}$. The set of lotteries on X_I is then denoted as P_I , and $p_I \in P_I$ denotes a specific marginal distribution of p on X_I. A conditional preference relation is a preference relation that is defined over lotteries in one set, while holding the outcome in a different set fixed. Given a fixed outcome in $X_{\bar{I}}$, an unconditional preference relation $\succeq$ on P can be expressed as a conditional preference relation $\succeq_{x_{\bar{I}}}$ on P_I. That is, rather than defining $\succeq$ over lotteries $(p_I, x_{\bar{I}}), (p'_I, x_{\bar{I}}) \in P$ with marginal probabilities $p_I, p'_I \in P_I$ on X_I and probability 1 for the outcome $x_{\bar{I}} \in X_{\bar{I}}$, we can define $\succeq_{x_{\bar{I}}}$ over the marginals $p_I, p'_I \in P_I$. The conditional preference relation $\succeq_{x_{\bar{I}}}$ thus restricts the unconditional preference relation $\succeq$ to those $p \in P$ that assign probability 1 to $x_{\bar{I}}$. Formally

$$p_I \succeq_{x_{\bar{I}}} p'_I \text{ if and only if } (p_I, x_{\bar{I}}) \succeq \left(p'_I, x_{\bar{I}}\right) \; \forall \; p_I, p'_I \text{ in } P_I.$$

2.2 Mutual preference and utility independence

Preferences over outcomes in one attribute set may or may not depend on the specific levels of the remaining attributes. If the preference order over levels in the attribute set X_I is independent of the outcome in a different attribute set $X_{\bar{I}}$, we say that X_I is preference independent of $X_{\bar{I}}$. Formally, preference independence (PI) can be defined as follows:

Definition 1 *(Preference independence)*
X_I is preference independent of $X_{\bar{I}}$ if and only if $\succeq_{x_{\bar{I}}} = \succeq_{x'_{\bar{I}}}$ *on* $X_I \; \forall \; x_{\bar{I}}, \, x'_{\bar{I}} \in X_{\bar{I}}$.

Note that preference independence is not a symmetric condition: Given that X_I is preference independent of $X_{\bar{I}}$ we cannot infer that $X_{\bar{I}}$ is preference independent of X_I and vice versa. A symmetric condition, namely mutual preference independence (MPI), is, however, easily constructed:

Definition 2 *(Mutual preference independence)*
X_I and $X_{\bar{I}}$ are mutually preference independent if and only if X_I is preference independent of $X_{\bar{I}}$ and $X_{\bar{I}}$ is preference independent of X_I.

Preferences $\succeq$ over X which satisfy MPI on the whole domain (i.e. each subset $X_I \in X$ is PI of its complement $X_{\bar{I}} \in X$) are representable by an additive utility function (Keeney and Raiffa 1976, theorem 3.6).

Preference independence restricts preferences that are defined over a set of deterministic attributes. The analogue for preferences defined over lotteries is utility independence. If the preference order over lotteries in P_I on X_I is independent of outcomes in $X_{\bar{I}}$, we say that X_I is utility independent of $X_{\bar{I}}$. The definition of utility independence (UI) mirrors that of preference independence. The difference is only in the set over which preferences are defined:

Definition 3 *(Utility independence)*
X_I is utility independent of $X_{\bar{I}}$ if and only if $\succeq_{x_{\bar{I}}} = \succeq_{x'_{\bar{I}}}$ *on* $P_I \; \forall \; x_{\bar{I}}, \, x'_{\bar{I}} \in X_{\bar{I}}$.

If utility independence holds, then all conditional preference relations $\succeq_{x_{\bar{I}}}$ on P_I preserve the same order among all $p_I \in P_I$. This includes degenerate lotteries that assign probability

1 to specific levels in the attribute set X_I. Thus, whenever X_I is utility independent of $X_{\bar{I}}$, it must also be true that X_I is preference independent of $X_{\bar{I}}$. The converse is not generally true.

Just like preference independence, utility independence is not a symmetric condition: Given that X_I is utility independent of $X_{\bar{I}}$ we cannot infer that $X_{\bar{I}}$ is utility independent of X_I and vice versa. The symmetric condition is called mutual utility independence (MUI):

Definition 4 *(Mutual utility independence)*
X_I and $X_{\bar{I}}$ are mutually utility independent if and only if X_I is utility independent of $X_{\bar{I}}$ and $X_{\bar{I}}$ is utility independent of X_I.

Preferences $\succeq$ over P which satisfy MUI on the whole domain (i.e. each subset $X_I \in X$ is UI of its complement $X_{\bar{I}} \in X$) and which comply with von Neumann and Morgenstern's expected utility axioms are representable by an additive or multiplicative utility function (Keeney and Raiffa 1976, theorem 6.1). A standard additive expected utility function is mutually utility independent on the whole domain.

2.3 Temporal context

Consider the aggregation of an infinite number of attributes x_t with $t = 1, 2, 3...$ which differ with respect to the period at which they occur. An intertemporal social welfare function constitutes such an aggregation. A distinctive feature of this aggregation is the temporal order of the attributes. Due to this order, assumptions of utility independence can be given a temporal interpretation. If utility independence is geared towards the past, we speak of history independence, if it is geared towards the future, we call it future independence.

To be more specific, define by $X = X_1 \times X_2 \times X_3 ...$ the space of possible consumption paths over an infinite horizon. History independence of preferences over lotteries P_t on an attribute set $X_t \in X$ requires that X_t is utility independent of each $X_\tau \in X$ with $\tau < t$. Likewise, future independence of preferences over lotteries P_t on X_t requires that X_t is utility independent of each X_τ with $\tau > t$. If for some t, τ with $t < \tau$, it holds that preferences over P_t on X_t are future independent and those over P_τ on X_τ are history independent, then it must also be true that X_t and X_τ are mutually utility independent.

If preferences over each P_t on $X_t \in X$ are both future and history independent, then each pair $X_t, X_\tau \in X$ with $t, \tau = 1, 2, ...\infty$ and $t \neq \tau$ is mutually utility independent. We then simply say that preferences over P on X are mutually utility independent on the whole domain.

Preferences which are history and future independent, which satisfy von Neumann and Morgenstern's expected utility axioms, and which are defined over lotteries on intertemporal consumption paths, are representable by an additive or multiplicative intertemporal utility function (Meyer 1976, theorem 9.2). Correspondingly, preferences represented by the standard additive (intertemporal) expected utility function are mutually utility independent on the entire domain.

In a temporal but deterministic context, Koopmans (1960) proved that several axioms, among them a crucial assumption on period independence, warrants the existence of the additive discounted utility model.[3] In the following I extend Koopmans' requirement for independence of preferences defined over deterministic consumption paths to a larger domain, namely to the domain of temporal lotteries.

3 Preferences: MUI and KP recursive

Intergenerational decision making involves allocating resources across many different generations. These generations differ with respect to their consumption level as well as with respect to the degree of consumption risk to which they are exposed. A decision maker who optimizes intertemporal welfare evaluates the consumption and risk levels according to his preferences, in particular according to his intertemporal elasticity of substitution (IES) and his degree of risk aversion. These two preference characteristics are entangled in the standard model of intertemporal choice under risk, namely in the additive expected utility model. To model the preferences of an intertemporal decision maker in a more flexible manner, I resort to the recursive utility representation of Kreps and Porteus (1978). A Kreps-Porteus (KP) recursive preference representation enables the disentanglement of a

[3] More specifically, Koopmans (1960) showed that stationary, time-consistent, period independent preferences over infinite deterministic consumption paths are represented by $U(x) = \sum_{t=1}^{\infty} \beta^{t-1} u(x_t)$. Under addition of a continuity axiom, he showed that the utility discount factor (rate of pure time preference) must be such that $0 < \beta < 1$.

decision makers' degree of risk aversion from the IES.

KP recursive preferences are defined over objects called temporal lotteries. The definitions of mutual preference and utility independence above, however, concern preferences that are defined over deterministic attributes or over lotteries on attribute sets. The results of Keeney and Raiffa (1976), Meyer (1976) and Koopmans (1960) on the decomposition of a utility function when preferences satisfy definitions 1, 2, 3, or 4, are therefore not directly applicable in the context considered here.

To study how mutual utility independence restricts a Kreps-Porteus recursive preference representation, mutual utility independence for preferences over temporal lotteries must be defined. To this end, denote the set of temporal lotteries by D and write a specific temporal lottery as $(x_1, \tilde{x}_2, \tilde{x}_3, ...) \in D$. A temporal lottery consists of a certain attribute x_1 for the initial period and (potentially) uncertain attributes $\tilde{x}_t$ for $t > 1$. Note that the set of degenerate temporal lotteries (deterministic consumption paths) X^∞ is a subset of D.[4] Mutual utility independence for preferences over temporal lotteries can be defined in the following way:

Definition 5 *(MUI for preferences over temporal lotteries)*
Preferences $\succeq$ over the set of temporal lotteries D are mutually utility independent if

$$\begin{array}{ccc} (x_1, \tilde{x}_2, ...\tilde{x}_{t-1}, x_t, \tilde{x}_{t+1}...) & \succeq & (x'_1, \tilde{x}'_2, ...\tilde{x}'_{t-1}, x_t, \tilde{x}'_{t+1}...) \\ & \Downarrow & \\ (x_1, \tilde{x}_2, ...\tilde{x}_{t-1}, x'_t, \tilde{x}_{t+1}...) & \succeq & (x'_1, \tilde{x}'_2, ...\tilde{x}'_{t-1}, x'_t, \tilde{x}'_{t+1}...) \end{array}$$

$\forall\ x_t, x'_t \in X_t$.

Definition 5 is now imposed on Kreps-Porteus recursive preferences. Denote by $\succeq^D$ a preference relation over the set of temporal lotteries D. Suppose $\succeq^D$ is KP recursive and let $U^D : D \to \mathbb{R}$ represent such preferences. Since U^D represents KP recursive preferences,

[4] For a more comprehensive discussion of temporal lotteries see Kreps and Porteus (1978), Epstein and Zin (1989), or Bommier and Le Grand (2014).

it must satisfy the recursion

$$U^D(x_1, m) = W\left(x_1, E_m\left[U^D\right]\right), \tag{1}$$

where $E_m[\cdot]$ is the expectation with respect to the probability measure m on D.[5]

Suppose in addition that the considered preference relation $\succeq^D$ satisfies mutual utility independence according to definition 5. Note that the assumption of MUI on D implies MUI on the subdomain $X^\infty \subseteq D$ as well. Given MUI of $\succeq^D$ on D, the form of U^D can be narrowed down in two steps.

First, I restrict the form of U^D such that it represents only preferences that are MUI on the subdomain $X^\infty \subseteq D$. To this end, I use Koopmans' (1960) representation result for period independent preferences. His definition of period independence accords to my definition of mutual utility independence.[6] Koopmans shows that a preference relation $\succeq^X$ over X^∞ which satisfies continuity, sensitivity, stationarity and mutual utility independence can be represented by an additive discounted utility function $U^X : X^\infty \to \mathbb{R}$:

$$U^X(x_1, x_2 \ldots) = u(x_1) + \beta U^X(x_2, x_3 \ldots). \tag{2}$$

Note that U^D and U^X represent the same (mutually utility independent) preferences on X^∞. Since U^D and U^X represent the same ordinal preferences, there exists some increasing ϕ such that $U^D = \phi\left(U^X\right)$ (Kihlstrom and Mirman 1974). Denoting by $W^D(x, y)$ and $W^X(x, y)$ the aggregators of U^D and U^X and using $U^D = \phi\left(U^X\right)$, we can write $W^D(x, y) = \phi\left(W^X\left(x, \phi^{-1}(y)\right)\right)$ and thus[7]

$$U^D(x_1, m) = \phi\left(u(x_1) + \beta\phi^{-1}\left(E_m U^D(x_2, m)\right)\right). \tag{3}$$

Equation (3) restricts the form of U^D such that it represents only preferences which satisfy

[5] See e.g. Bommier and Le Grand (2014).

[6] Meyer (1976) shows that the combination of Koopmans' postulates on stationarity and period independence (Koopmans' postulate 3') induce complete pairwise preferential independence. Complete pairwise preferential independence corresponds to the definition of mutual utility independence on the domain X^∞ (see section 3.6.3 in Keeney and Raiffa 1976).

[7] Let $W^X(x, y) = u(x) + \beta y$ such that $W^D(x, y) = \phi\left(u(x) + \beta\phi^{-1}(y)\right)$. For $U^D(x_1, m) = W^D\left(x_1, E_m[U^D]\right)$ this yields equation (3).

MUI on X^{∞}.

Second, I restrict the form of U^D further such that it represents only preferences that are MUI on the entire domain D. To this end, one needs to restrict ϕ in such a way that U^D represents preferences with constant absolute risk aversion. I show this in appendix A.1. The implications for a renormalized ordinal KP recursive utility function $U = \phi^{-1}\left(U^D\right)$ are summarized in theorem 1.

Theorem 1 *(Representation of KP recursive preferences that satisfy MUI)*
Consider a decision maker with preferences that satisfy the Kreps-Porteus recursion (equation 1). Suppose that these preferences are mutually utility independent over the set of temporal lotteries D (definition 5). Such preferences can be represented by a utility function $U : D \to \mathbb{R}$ of the following form:

$$U\left(x_1, m\right) = u\left(x_1\right) - \frac{\beta}{k} \ln\left(E_m \exp\left(-kU\left(x_2, m\right)\right)\right). \tag{4}$$

Equation (4) is the constant absolute risk aversion form of Hansen and Sargent's (1995) Risk-Sensitive (RS) preferences. The parameter k measures the decision maker's degree of temporal risk aversion. We say that the decision maker is temporally risk averse if $k > 0$ and temporally risk loving if $k < 0$. Temporal risk aversion can be understood as aversion towards risk on continuation utility, which is given by $U\left(x_2, m\right)$ in equation (4).

For $k = 0$, equation (4) nests the additive (intertemporal) expected utility function

$$U\left(x_1, m\right) = u\left(x_1\right) + \beta E_m U\left(x_2, m\right). \tag{5}$$

A decision maker with $k = 0$, i.e. an additive expected utility decision maker, is called temporally risk neutral. Such a decision maker is neutral towards risk on continuation utility. Aversion towards risk on consumption x_t is solely governed by the curvature of the felicity function $u\left(x_t\right)$, which simultaneously defines the intertemporal elasticity of substitution.

The implications of the mutual utility independence assumption become obvious if one considers the case of independently distributed risk on the attributes $\tilde{x}_t$. If the attributes are statistically independent, the assumption of mutual utility independence on KP recursive preferences implies the additive separability of the respective utility function. In particular, if preferences are represented by equation (4) and risk on consumption $\tilde{x}_t$ is independently distributed, the utility function can be written as[8]

$$U(x_1, \hat{x}_2, ...) = u(x_1) + \beta \sum_{t=2}^{\infty} \beta^{t-2} u(\hat{x}_t), \tag{6}$$

where $\hat{x}_t$ is certainty equivalent consumption in t. For the RS decision maker under consideration, $\hat{x}_t$ is derived from $u(\hat{x}_t) = -\frac{1}{k} \ln (E_{t-1} \exp(-ku(\tilde{x}_t)))$. If lotteries on $\tilde{x}_t$ are degenerate (i.e. if consumption is deterministic), then (6) is equivalent to Koopmans' (1960) additive discounted utility function, i.e. equation (2).

I am not the first to connect assumptions of mutual utility independence and Kreps-Porteus recursive preferences. Bommier and Le Grand (2014) remark that their Kreps-Porteus recursive preference specification under scrutiny, namely Risk-Sensitive preferences, satisfies mutual utility independence. Above I approached the issue from a different angle, however. I showed more formally that Kreps-Porteus recursive preferences which satisfy mutual utility independence must be of the Risk-Sensitive type.[9]

4 Implications for discounting: The horizon effect

I showed above that Kreps-Porteus recursive preferences which satisfy mutual utility independence are restricted to a specific parametric form, namely to that of Risk-Sensitive preferences. In this section I analyze the instantaneous consumption discount rate of a decision maker with such preferences.

An instantaneous consumption discount rate $DR_{1,2}$ compares the effects on intertemporal utility $U(x)$ when consumption in the first and in the second period are marginally changed:

$$DR_{1,2} = -\ln \frac{\partial U(x) / \partial x_2}{\partial U(x) / \partial x_1}. \tag{7}$$

[8] See appendix B.1.

[9] Related to but quite different from my approach is Traeger (2012). In a finite (rolling) horizon framework with Kreps-Porteus recursive preferences, he derives a constant absolute risk aversion parameterisation of Kreps-Porteus recursive preferences (Risk-Sensitive preferences) from an assumption denoted as 'coinciding last outcome independence'.

In the following I show that the instantaneous consumption discount rate of a decision maker with RS preferences is subject to an effect which is not present in the well known Ramsey Equation and its extensions. This effect is denoted as the *horizon effect*.

To avoid confusion, note that Gollier (2002a, 2002b) also refers to an effect on the consumption discount rate that is connected to the time horizon. This effect is different from what I have in mind, however. In Gollier's contributions, 'horizon' refers to the time horizon *between* the present period and the period to which the discount rate applies. Hence, his understanding of the horizon is related to the term structure of the consumption discount rate. In the present paper, the object of interest is the instantaneous (or one-period ahead) consumption discount rate, in which's context 'horizon' refers to the time horizon *after* the period to which the discount rate applies.[10]

4.1 Defining the horizon effect

A horizon effect is present whenever the consumption discount rate is affected by circumstances that realize only after the period for which one discounts. For the instantaneous discount rate $DR_{1,2}$ (equation 7) this is the case if the value of period 2 consumption relative to that of consumption in period 1 is subject to circumstance in periods $t \geq 3$.

To formalize the horizon effect, I compare the instantaneous consumption discount rate in two situations, A and B. In both situations I consider a decision maker whose preferences are defined over a temporally infinite domain.

In situation A, the decision maker faces a world in which humanity exists in periods $t = 1, 2, ...\bar{T}$ (with $\bar{T} > 2$), and does not exist in periods $t = \bar{T}+1, \bar{T}+2, ...\infty$. Since period $\bar{T}$ is the last period in which humanity exists, we can think of $\bar{T}$ as the length of the decision maker's temporal horizon. While humanity exists, it consumes x_t in each period t and derives felicity $u(x_t)$ from this consumption. In $t > \bar{T}$, i.e. when humanity does not exist, we assign zero felicity: $u(-) = 0$.[11] The instantaneous consumption discount

[10] Closer to my understanding of the horizon effect is Traeger (2011). He also points to the fact that the 'planning horizon' after the period for which one discounts may affect the discount rate. While Traeger is aware of the existence of the (planning) horizon effect in a very general Kreps-Porteus recursive setting, he does not study it in detail. In the contrary, as I discuss below, he eliminates the effect by employing an Epstein-Zin parameterization of Kreps-Porteus preferences with homogeneous felicity.

[11] The '$-$' stands for the consumption level of non-existent people. Note that this is different from just assuming that existing people have zero consumption. For an enlarged discussion of this point see Bommier

rate that applies to this situation is denoted $DR_{1,2}^{\bar{T}>2}$. This is the rate at which the value of consumption in period $t = 2$ is compared to the value of consumption in period $t = 1$, given a decision maker with a horizon that extends the period to which the discount rate applies, i.e. a decision maker with $\bar{T} > 2$. Assuming that such a decision maker has KP recursive preferences yields the instantaneous consumption discount rate[12]

$$\begin{aligned} DR_{1,2}^{\bar{T}>2} &= -\ln\beta - \ln \frac{E_1\left[\frac{\phi'(U_2)}{\phi'\left(\phi^{-1}(E_1[\phi(U_2)])\right)} u'\left(\tilde{x}_2\right)\right]}{u'\left(x_1\right)} \\ & \text{with } U_t = u\left(\tilde{x}_t\right) + \beta\phi^{-1}\left(E_t\left[\phi\left(U_{t+1}\right)\right]\right)\ \forall\ t = 2, 3, ...\bar{T} - 1. \end{aligned} \tag{8}$$

Note that the discounting equation (8) may be subject to circumstances that apply to periods $t \geq 3$ since the continuation utility U_2 is a function of these values.

In situation B, humanity exists only in periods $t = 1, 2$ and does not exist in periods $t \geq 3$. Hence, period $t = 2$ is the last period in which humanity exists, and thus defines the decision maker's horizon as $\bar{T} = 2$. The respective instantaneous consumption discount rate is denoted $DR_{1,2}^{\bar{T}=2}$, which, given a KP recursive decision maker, is written as

$$DR_{1,2}^{\bar{T}=2} = -\ln\beta - \ln \frac{E_1\left[\frac{\phi'(u(\tilde{x}_2))}{\phi'\left(\phi^{-1}(E_1[\phi(u(\tilde{x}_2))])\right)} u'\left(\tilde{x}_2\right)\right]}{u'\left(x_1\right)}. \tag{9}$$

Note that equation (9) is independent of values in periods $t \geq 3$.[13]

In both equations, β is the utility discount factor. The term $(-\ln\beta)$ therefore constitutes the rate of pure time preference (the utility discount rate).

Given the description of situations A and B, we can define the horizon effect more formally:

Definition 6 *(Horizon effect)*

(2013). He considers preferences that are defined over a finite lifetime, but with an infinite number of possibilites for the length of this lifetime.

[12] Using $U = \phi^{-1}\left(U^D\right)$ on the Kreps-Porteus recursive preference representation (3) and deriving $DR_{1,2}$ as defined in (7) for the resulting utility function yields (8) if $\bar{T} > 2$ and (9) if $\bar{T} = 2$.

[13] In situations A and B, we could alternatively assume that humanity exists in $t > \bar{T}$ but has consumption $\tilde{x}_t$ which is not correlated with consumption in period 2. The implied discounting functions would be the same as in the case where we assume that generations in $t > \bar{T}$ do not exist.

The instantaneous consumption discount rate $DR_{1,2}^{\bar{T}>2}$ *is subject to a horizon effect whenever*

$$DR_{1,2}^{\bar{T}>2} \neq DR_{1,2}^{\bar{T}=2}.$$

The comparison of equations (8) and (9) in light of definition 6 reveals that the discount rate of a KP recursive decision maker is subject to a horizon effect whenever

$$E_1\left[\frac{\phi'(U_2)}{\phi'\left(\phi^{-1}\left(E_1\left[\phi\left(U_2\right)\right]\right)\right)}u'\left(\tilde{x}_2\right)\right] \neq E_1\left[\frac{\phi'\left(u\left(\tilde{x}_2\right)\right)}{\phi'\left(\phi^{-1}\left(E_1\left[\phi\left(u\left(\tilde{x}_2\right)\right)\right]\right)\right)}u'\left(\tilde{x}_2\right)\right]. \qquad (10)$$

The fractions on both sides of this inequality twist the statistical probability of a given state of the world to account for risk aversion with respect to risk on the continuation utility. The continuation utility is U_2 in situation A and $u(\tilde{x}_2)$ in situation B. I refer to these fractions as *risk aversion adjustment factors*.

Equation (10) clarifies that a horizon effect may exist whenever the risk aversion adjustment factors of a given state of the world are not equivalent for $DR_{1,2}^{\bar{T}>2}$ and $DR_{1,2}^{\bar{T}=2}$. Note that the adjustment factors of a decision maker with additive expected utility preferences are 1 in each state of the world since $\phi(\cdot)$ is linear in this case. This implies that equation (10) holds with equality, and it follows that the discount rate of an additive expected utility decision maker is never subject to a horizon effect.

4.2 The discount rate of a Risk-Sensitive decision maker

In theorem 1 I stated that the preferences of a KP recursive and mutually utility independent decision maker are representable by the Risk-Sensitive utility function, as specified in equation (4). The discount rate of a KP recursive decision maker is thus restricted to a parametric form with $\phi(z) = -\exp(-kz)$, which represents constant absolute risk aversion with respect to continuation utilities.

For situation A (equation 8), this implies that the discount rate of a Risk-Sensitive decision maker with $\bar{T} > 2$ is given by

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln\frac{E_1\left[\frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]}u'(\tilde{x}_2)\right]}{u'(x_1)} \tag{11}$$
$$\text{with } U_t = u(\tilde{x}_t) - \frac{\beta}{k}\ln\left(E_t\left[\exp\left(-kU_{t+1}\right)\right]\right)\ \forall\ t = 2, 3, ...\bar{T}-1.$$

The discount rate that corresponds to situation B (equation 9), and thus to a decision maker with $\bar{T} = 2$, is written as

$$DR_{1,2}^{\bar{T}=2} = -\ln\beta - \ln\frac{E_1\left[\frac{\exp(-ku(\tilde{x}_2))}{E_1[\exp(-ku(\tilde{x}_2))]}u'(\tilde{x}_2)\right]}{u'(x_1)}. \tag{12}$$

Equations (11) and (12) define instantaneous consumption discount rates for a decision maker with KP recursive mutually utility independent (equivalently: Risk-Sensitive) preferences. Under (11), the decision maker faces a world in which humanity exists in periods $t = 1, 2, ...\bar{T}$. Under (12), the decision maker faces a world in which humanity only exists in the first two periods. In each period $t \geq 2$ that is taken into account in (11) and (12), humanity has possibly uncertain consumption $\tilde{x}_t$. Equation (11) is the subject under scrutiny in the remaining analysis, equation (12) serves as a benchmark to determine the existence, the direction, and the size of the horizon effect.

Before I go to the main analysis, let me point to a number of conditions under which the existence of a horizon effect acting on (11) can be excluded. In the next section I will assume that these assumptions are not met, hence I will consider a setting in which a horizon effect may exist. $DR_{1,2}^{\bar{T}>2}$ is free from a horizon effect ($DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$) if $\beta = 0$, if $u(x_t)$ is linear, if there is no risk in period 2, if $k = 0$, or if the risk on $\tilde{x}_t$ is independently distributed. I discuss these conditions in more detail in appendix B.2. Note however, that the case with independently distributed risk on consumption levels $\tilde{x}_t$ has already been discussed earlier. With independent risk on consumption levels, the recursive utility function of a RS decision maker can be written in a temporally additive manner (see equation 6), which immediately implies that the instantaneous consumption discount rate is a function of circumstances in periods 1 and 2 only, i.e. it is not subject to a horizon effect.

4.3 Existence and direction of the horizon effect

I examine the instantaneous consumption discount rate $DR_{1,2}^{\bar{T}>2}$ of a temporally risk averse Risk-Sensitive decision maker under a set of assumptions that are standard in the discounting literature. In particular, I assume that the decision maker has at least some valuation for generations living in $t \geq 2$ ($\beta > 0$), that the felicity function is concave and characterized by constant elasticity of substitution (CES), and that consumption *growth* is risky and independently distributed. Note that the riskiness of consumption growth implies that risk on consumption itself cannot be independently distributed.

The discount rate of the decision maker under consideration is specified in (11) with $k > 0$ and $0 < \beta < 1$. The consumption growth rate $g_t = \frac{x_t}{x_{t-1}} - 1 > -1 \ \forall \ t$ is subject to independently distributed risk. Generation t obtains CES felicity from $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ with $\rho < 1$, where the intertemporal elasticity of substitution is defined by $IES = (1-\rho)^{-1}$. Given this setting, I examine how the discount rate $DR_{1,2}^{\bar{T}>2}$ depends on the length of the horizon $\bar{T} > 2$. In particular, I prove in appendix A.2 that the horizon effect reduces the discount rate $DR_{1,2}^{\bar{T}>2}$ relative to $DR_{1,2}^{\bar{T}=2}$. This finding is formalized in proposition 1.

Proposition 1 *(Existence and direction of the horizon effect)*
Consider the discount rate of a RS decision maker (equation 11). Assume $k > 0$, $0 < \beta < 1$ and $\tilde{g}_t > -1 \ \forall \, t \geq 2$. The horizon effect exists and reduces the discount rate, i.e. $DR_{1,2}^{\bar{T}>2} < DR_{1,2}^{\bar{T}=2}$, for either of the two following specifications:

1. $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ *($\rho < 1$, $IES > 0$)*
   where $\tilde{g}_2$ is risky and g_t is deterministic $\forall \ t \geq 3$

2. $u(x_t) = \ln x_t$ *($\rho = 0$, $IES = 1$)*[14]
   where $\tilde{g}_t$ is risky and independently distributed $\forall \ t \geq 2$

Note that statement 1 in proposition 1 still holds if one employs the more common CES felicity function $u(x_t) = \frac{x_t^\rho}{\rho}$. This is because the discount rate of a RS decision maker is invariant towards the addition of the constant $-\frac{1}{\rho}$ to felicity.

Proposition 1 states that the instantaneous consumption discount rate of a RS decision

[14] If $IES = 1$ ($\rho = 0$) we get $\lim_{\rho \to 0} \frac{x_t^\rho - 1}{\rho} = \ln x_t$, and thus $u(x_t) = \ln x_t$.

maker in a standard discounting setting depends on the horizon after the period for which one discounts, i.e. on the horizon after period 2. The standard practise of cutting off the horizon after the period of discount, i.e. looking at $DR_{1,2}^{\bar{T}=2}$ rather than $DR_{1,2}^{\bar{T}>2}$ as in Gollier (2002a) and Hector (2013), is therefore problematic in the context considered here. How problematic it is depends on the size of the horizon effect, which I elaborate on in the next section.

On first sight, the existence of the horizon effect may seem to be at odds with the assumption of mutual utility independence. Imposing mutual utility independence on preferences, and hence imposing history and future independence, leads to a discount rate that explicitly depends on the future through the horizon effect. On closer inspection, this result is not surprising. To see this, recall that the combination of MUI and KP recursivity, i.e. assuming RS preferences, implies the additive separability of the decision maker's utility function if risk on consumption levels $\tilde{x}_t$ is independently distributed. A discount rate that is derived from such an additively separable utility function is not subject to a horizon effect. The independence of preferences over risk in period 2 from the consumption levels in $t \neq 2$, *together* with the statistical independence of risk on consumption levels, imply the absence of a horizon effect. The horizon effect enters the stage only as we give up the statistical independence of risk on consumption levels and instead assume independently distributed risk on growth. Risk on consumption growth in period 2 (whether independently distributed or not) goes along with correlated risk on consumption levels in each period $t \geq 2$. Hence, risk in period 2 does not only affect the riskiness of period 2 consumption, but also affects the riskiness of consumption in $t = 3, 4, ...\bar{T}$ and thereby increases the risk on (continuation) utility U_2. The more periods are aggregated in U_2, i.e. the longer the horizon is, the bigger is this risk on continuation utility in absolute terms. A Risk-Sensitive decision maker is averse towards risk on continuation utility and thus adjusts the discount rate in accordance with the size of this risk. This is eventually reflected in the horizon effect.

The technicalities behind the horizon effect can be sketched by a simple example. Suppose for simplicity that $\bar{T} = 3$ is the last period in which a generation exists. Suppose furthermore that only second period consumption growth $\tilde{g}_2$ is risky, whereas consumption growth in the third period, g_3, is deterministic and thus independent of the risk in period 2. Risk on period 3 consumption levels is not independent of the risk on period 2 consumption levels, however, both are functions of $\tilde{g}_2$: $\tilde{x}_2 = (1 + \tilde{g}_2) x_1$, $\tilde{x}_3 = (1 + \tilde{g}_2)(1 + g_3) x_1$. In this setting, all uncertainty resolves in period 2, and second period continuation utility can thus be written as $U_2 = u(\tilde{x}_2) + \beta u(\tilde{x}_3)$. Plugging U_2 into equation (11) then yields the instantaneous consumption discount rate of the Risk-Sensitive decision maker with $\bar{T} = 3$:

$$DR_{1,2}^{\bar{T}=3} = -\ln\beta - \ln\frac{E_1\left[\frac{\exp(-ku(\tilde{x}_2)-k\beta u(\tilde{x}_3))}{E_1[\exp(-ku(\tilde{x}_2)-k\beta u(\tilde{x}_3))]}u'(\tilde{x}_2)\right]}{u'(x_1)}.$$

Since $\tilde{x}_3$ is correlated with $\tilde{x}_2$, it cannot be taken out of the expectation operator, which implies that circumstances in period 3 affect the risk aversion adjustment factor (the fraction in the numerator). This eventually implies the presence of the horizon effect. In contrast, if $\tilde{x}_2$ and $\tilde{x}_3$ were uncorrelated, then the risk aversion adjustment factor would be independent of circumstances in period 3, and there would thus be no horizon effect.[15]

5 Analytical solution: A closer look at the horizon effect

In order to improve the understanding of the horizon effect and to illustrate its significance in a numerical example, I derive an analytical solution for the instantaneous consumption discounting equation of a Risk-Sensitive decision maker with $\bar{T} > 2$, i.e. for equation (11). This analytical solution, which allows for an immediate comparison with the Ramsey Equation and its previous extensions, will be denoted as the Extended Ramsey Equation for Risk-Sensitive preferences.

Throughout this section I assume that uncertain consumption growth rates $\tilde{g}_t$ are not only independently but also normally distributed at each point in time, i.e. $\tilde{g}_t \sim N\left(\mu_t, \sigma_t^2\right)$. Furthermore, I assume that the intertemporal elasticity of substitution equals one ($IES = 1$, $\rho = 0$), in which case the CES felicity function $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ takes the logarithmic form $u(x_t) = \ln x_t$.[16] The utility discount factor β will be substituted by the utility discount rate (the rate of pure time preference) $\delta = -\ln\beta$, since this notation is more common in

[15] In this case, the continuation utility would be given by $U_2 = u(\tilde{x}_2) - \frac{\beta}{k}\ln\left(E_2\left[\exp\left(-ku(\tilde{x}_3)\right)\right]\right)$, where the last term is independent of period 2 information, and would thus cancel out in the risk aversion adjustment factor.

[16] See footnote 14.

the discounting literature.

5.1 The Extended Ramsey Equation for Risk-Sensitive preferences

The instantaneous consumption discount rate of a Risk-Sensitive decision maker with horizon $\bar{T} > 2$ (situation A) is defined by equation (11). Imposing the assumptions listed in the last paragraph on this equation permits an analytical solution as derived in appendix B.3. This analytical solution is denoted as the Extended Ramsey Equation for Risk-Sensitive preferences:

Definition 7 *(Extended Ramsey Equation for Risk-Sensitive preferences)*
Define $\delta = -\ln \beta$ and assume $\tilde{g}_t \sim N\left(\mu_t, \sigma_t^2\right)$ and $IES = 1$ $(\rho = 0)$. Then the instantaneous consumption discount rate of a Risk-Sensitive decision maker with $\bar{T} > 2$ (equation 11) can be written as

$$DR_{1,2}^{\bar{T}>2} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k \sum_{\tau=0}^{\bar{T}-3} \exp\left(-\delta\right)^{\tau+1}, \tag{13}$$

which is denoted as the'Extended Ramsey Equation for Risk-Sensitive preferences'.

In comparison, the analytical solution for the consumption discount rate of a Risk-Sensitive decision maker with horizon $\bar{T} = 2$ (situation B) is derived from equation (12) as[17]

$$DR_{1,2}^{\bar{T}=2} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k. \tag{14}$$

The difference between equations (13) and (14) constitutes the horizon effect that acts on $DR_{1,2}^{\bar{T}>2}$. The horizon effect drives a wedge between the discount rate in a setting with horizon $\bar{T} > 2$, and that in a setting with $\bar{T} = 2$:

$$DR_{1,2}^{\bar{T}>2} - DR_{1,2}^{\bar{T}=2} = -\frac{\sigma_2^2}{2} 2k \sum_{\tau=0}^{\bar{T}-3} \exp\left(-\delta\right)^{\tau+1}. \tag{15}$$

Equation (15) confirms statement 2 of proposition 1 for a setting with normally distributed risk on consumption growth. Given a temporally risk averse $(k > 0)$ Risk-Sensitive agent

[17] Defining $\delta = -\ln \beta$ and assuming $\tilde{g}_t \sim N\left(\mu_t, \sigma_t^2\right)$, $IES = 1$ as before.

with a horizon length that extends the period to which the discount rate applies ($\bar{T} > 2$), we find that the instantaneous consumption discount rate is decreased by the horizon effect ($DR_{1,2}^{\bar{T}>2} < DR_{1,2}^{\bar{T}=2}$) whenever second period growth is risky ($\sigma_2^2 \neq 0$). The longer is the horizon $\bar{T}$, the bigger is the absolute size of the horizon effect, and the smaller is therefore the instantaneous consumption discount rate of a strictly temporally risk averse Risk-Sensitive decision maker.

5.2 Comparison to the literature

The following list of equations collects the Ramsey Equation and its previous extensions and contrasts them to the Extended Ramsey Equation for Risk-Sensitive preferences. All equations are presented for the case where $IES = 1$. RE refers to *Ramsey Equation*, ERE to *Extended Ramsey Equation*, ERE-EZ to *Extended Ramsey Equation for Epstein-Zin preferences*, and ERE-RS to *Extended Ramsey Equation for Risk-Sensitive preferences.*

$$\text{RE:} \qquad DR_{1,2}^{\bar{T}>2} = \delta + \mu_2 \tag{16}$$

$$\text{ERE:} \qquad DR_{1,2}^{\bar{T}>2} = \delta + \mu_2 - \frac{\sigma_2^2}{2} \tag{17}$$

$$\text{ERE-EZ:} \qquad DR_{1,2}^{\bar{T}>2} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}(-2\alpha) \tag{18}$$

$$\text{ERE-RS:} \qquad DR_{1,2}^{\bar{T}>2} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}2k \quad - \quad \frac{\sigma_2^2}{2}2k \sum_{\tau=0}^{\bar{T}-3} \exp(-\delta)^{\tau+1} \tag{19}$$

The Ramsey Equation (equation 16; Ramsey 1928) constitutes the most widely accepted organizing principle for deterministic consumption discounting in an intergenerational context (Arrow et al. 2012). It is derived from the discounted utility model (with CES felicity), i.e. from the prevalent framework for the representation of preferences over deterministic consumption paths. Since consumption growth is deterministic in the setting of the discounted utility model, we have $\mu_2 = g_2$ and $\sigma_2^2 = 0$. The first term in (16) represents discounting for pure time preference (utility discounting at rate δ).[18] The second term is called the wealth effect. The wealth effect accounts for consumption discounting due to differences in the consumption levels in period 1 and 2. An increase in period 2 consumption is less

[18] The utility discount rate is sometimes also refered to as the rate of impatience or the rate of empathic distance.

valuable than an increase in present consumption if people alive in period 2 are richer than people alive in period 1, i.e. if $\mu_2 > 0$, and if the decision maker is averse towards such consumption inequalities.[19]

The Extended Ramsey Equation (equation 17; see e.g. Gollier 2002a, 2002b, Gollier 2011) extends the Ramsey Equation to a world with risk on the consumption growth rate $\tilde{g}_2$. It is derived from the additive expected utility model (with CES felicity and normally distributed risk on consumption growth), i.e. from the prevalent framework for the representation of preferences over lotteries on consumption paths. The third term in (17) reduces the consumption discount rate according to the decision maker's aversion towards second period risk, and thus accounts for an increased valuation of future consumption out of a precautionary savings motive. Note that risk aversion is measured by the inverse of the IES in this setting, i.e. risk aversion and the intertemporal elasticity of substitution are entangled. Since risk aversion cannot be too high in a setting in which risk aversion and the IES are entangled, the last term is small for moderate sizes of σ_2^2.[20] Due to the entanglement of risk aversion and the IES, risk and risk aversion typically play a negligible role in context of the Extended Ramsey Equation.

The Extended Ramsey Equation for Epstein-Zin preferences (equation 18) yields a consumption discount rate for the Epstein-Zin specification of Kreps-Porteus recursive preferences, which postulates constant relative risk aversion with respect to continuation utilities. Equation 18 can be found in Traeger (2011, 2014), who derives this equation in an Epstein-Zin preference setting for the $IES = 1$ ($\rho = 0$) subcase of CES felicity $u(x_t) = \frac{x_t^\rho}{\rho}$, given

[19] The decision maker is averse towards such consumption inequalities whenever $IES > 0$, which is given here since we have $IES = 1$.

[20] Note that this inflexibility of the additive expected utility model is unconnected to our assumption $IES = 1$, which was merely imposed for the comparison to the Extended Ramsey Equation for RS preferences. This is may be clearer if we write the Extended Ramsey Equation without assuming $IES = 1$. Then, given an additive expected utility function with CES felicity and normally distributed growth risk, the Extended Ramsey Equation is written as

$$DR_{1,2}^{\bar{T}\geq 2} = \delta + IES^{-1}\mu_2 - \frac{\sigma_2^2}{2}RRA^2$$

where the degree of relative risk aversion (RRA) is linked to the intertemporal elasticity of substitution through the relationship $RRA = IES^{-1}$.

normally distributed growth risk.[21,22] Similar to the Risk-Sensitive specification of Kreps-Porteus recursive preferences, the Epstein-Zin decision maker can be averse towards risk on continuation utility. Traeger denotes this aversion towards risk on continuation utility in the Epstein-Zin specification of KP recursive preferences as *relative intertemporal risk aversion*, and explains that it is a function of ρ (and thus of $IES = (1-\rho)^{-1}$) as well as of Arrow-Pratt risk aversion $(1-\alpha)$. Whenever $\alpha < \rho$, a decision maker with Epstein-Zin preferences is more risk averse than a decision maker whose preferences are representable by the additive expected utility model. Since $\rho = 0$ ($IES = 1$) in the setting that pertains to equation (18), we must have that $\alpha < 0$ whenever we assume that the decision maker is strictly (inter)temporally risk averse, and thus more risk averse than an additive expected utility decision maker. With $\alpha < 0$, the last term in (18) is negative, and the consumption discount rate that pertains to (18) is thus decreased relative to the consumption discount rate that pertains to (17). This reduction in the consumption discount rate can be interpreted as an increase in the precautionary savings motive due to increased risk aversion relative to the additive expected utility model. Note that if $\alpha = \rho$, the Epstein-Zin model nests the additive expected utility model, such that the last term in (18) vanishes, which implies that (18) is equivalent to (17). Note furthermore that equation (18) is not subject to a horizon effect, i.e. there is no fifth term. This is true regardless of the number of periods (the horizon) taken into account in the underlying decision problem. In fact, the planning horizon $\bar{T}$ of the setting in which Traeger (2011) derives equation (18) is finite but exceeds the period to which the discount rate applies (here: period 2). My calculations in appendix B.4 (which are not restricted to the case with $IES = 1$) confirm the absence of the horizon effect in the discounting function of a decision maker with Epstein-Zin preferences as employed in Traeger (2011, 2014).[23]

[21] See Traeger (2011) for a derivation of equation (18) in a multiperiod setting or Traeger (2014) for a derivation in a two period setting. Note that Traeger refers to this equation as the *consumption discount rate in the isoelastic setting with intertemporal risk aversion* rather than as the *Extended Ramsey Equation for EZ preferences.*

[22] The particular recursive utility function is specified by $U_t = \frac{x_t^\rho}{\rho} + \beta \left(E_t U_{t+1}^{\frac{\alpha}{\rho}} \right)^{\frac{\rho}{\alpha}}$.

[23] Let me point to an apparent inconsistency that stands out when we compare the Extended Ramsey Equations for RS and EZ preferences with $IES = 1$. To see this apparent inconsistency, note that the Risk-Sensitive and the Epstein-Zin specification of Kreps-Porteus recursive preferences should be equivalent if $u(x_t) = \ln x_t$, i.e. if $IES = 1$. One would thus expect to find the same instantaneous consumption discount rate for both specifications in this special case. What I find here, instead, is that the Extended Ramsey Equation for RS preferences is subject to a horizon effect, whereas the Extended Ramsey Equation for EZ preferences, as stated in Traeger (2011), is free from a horizon effect. The cause of this apparent

The Extended Ramsey Equation for Risk-Sensitive preferences (13/19) corresponds to equation (16) if there is no risk on second period growth ($\sigma_2^2 = 0$), and to equation (17) if the RS decision maker is temporally risk neutral ($k = 0$). However, given that the Risk-Sensitive decision maker is strictly temporally risk averse ($k > 0$), the instantaneous consumption discount rate is decreased by two additional terms. First, the fourth term in (13/19) accounts for the increased risk aversion of the RS decision maker relative to the risk aversion of an additive expected utility decision maker. Given $k > 0$, the RS decision maker is more risk averse than the additive expected utility decision maker, which implies an amplification of the precautionary savings motive in the presence of risk on second period consumption, and thus a decrease in the consumption discount rate. This effect is similar to the effect of relative intertemporal risk aversion in the setting of Traeger (2011, 2014). Second, the fifth term in (13/19)–the horizon effect–reduces the instantaneous consumption discount rate due the correlation in consumption *levels* from period 2 onwards, as implied by risk on second period consumption *growth*. The strictly temporally risk averse Risk-Sensitive agent is averse towards this correlation in consumption levels, and therefore values second period consumption more, i.e. discounts it less. Put differently, a strictly temporally risk averse RS agent differentiates between independent risk on consumption levels in period 2 and onwards (recall that there would be no horizon effect in this case) and risk on second period consumption growth which goes along with correlation in the risk on consumption levels. The presence of correlated consumption risk from period 2 onwards amplifies the RS agents precautionary savings motive, and thus implies a decrease in the instantaneous consumption discount rate.

These comparisons with the Ramsey Equation and its previous extensions highlight the novelty of the horizon effect. The fifth term in (13/19), which constitutes the horizon effect, is unique to the consumption discount rate $DR_{1,2}^{\bar{T}>2}$ of a decision maker with Risk-

inconsistency is that a homogeneous CES felicity function, $u(x_t) = \frac{x_t^\rho}{\rho}$, is employed for the derivation of (18). The homogeneity of this function eliminates the horizon effect in the EZ case, as is evident from the calculations in appendix B.4. A logarithmic felicity function in the contrary, which is often treated as the limit of $u(x_t) = \frac{x_t^\rho}{\rho}$ when $IES = 1$, is not homogeneous. In fact, $u(x_t) = \ln x_t$ is not the limit of the homogeneous CES function $u(x_t) = \frac{x_t^\rho}{\rho}$, but rather the limit of the non-homogeneous CES function $u(x_t) = \frac{x_t^\rho - 1}{\rho}$. These two specifications are often used interchangeably since the addition of the constant $-\frac{1}{\rho}$ to $\frac{x_t^\rho}{\rho}$ does not change preferences over x_t. What the addition of this constant does, however, is to eliminate the homogeneity of $u(x_t)$.

Sensitive preferences. Since the rate of pure time preference δ is usually considered to be small, the horizon effect may be quite significant, even for moderate degrees of temporal risk aversion. I enlarge upon this point in the next section.

5.3 The twofold role of the rate of pure time preference

The previous comparison to the literature illustrates that in absence of the horizon effect, the connection between the rate of pure time preference δ (the utility discount rate) and the consumption discount rate is a one to one relationship. Increasing δ augments the consumption discount rate $DR_{1,2}^{\bar{T}}$ by the same amount. In this case, the task of the rate of pure time preference is solely to discount the felicity of the people in the period to which the consumption discount rate applies (period 2). In the Ramsey Equation and its previous extensions (equations 16, 17 and 18), all of which are not subject to a horizon effect, δ takes on this single role.

If the consumption discount rate is subject to a horizon effect as in the Extended Ramsey Equation for RS preferences, the role of the rate of pure time preference is twofold. As in the Ramsey Equation and its previous extensions, δ accounts for discounting second period felicity. This role is assumed by the first term on the right hand side of equation (13). Yet δ also appears in the term which defines the horizon effect, namely in the last term on the right hand side of equation (13). The impact of δ in this role is such that the absolute magnitude of the horizon effect decreases as δ increases. In a sense, the horizon effect is discounted more strongly as δ increases. A smaller absolute magnitude of the horizon effect then implies a bigger consumption discount rate, since the horizon effect impacts $DR_{1,2}^{\bar{T}>2}$ negatively. Corollary 1 summarizes this twofold role of the rate of pure time preference in the Extended Ramsey Equation for Risk-Sensitive preferences.

Corollary 1 *(A twofold role of the rate of pure time preference)*
Consider the Extended Ramsey Equation for Risk-Sensitive preferences (equation 13). The rate of pure time preference affects $DR_{1,2}^{\bar{T}>2}$ positively through two distinct terms:

term 1 of (13): the bigger δ is, the more is period 2 felicity discounted

term 5 of (13): the bigger δ is, the smaller is the absolute value of the horizon effect

5.4 A numerical illustration of the horizon effect

To gain insights on the magnitude and thus on the significance of the horizon effect, I conduct a numerical illustration for which I assume a descriptive approach to social cost-benefit analysis.[24] The descriptive approach holds that the preference parameters that characterize a fictive social agent should be deduced from the revealed preferences of the non-fictive members of society, i.e. from the choices of private agents. Private agents' preferences for the allocation of consumption over time and over states of the world are revealed through the choices they make in the financial markets, and are reflected in the ensuing equilibrium prices. In particular, the risk-free rate of return r_f (the return to an asset with certain payoff and price 1) reflects private agents' valuation of future consumption relative to present consumption, and thus reflects their underlying consumption discount rate. Hence, we can exploit the equilibrium relationship $r_f = DR_{1,2}$ in order to elicit a private agent's preference parameters.[25]

For the numerical analysis at hand I consider two different agents, namely a private and a social agent. The two agents are assumed to be equivalent with respect to their preference parameters: Both agents are of the Risk-Sensitive type and are characterized by the (equivalent) preference tuple (δ, IES, k^*). However, I postulate an important distinction between the private and the social agent. The private agent's valuation of second period consumption (his instantaneous consumption discount rate) is based on a utility function with a horizon $\bar{T}^P$ which reflects his remaining lifetime. The social agent's discounting function, in contrast, is assumed to reflect a much longer horizon $\bar{T}^S \geq \bar{T}^P$, since the social agent takes into account the lifetime of the young and the yet unborn generations when assessing the value of future consumption.

The setting on which the numerical analysis builds is that of an economy with a risk-free

[24] A major disagreement (see e.g. Arrow et al. 1996) in social cost-benefit analysis concerns the question whether the preference parameters of a social decision maker should be deduced from the revealed preferences of private agents (the descriptive approach), or whether the parameterization of a social decision maker's utility function should be guided by ethical principles (the prescriptive approach).

[25] In standard asset pricing theory (see e.g. Cochrane 2005) the risk-free rate of return is defined by the fundamental pricing equation for an asset with certain payoff and price $p_t = 1$, i.e. by $p_t = 1 = E\left[M_{t+1}\left(1 + r_f\right)\right]$, where M_{t+1} is called a pricing kernel. The pricing kernel is defined by $E\left[M_{t+1}\right] = \frac{\partial U(x)/\partial x_{t+1}}{\partial U(x)/\partial x_t}$, which is simply the marginal rate of substitution between consumption in periods t and $t+1$. Combining the two equations and setting $t = 1$ yields an equation for the risk-free rate of return, and reveals its equivalence to the instantaneous consumption discount rate: $r_f = -\ln \frac{\partial U_1(x)/\partial x_2}{\partial U_1(x)/\partial x_1} = DR_{1,2}$.

rate of return of $r_f = 2.4\%$ and *iid* normal risk on future consumption growth such that $\tilde{g}_t \sim N\left(\mu_t = 1.8\%, \sigma_t^2 = (3.6\%)^2\right)$. The value for r_f corresponds to the average annual real rate of return to 10-year US Treasury notes over the period 1872-2013, which I derived from the data collected on the homepage of Robert Shiller.[26] The values for the mean and the standard deviation of consumption growth are the estimates of Kocherlakota (1996), which he obtained from annual US data for the period 1889-1978. The private agent's horizon is set to $\bar{T}^P = 30$ years; the analysis for the social agent is conducted for a domain of integer valued horizon lengths $\bar{T}^S \in [30, ...200]$ years. The analysis is carried out for two different values of the rate of pure time preference, namely a rather high annual value of $\delta = 1.5\%$, and a rather low annual value of $\delta = 0.7\%$.[27] To keep this numerical analysis simple, I assume $IES = 1$, which enables us to conduct the numerical analysis by means of the Extended Ramsey Equation for RS preferences.[28]

Given the equilibrium relationship $r_f = DR_{1,2}^{\bar{T}^P}$, we can calibrate the private agent's preference model to the risk-free rate of return. In particular, since $IES = 1$, we can employ a rearranged version of the private agent's Extended Ramsey Equation for RS preferences (equation 13 with $\bar{T} = \bar{T}^P$) to determine the degree of temporal risk aversion k^*:

$$k^* = \frac{\mu_2 - r_f + \delta - \frac{\sigma_2^2}{2}}{\sigma_2^2 \left(1 + \sum_{\tau=0}^{\bar{T}^P - 3} \exp(-\delta)^{\tau+1}\right)}, \tag{20}$$

Solving (20) with $r_f = 2.4\%$, $\mu_2 = 1.8\%$, $\sigma_2 = 3.6\%$ and $\bar{T}^P = 30$ yields $k^* \approx 0.27$ for $\delta = 1.5\%$, and $k^* \approx 0.01$ for $\delta = 0.7\%$. Hence, the Risk-Sensitive private agent, and thus the Risk-Sensitive social agent, are characterized by $(\delta = 1.5\%, IES = 1, k^* = 0.27)$ or by $(\delta = 0.7\%, IES = 1, k^* = 0.01)$.

The fact that the endogenously determined values for k^* are increasing in δ is connected to

[26] http://www.econ.yale.edu/~shiller/data.htm. The data contains the consumer price index as well as the nominal 10-year Treasury rate for the period 1871-2013. Hence we can derive the inflation rates in the years 1872-2013, the real returns in the respective years, and the average rate of return over the respective period.

[27] The high value for the rate of pure time preference corresponds to the utility discount rate in Nordhaus' (2008) well-known assessment of optimal climate policy in the integrated assessment model DICE. Nordhaus' value for the utility discount rate is sometimes critisized for being too high. In particular Stern (2007), who advocates a prescriptive approach to social cost-benefit analysis of climate policy, employs a much lower utility discount rate of $\delta = 0.1\%$.

[28] If $IES \neq 1$ the instantaneous consumption discount rate of the private and the social agent can be derived from equation (11) through a more involved recursive computational analysis. I have conducted such an analysis in a different paper, which is available upon request.

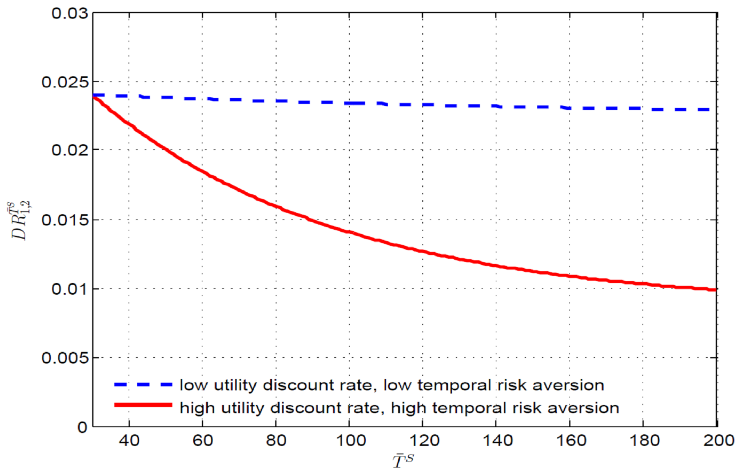

Figure 1: The instantaneous consumption discount rate of a Risk-Sensitive agent as a function of his horizon.

the twofold role of the rate of pure time preference. In view of the two channels outlined in corollary 1 and in view of the Extended Ramsey Equation for RS preferences, it is clear that increases in δ must be counterbalanced by increases in k^* if the instantaneous consumption discount rate of the private agent is calibrated such that it matches the constant risk-free rate or return.

Employing the economic variables and the values for δ and k^* in the social agent's Extended Ramsey Equation for RS preferences (equation 13 with $\bar{T} = \bar{T}^S$ and $\bar{T}^S \in [30, ...200]$) yields the instantaneous consumption discount rate as a function of $\bar{T}^S$. The dependence of the social agent's instantaneous consumption discount rate $DR_{1,2}^{\bar{T}^S}$ on his horizon $\bar{T}^S$, given different pairs (δ, k^*), is depicted in figure 1.

The (blue) dotted line in figure 1 corresponds to the instantaneous consumption discount rate of a RS decision maker with preference tuple $(\delta = 0.7\%, IES = 1, k^* = 0.01)$; the red (straight) line corresponds to the discount rate of a social decision maker with preferences specified by $(\delta = 1.5\%, IES = 1, k^* = 0.27)$. For both specifications, we can observe how the assumption on the private agent's horizon is reflected in the social agent's discount rate. For a horizon $\bar{T}^S = \bar{T}^P = 30$ years, the social agent is exactly equivalent to the private agent, and his discount rate therefore corresponds to the risk-free rate of return, i.e. $DR_{1,2}^{\bar{T}^S=30} = r_f = 2.4\%$. If the horizon of the social agent is higher than that of the private agent $(\bar{T}^S > \bar{T}^P = 30)$, we observe that the social agent's discount rate lies below the risk-free rate of return.

Given the low δ, low k^* specification (blue dotted line), the horizon has an almost negligible effect on the consumption discount rate of the Risk-Sensitive social agent. Under this specification, a social agent with horizon $\bar{T} = 200$ discounts second period consumption at $DR_{1,2}^{\bar{T}^S=200} \approx 2.3\%$, i.e. the horizon effect decreases the discount rate only by about 0.1% relative to the risk-free rate of return. However, given the high δ, high k^* specification (red straight line), the impact of the horizon effect can be very big if long horizons $\bar{T}^S$ are considered. Under this specification, a social agent with horizon $\bar{T} = 200$ years is characterized by an instantaneous consumption discount rate of $DR_{1,2}^{\bar{T}^S=200} \approx 1\%$, which lies significantly below the risk-free rate of return.

A careless glimpse on figure 1 may suggest that the absolute value of the horizon effect is increasing in δ. This inference is misleading, however. A high δ implies a high value for the endogenously determined degree of temporal risk aversion k^*, as explained above. It is this high degree of k^* which brings about a more pronounced horizon effect, and thus a lower consumption discount rate.[29]

This numerical analysis suggests that the horizon effect may reduce the instantaneous consumption discount rate to levels that lie significantly below the risk-free rate of return. Temporally risk averse Risk-Sensitive agents take into account that the risk on period 2 consumption growth translates into correlated risk on the consumption levels in periods $2, 3, ...\bar{T}$. Thus, if a bad state of the world (a low consumption growth rate) realizes in period 2, an entirely lower remaining consumption path will be implied. The private Risk-Sensitive agent only fears 30 years of consecutively low consumption levels. The risk-free rate of return reflects this prospect of a relatively short time-span of suffering. The social agent, in contrast, takes into account that his protégés (society) may have to suffer for a considerably longer time-span. As a result of this long-term perspective, he discounts second period consumption at a rate below r_f, that is, he values future consumption more than implied by the risk-free rate.

[29] In fact, increasing δ for a constant degree of temporal risk aversion has just the opposite effect, i.e. it diminishes the horizon effect, as already discussed in context of the twofold role of the rate of pure time preference (corollary 1).

6 Conclusion

It is widely acknowledged that the entanglement of risk aversion and the intertemporal elasticity of substitution constitutes a problematic shortcoming of the additive expected utility model. This model constitutes the standard framework for intergenerational consumption discounting in presence of risk, and is the basis for the Extended Ramsey Equation. In a series of recent contributions on consumption discounting in the context of climate policy assessment, attention has shifted to the more flexible Kreps-Porteus recursive model, in which a disentanglement of risk aversion and the IES can be achieved. In particular, attention has shifted to a very specific parameterization of the Kreps-Porteus recursive model, namely to the Epstein-Zin preference model which postulates constant relative risk aversion with respect to continuation utilities (see e.g. Traeger 2011, 2014). The prevalence of the Epstein-Zin preference model in recursive approaches to social cost-benefit analysis mirrors its popularity in the asset pricing literature, which is often attributed to the homotheticity of the Epstein-Zin utility function. The question whether such homotheticity is an eligible assumption in the context of social cost-benefit analysis, and in particular in context of the assessment of optimal climate policies, remains yet to be discussed.

If we let go of homotheticity but impose mutual utility independence on the Kreps-Porteus recursive model, then, as formally shown in this paper, the utility function of the respective decision maker must be of the Risk-Sensitive form, i.e. be characterized by constant absolute risk aversion with respect to continuation utilities. Given that risk on consumption *levels* is independently distributed, the assumption of mutual utility independence is ultimately reflected in the fact that the instantaneous consumption discount rate is a function of circumstances in periods 1 and 2 only. However, if we impose the less restrictive assumption of independent risk on consumption *growth*, and thus allow for correlation in consumption levels, the instantaneous consumption discount rate of a Risk-Sensitive decision maker may be subject to circumstance after the period to which the discount rate applies, and in particular may be a function of the temporal horizon of the Risk-Sensitive agent.

I showed in this paper that the instantaneous consumption discount rate of a temporally risk averse Risk-Sensitive agent with horizon $\bar{T} > 2$ is decreased by a horizon effect under standard discounting assumptions. The horizon effect can be interpreted as an increase in the decision maker's precautionary savings motive, which arises from the fact that the Risk-Sensitive agent accounts for the correlation in consumption levels, as implied by risk on period 2 growth. Hence, when evaluating the risk in period 2 and adjusting the consumption discount rate accordingly, the Risk-Sensitive agent discriminates between second period risk that is unrelated to the risk on the remaining consumption path, and second period risk that is correlated with the remaining consumption path. Given that he is temporally risk averse, he applies higher precaution in a world with correlated consumption levels.

By means of an analytical solution for the instantaneous consumption discount rate–the Extended Ramsey Equation for Risk Sensitive preferences–I took a closer look at the horizon effect. First, I highlighted that the importance of the rate of pure time preference is amplified in presence of a horizon effect. Second, I employed the Extended Ramsey Equation for RS preferences for a numerical illustration, and argued that–in context of social cost-benefit analysis–the horizon effect may provide a rationale for discounting monetary values at a rate below the risk-free rate of return. The assumption which underlies this claim is that a social decision maker is characterized by a horizon that extends that of private agents, whereas it is the relatively short horizon of private agents that is reflected in the equilibrium prices on financial markets.

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A Proofs

A.1 Proof of theorem 1

In this proof I show that Kreps-Porteus recursive preferences which satisfy mutual utility independence are representable by (4). First, I denote by ${}_3x = (x_3, x_4, x_5, ...)$, ${}_3x' = (x'_3, x'_4, x'_5, ...)$ two specific deterministic consumption paths (outcomes) in ${}_3X = X_3 \times X_4 \times X_5 \times ....$ Using this notation, I consider the specific temporal lotteries $(x_1, p_2,\ {}_3x)$, $(x_1, p'_2,\ {}_3x)$, $(x'_1, p_2,\ {}_3x')$, $(x'_1, p'_2,\ {}_3x') \in D$ where x_1 and x'_1 are two specific levels in X_1 and $p_2, p'_2 \in P_2$ are two specific lotteries over X_2.

Second, I consider a decision maker with KP recursive preferences that are defined on D and satisfy mutual utility independence. Denote these preferences as $\succeq^D$. By the definition of mutual utility independence for temporal lotteries (definition 5) it must be true that

$$(x_1, p_2,\ {}_3x) \succeq^D \left(x_1, p'_2,\ {}_3x\right) \iff \left(x'_1, p_2,\ {}_3x'\right) \succeq^D \left(x'_1, p'_2,\ {}_3x'\right).$$

Employing the notion of conditional preferences we can equivalently write

$$\succeq^D_{x_1, {}_3x} \quad = \quad \succeq^D_{x'_1, {}_3x'} \text{ on } p_2, p'_2 \in P_2, \text{ for (all) } \left(x_1,\ {}_3x\right), \left(x'_1,\ {}_3x'\right) \in X_1 \times\ {}_3X. \tag{21}$$

Now let U^D represent $\succeq^D$ and consider $U^D(x_1, m) = W^D\left(x_1, E_m[U^D]\right)$. Given the temporal lottery $(x_1, p_2,\ {}_3x)$, we write $U^D(x_1, p_2,\ {}_3x) = W^D\left(x_1, E_m[U^D(x_2,\ {}_3x)]\right)$ where $U^D(x_2,\ {}_3x) = \phi\left(U^X(x_2,\ {}_3x)\right)$ and $U^X(x_2,\ {}_3x) = u(x_2) + \beta U^X({}_3x)$. Now let $v(p_2) = E_m\phi\left(u(x_2) + \beta U^X({}_3x)\right)$ represent $\succeq^D_{x_1, {}_3x}$. By (21) (i.e. by MUI), $v(p_2)$ represents $\succeq^D_{x_1, {}_3x}$ $\forall$ ${}_3x \in {}_3X$ (and $\forall\ x_1 \in X_1$). Put differently, a certainty equivalent $\hat{x}_2$ (as could be derived from $v(p_2)$), which makes a decision maker with $\succeq^D_{x_1, {}_3x}$ indifferent to receiving the lottery p_2, is independent of the specific level of ${}_3x$. This just means that a decision maker with preferences $\succeq^D_{x_1, {}_3x}$ is constantly absolute risk averse, which in turn implies $\phi(z) = -\exp(-kz)$. Using $\phi(z) = -\exp(-kz)$ in (3) and renormalizing by $U = \phi^{-1}\left(U^D\right)$ yields (4).

A.2 Proof of proposition 1

In this section I proof that the horizon effect diminishes the discount rate under the conditions stated in proposition 1. The proof employs the notions of comonotonicity and countercomonotonicity which are defined as follows.

Definition 8 *(Strict comonotonicity and strict countercomonotonicity)*
Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a single random variable $\tilde{z}$:

$$(Z_1, Z_2) = (g_1(\tilde{z}), g_2(\tilde{z})).$$

If g_1 and g_2 are strictly increasing in $\tilde{z}$, then Z_1 and Z_2 are called comonotonic.
If g_1 is strictly increasing and g_2 is strictly decreasing in $\tilde{z}$, or vice versa, then Z_1 and Z_2 are called countercomonotonic.

Furthermore, the proof uses a lemma that I refer to as the *risk aversion adjusted covariance inequality.* Before stating the lemma, I define formally what I mean by a *risk aversion adjusted probability*, a *risk aversion adjusted expectation operator* and a *risk aversion adjusted covariance.* A prolonged discussion of these concepts can be found in Hector (2013).

A risk aversion adjusted probability twists the statistical probability of a given state of the world $\omega = 1, ...N$ to account for temporal risk aversion with respect to continuation utility. First, define π^ω as the product of a statistical probability l^ω and the risk aversion adjustment factor (see section 4) in the respective state ω:

$$\pi^\omega = l^\omega \frac{\phi'(U_2^\omega)}{\phi'\left(\phi^{-1}(E_1[\phi(U_2)])\right)}.$$

Second, note that π^ω can be interpreted as a probability whenever $0 \leq \pi^\omega \leq 1 \; \forall \; \omega$ and $\sum_{\omega=1}^N \pi^\omega = 1$. Lastly, note already that if $\phi(z) = -\exp(-kz)$ (which is the case for Risk-Sensitive preferences), then $\pi^\omega = l^\omega \frac{\exp\left(-kU_2^\omega\right)}{E_1[\exp(-kU_2)]}$, which satisfies the conditions for the interpretation of π^ω as a probability.

A risk aversion adjusted expectation operator for a random variable $\tilde{z}$ and some function $g(\tilde{z})$ is then defined as

$$E_\pi[g(\tilde{z})] = \sum_{\omega=1}^N \pi^\omega g(z^\omega)$$

This expectation operator employs risk aversion adjusted probabilities in the place of statistical probabilities.

Finally, a risk aversion adjusted covariance between two random variables or functions $g_1(\tilde{z}_1)$ and $g_2(\tilde{z}_2)$ is a covariance which is constructed from risk aversion adjusted expectation operators:

$$cov_{\pi}\left[g_1\left(\tilde{z}_1\right), g_2\left(\tilde{z}_2\right)\right] = E_{\pi}\left[g_1\left(\tilde{z}_1\right) g_2\left(\tilde{z}_2\right)\right] - E_{\pi}\left[g_1\left(\tilde{z}_1\right)\right] E_{\pi}\left[g_2\left(\tilde{z}_2\right)\right].$$

We are now ready to state a lemma on the risk aversion adjusted covariance inequality. A proof of this lemma, which is a close analogue to theorem 43 in Hardy et al. (1934), is contained in Hector (2013).

Lemma 1 *(Risk aversion adjusted covariance inequality).*
Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a single random variable $\tilde{z}$. If Z_1 and Z_2 are strictly comonotonic, then

$$cov_{\pi}\left[Z_1, Z_2\right] > 0.$$

The inequality is reversed if Z_1 and Z_2 are strictly countercomonotonic.

Let us now turn to the actual proof of proposition 1. The decision maker under consideration has mutually utility independent KP recursive preferences. His instantaneous discount rate for a setting with horizon $\bar{T} > 2$ is thus given by (11). Assume that $k > 0$, $0 < \beta < 1$ and $g_t > -1 \ \forall \ t \geq 2$.

To prove statement 1 of proposition 1, assume furthermore that felicity is given by $u\left(x_t\right) = \frac{x_t^{\rho}-1}{\rho}$ with $\rho < 1$ and that only second period consumption growth $\tilde{g}_2$ is risky. The consumption growth rate in $t \geq 3$ is deterministic.

For the proof of statement 2 of proposition 1, assume that $IES = 1$ ($\rho = 0$), which yields $\lim_{\rho \to 0} \frac{x_t^{\rho}-1}{\rho} = \ln x_t$ such that felicity is given by $u\left(x_t\right) = \ln\left(x_t\right)$. Furthermore assume that consumption growth $\tilde{g}_t$ in $t \geq 2$ is risky and independently distributed.

<u>proof of statement 1</u>
Suppose $u\left(x_t\right) = \frac{x_t^{\rho}-1}{\rho}$ with $\rho < 1$ and only period 2 growth is uncertain. Continuation utility U_2 can be rewritten in a simple manner since all risk resolves in period 2:

$$U_2 = u\left(\tilde{x}_2\right) + \sum_{t=3}^{\bar{T}} \beta^{t-2} u\left(\tilde{x}_t\right). \tag{22}$$

With $\tilde{x}_2 = \left(1 + \tilde{g}_2\right) x_1$, felicity in $t \geq 3$ can be written as

$$u(\tilde{x}_t) = u\left(\tilde{x}_2 \prod_{\tau=3}^{t}(1+g_\tau)\right) = \frac{\tilde{x}_2^\rho}{\rho}\left[\prod_{\tau=3}^{t}(1+g_\tau)\right]^\rho - \frac{1}{\rho}. \tag{23}$$

Plugging the felicity function (23) into the continuation utility U_2 (22), and (22) into the discounting function (11), yields $DR_{1,2}^{\bar{T}>2}$ as

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln E_1\left[\frac{\exp\left(-ku(\tilde{x}_2) - \frac{k}{\rho}\tilde{x}_2^\rho h(\bar{T})\right)}{E_1\left[\exp\left(-ku(\tilde{x}_2) - \frac{k}{\rho}\tilde{x}_2^\rho h(\bar{T})\right)\right]}\frac{u'(\tilde{x}_2)}{u'(x_1)}\right] \tag{24}$$

$$\text{where } h(\bar{T}) = \sum_{t=3}^{\bar{T}}\left(\beta^{t-2}\left[\prod_{\tau=3}^{t}(1+g_\tau)\right]^\rho\right).$$

In the next step I study how $DR_{1,2}^{\bar{T}>2}$ changes as the horizon $\bar{T}$ changes. To this end I would need to examine the derivative of $h(\bar{T})$ with respect to $\bar{T}$. However, as the domain of $h(\bar{T})$ is discrete, $h'(\bar{T})$ does not exist. Thus I define a function $\hat{h}(\bar{T}) : \mathbb{R}^+ \to \mathbb{R}$ with $\hat{h}(\bar{T}) = h(\bar{T})\ \forall\, \bar{T} \in \mathbb{N}$. The function $\hat{h}(\bar{T})$ is assumed to constitute a smooth interpolation between the discrete points defined by $h(\bar{T})$ at all $\bar{T} \in \mathbb{N}$. I then examine the derivative of $\hat{h}(\bar{T})$ rather than that of $h(\bar{T})$. Since $\hat{h}(\bar{T})$ is strictly increasing, its derivative $\hat{h}'(\bar{T})$ is positive.

Substituting all $h(\bar{T})$ by $\hat{h}(\bar{T})$ and taking the derivative of (24) with respect to $\bar{T}$ yields

$$\frac{\partial DR_{1,2}^{\bar{T}>2}}{\partial\bar{T}} = \frac{E_1\left[f(\tilde{g}_2)\,u'(\tilde{x}_2)\left(\frac{k}{\rho}\tilde{x}_2^\rho\hat{h}'(\bar{T})\right)\right]}{E_1\left[f(\tilde{g}_2)\,u'(\tilde{x}_2)\right]} - \frac{E_1\left[f(\tilde{g}_2)\left(\frac{k}{\rho}\tilde{x}_2^\rho\hat{h}'(\bar{T})\right)\right]}{E_1\left[f(\tilde{g}_2)\right]}$$

$$\text{where } f(\tilde{g}_2) = \exp\left(-ku(\tilde{x}_2) - \frac{k}{\rho}\tilde{x}_2^\rho\hat{h}(\bar{T})\right).$$

Thus

$$\frac{\partial DR_{1,2}^{\bar{T}>2}}{\partial\bar{T}} \gtreqless 0$$

whenever

$$\frac{E_1\left[f(\tilde{g}_2)\,u'(\tilde{x}_2)\left(\frac{k}{\rho}\tilde{x}_2^\rho\hat{h}'(\bar{T})\right)\right]}{E_1\left[f(\tilde{g}_2)\,u'(\tilde{x}_2)\right]} - \frac{E_1\left[f(\tilde{g}_2)\left(\frac{k}{\rho}\tilde{x}_2^\rho\hat{h}'(\bar{T})\right)\right]}{E_1\left[f(\tilde{g}_2)\right]} \gtreqless 0.$$

After multiplying the last equation with $E_1\left[f\left(\tilde{g}_2\right)u'\left(\tilde{x}_2\right)\right]/E_1\left[f\left(\tilde{g}_2\right)\right]$ we can write

$$\left[\begin{array}{c} E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]}u'\left(\tilde{x}_2\right)\left(\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right)\right]- \\ E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]}u'\left(\tilde{x}_2\right)\right]E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]}\left(\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right)\right] \end{array}\right] \gtreqless 0. \tag{25}$$

Now note that $\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} = \frac{\exp\left(-ku(\tilde{x}_2)-\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}\left(\bar{T}\right)\right)}{E_1\left[\exp\left(-ku(\tilde{x}_2)-\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}\left(\bar{T}\right)\right)\right]} = \frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]}$ is the risk aversion adjustment factor as mentioned earlier. We can thus rewrite (25) in terms of a risk aversion adjusted expectation operator $E_{1,\pi}$:

$$E_{1,\pi}\left[u'\left(\tilde{x}_2\right)\left(\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right)\right]-E_{1,\pi}\left[u'\left(\tilde{x}_2\right)\right]E_{1,\pi}\left[\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right]\gtreqless 0. \tag{26}$$

Equation (26) is the risk aversion adjusted covariance between $u'\left(\tilde{x}_2\right)$ and $\left(\frac{k}{\rho}\tilde{x}_2^{\rho}h'\left(\bar{T}\right)\right)$, both of which are functions of the single random variable $\tilde{g}_2$. By lemma 1, the sign of the risk aversion adjusted covariance can be determined from the comonotonicity characteristics of $u'\left(\tilde{x}_2\right)$ and $\left(\frac{k}{\rho}\tilde{x}_2^{\rho}h'\left(\bar{T}\right)\right)$. The comonotonicity characteristics in turn are determined by the derivatives of $u'\left(\tilde{x}_2\right)$ and $\left(\frac{k}{\rho}\tilde{x}_2^{\rho}h'\left(\bar{T}\right)\right)$ with respect to the random variable $\tilde{g}_2$. Here we have

$$\begin{aligned} \frac{\partial u'\left(\tilde{x}_2\right)}{\partial\tilde{g}_2} &= u''\left(\tilde{x}_2\right)x_1<0 \\ \frac{\partial\left(\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right)}{\partial\tilde{g}_2} &= k\tilde{x}_2^{-\rho}\hat{h}'\left(\bar{T}\right)x_1>0, \end{aligned}$$

Hence, $u'\left(\tilde{x}_2\right)$ and $\left(\frac{k}{\rho}\tilde{x}_2^{\rho}\hat{h}'\left(\bar{T}\right)\right)$ are countercomonotonic by definition 8. By lemma 1, countercomonotonicity implies a negative risk aversion adjusted covariance (equation 26), which in turn implies $\frac{\partial DR_{1,2}^{\bar{T}\geq 2}}{\partial\bar{T}}<0$.

<u>proof of statement 2</u>

Suppose $u\left(x_t\right)=\ln x_t$ and consumption growth $\tilde{g}_t$ in $t\geq 2$ is uncertain and independently distributed. Starting with the continuation utility in $\bar{T}$, I plug $U_{\bar{T}}$ into $U_{\bar{T}-1}$, $U_{\bar{T}-1}$ into

$U_{\bar{T}-2}$ and so on until I arrive in period $t=2$:

$$U_2 \;=\; \left(\sum_{\tau=2}^{\bar{T}} \beta^{\tau-2}\right) \ln\left(\tilde{x}_2\right) - \frac{1}{k} q\left(\tilde{g}_\tau\right) \tag{27}$$

$$\text{where } q\left(\tilde{g}_\tau\right) = \sum_{\tau=3}^{\bar{T}} \left(\beta^{\tau-2} \ln E_{\tau-1}\left[\left(1+\tilde{g}_\tau\right)^{-k\sum_\tau^{\bar{T}} \beta^{\bar{T}-\tau}}\right]\right).$$

Equation (27) exposes the additive separability of U_2 into a first term which collects $\tilde{g}_2$ (note that $\tilde{x}_2 = \left(1+\tilde{g}_2\right) x_1$) and a second term, namely $\left(-k^{-1} q\left(\tilde{g}_\tau\right)\right)$, which collects $\tilde{g}_t$ for $t \geq 3$. The latter term is independent of risk that reveals in period 2. Hence, upon plugging the continuation utility (27) into discounting equation (11), all terms containing $q\left(\tilde{g}_\tau\right)$ can be taken out of the expectation operator E_1 and subsequently cancel out. The instantaneous discount rate for a horizon $\bar{T} > 2$ is thus

$$DR_{1,2}^{\bar{T}>2} \;=\; -\ln\beta - \ln E_1\left[\frac{\exp\left(-kh\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)}{E_1\left[\exp\left(-kh\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right]} \frac{u'\left(\tilde{x}_2\right)}{u'\left(x_1\right)}\right] \tag{28}$$

$$\text{where } h\left(\bar{T}\right) = \sum_{\tau=2}^{\bar{T}} \beta^{\tau-2}.$$

Equation (28) depends on the length of the horizon $\bar{T}$ through $h\left(\bar{T}\right)$.

The direction of this dependency is studied by taking the derivative of $DR_{1,2}^{\bar{T}>2}$ with respect to $\bar{T}$. As in the proof of statement 1, I substitute $h\left(\bar{T}\right)$ by its continuous analogue $\hat{h}\left(\bar{T}\right)$. Then,

$$\frac{\partial DR_{1,2}^{\bar{T}>2}}{\partial \bar{T}} \;=\; \frac{E_1\left[f\left(\tilde{g}_2\right) u'\left(\tilde{x}_2\right)\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right]}{E_1\left[f\left(\tilde{g}_2\right) u'\left(\tilde{x}_2\right)\right]} - \frac{E_1\left[f\left(\tilde{g}_2\right)\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right]}{E_1\left[f\left(\tilde{g}_2\right)\right]}$$

$$\text{where } f\left(\tilde{g}_2\right) = \exp\left(-k\hat{h}\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right).$$

The direction of the inequality $\frac{\partial DR_{1,2}^{\bar{T}>2}}{\partial \bar{T}} \gtreqless 0$ is then equivalent to the direction of the inequality

$$\left[\begin{array}{c} E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'\left(\tilde{x}_2\right)\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right] - \\ E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'\left(\tilde{x}_2\right)\right] E_1\left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]}\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right] \end{array}\right] \gtreqless 0,$$

which, as in the precedent proof, can be stated as a risk aversion adjusted covariance:

$$E_{1,\pi}\left[u'\left(\tilde{x}_2\right)\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right]-E_{1,\pi}\left[u'\left(\tilde{x}_2\right)\right]E_{1,\pi}\left[k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right]\gtreqless 0. \tag{29}$$

Since

$$\begin{aligned}\frac{\partial u'\left(\tilde{x}_2\right)}{\partial\tilde{g}_2} &= u''\left(\tilde{x}_2\right)x_1<0\\ \frac{\partial\left(kh'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)}{\partial\tilde{g}_2} &= kh'\left(\bar{T}\right)\frac{1}{\left(1+\tilde{g}_2\right)}>0,\end{aligned}$$

$u'\left(\tilde{x}_2\right)$ and $\left(k\hat{h}'\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)$ are countercomonotonic according to definition 8. By lemma 1 it is then implied that equation (29) is negative and hence $\frac{\partial DR_{1,2}^{\bar{T}>2}}{\partial\bar{T}}<0$.

B Derivations

B.1 Risk-Sensitive preferences for independently distributed $\tilde{x}_t$

Suppose preferences are represented by (4) and $\tilde{x}_t$ for $t>1$ is risky and independently distributed. Plugging continuation utilities $U\left(x_2,m\right)$, $U\left(x_3,m\right)$,... into the initial utility function $U\left(x_1,m\right)$ yields

$$\begin{aligned}&U\left(x_1,m\right)=u\left(x_1\right)-...\\ &...\frac{\beta}{k}\ln\left(E_m\left[\exp\left(-ku\left(\tilde{x}_2\right)\right)\left(E_m\left[\exp\left(-ku\left(\tilde{x}_3\right)\right)\left(E_m\exp\left(-k\left(...\right)\right)\right)^{\beta}\right]\right)^{\beta}\right]\right).\end{aligned}$$

Since $\tilde{x}_t$ is independently distributed, this can be written as

$$\begin{aligned}&U\left(x_1,m\right)=u\left(x_1\right)-...\\ &...\frac{\beta}{k}\ln E_m\exp\left(-ku\left(\tilde{x}_2\right)\right)-\frac{\beta^2}{k}\ln\left(E_m\exp\left(-ku\left(\tilde{x}_3\right)\right)\right)-\frac{\beta^3}{k}\ln\left(E_m\exp\left(-k\left(...\right)\right)\right).\end{aligned}$$

Now note that the terms $-\frac{1}{k}\ln\left(E_m\exp\left(-ku\left(\tilde{x}_t\right)\right)\right)$ can be substituted for by $u\left(\hat{x}_t\right)$ since they determine certainty equivalent consumption $\hat{x}_t$. Thus we can further simplify the last equation and write

$$U\left(x_1,m\right)=u\left(x_1\right)+\beta u\left(\hat{x}_2\right)+\beta^2u\left(\hat{x}_3\right)+...=u\left(x_1\right)+\beta\sum_{t=2}^{\infty}\beta^{t-2}u\left(\hat{x}_t\right).$$

B.2 Absence of the horizon effect

In this section I discuss conditions under which the instantaneous consumption discount rate of a decision maker with Risk-Sensitive preferences (equation 11) is not subject to a horizon effect. Although these conditions are fairly obvious, I evolve on them to facilitate the general understanding of the horizon effect.

The requirement for the absence of the horizon effect is that $DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$. This requirement is met under the following specifications.

(i) $\beta = 0$

If $\beta = 0$, the utility function of a RS decision maker (equation 4) is $U_1 = u(x_1)$, independently of the length of the horizon $\bar{T}$ taken into account. The respective decision maker has no valuation for generations living in $t \geq 2$ and therefore applies an infinite discount rate to period 2 consumption values. This is true irrespective of the existence of generations in periods $t \geq 3$. Hence the discount rates $DR_{1,2}^{\bar{T}>2}$ and $DR_{1,2}^{\bar{T}=2}$ are equivalent and there is no horizon effect.

(ii) $u(x_t)$ linear

If $u(x_t)$ is linear, $u'(x_t)$ is a constant and thus independent of x_t (which may or may not be risky). Thus one can write $u'(x_1) = u'(\tilde{x}_2) = c$ which reduces (11) to

$$DR_{1,2}^{\bar{T}>2} = -\ln \beta - \ln E_1 \left[\frac{\exp(-kU_2)}{E_1 [\exp(-kU_2)]} \right] = -\ln \beta.$$

This is equivalent to $DR_{1,2}^{\bar{T}=2}$ under linear $u(x_t)$. Hence $DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$.

Intuitively, the absence of the horizon effect is explained by the absence of (risk aversion adjusted) probabilities. Since risk on $u'(\tilde{x}_2)$ plays no role if $u(x_t)$ is linear, there is no role for probabilities or risk aversion adjusted probabilities. The horizon of the decision problem, which enters the discounting equation through the adjustment factor, has therefore no effect on the discount rate.

(iii) no risk in period 2

If there is no risk in period 2 (but potentially in periods $t > 2$), the risk aversion adjustment factor in equation (4) can be written as

$$\frac{\exp(-kU_2)}{E_1 [\exp(-kU_2)]} = \frac{\exp(-ku(x_2)) \cdot \exp(\beta \ln(E_2 [\exp(-kU_3)]))}{\exp(-ku(x_2)) \cdot \exp(\beta \ln(E_2 [\exp(-kU_3)]))} = 1.$$

The expectation operator E_1 can be neglected since x_2 is certain and since the uncertain continuation utility U_3 is transformed into a certainty equivalent by the expectation operator E_2. Without the expectation operator E_1, the numerator and the denominator cancel each other out. The discount rate is then simply

$$DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2} = -\ln\beta - \ln\left[\frac{u'(x_2)}{u'(x_1)}\right].$$

The intuition is as in the case where $u(x_t)$ is linear. If there is no risk on $u'(x_2)$, then there is no role for risk aversion adjustment factors and thus no channel through which the horizon $t \geq 3$ could enter the discounting function.

(iv) $k = 0$

If $k = 0$, the risk aversion adjustment factor $\frac{\exp(-kz)}{E_1[\exp(-kz)]}$ equals one. Thus,

$$DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2} = -\ln\beta - \ln E_1\left[\frac{u'(\tilde{x}_2)}{u'(x_1)}\right].$$

The intuitive explanation is that $k = 0$ restricts $DR_{1,2}^{\bar{T}>2}$ to the discount rate of a decision maker who is temporally risk neutral, i.e. a decision maker whose preferences are representable by an additive expected utility function. A discount rate that is derived from an additive utility function only depends on values of the present period and values of the period that is discounted. This means that the instantaneous discount rate $DR_{1,2}^{\bar{T}>2}$ is independent of values in periods $t \geq 3$, and thus not subject to a horizon effect.

(v) risk on $\tilde{x}_t$ independently distributed

If risk on $\tilde{x}_t$ is independently distributed, then the discounting equation (11) of the Risk-Sensitive decision maker can be written as

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln E_1\left[\frac{\exp\left(-ku\left(\tilde{x}_2\right)\right)\cdot\exp\left(\beta\ln\left(E_2\left[\exp\left(-kU_3\right)\right]\right)\right)}{E_1\left[\exp\left(-ku\left(\tilde{x}_2\right)\right)\right]\cdot\exp\left(\beta\ln\left(E_2\left[\exp\left(-kU_3\right)\right]\right)\right)}\frac{u'(\tilde{x}_2)}{u'(x_1)}\right].$$

The exponential function containing U_3 was taken out of the expectation operator E_1 since the risk contained in U_3 is independent of period 2 information. As this exponential appears in the numerator as well as in the denominator, it cancels out and we get $DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$. The absence of a horizon effect for independently distributed $\tilde{x}_t$ is a direct consequence of the mutual utility independence of the decision maker. I already showed in equation (6) of section **??** that a RS decision maker who faces independently distributed $\tilde{x}_t$ has an additive utility function. As with $k=0$, the additivity of the decision maker's utility function implies the absence of a horizon effect.

B.3 Analytical solution for the discount rate

Consider a RS decision maker with discount rate (11). Assume consumption growth $\tilde{g}_t$ is risky and independently distributed in all $t \geq 2$ where $\tilde{g}_t > -1 \ \forall \ t \geq 2$. Furthermore assume $IES = 1$ ($\rho = 0$) such that $u(x_t) = \ln x_t$. In the proof of statement 2 of proposition 1, I already showed that the instantaneous discount rate in this setting can be written as

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln E_1\left[\frac{\exp\left(-kh\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)}{E_1\left[\exp\left(-kh\left(\bar{T}\right)\ln\left(\tilde{x}_2\right)\right)\right]}\frac{u'\left(\tilde{x}_2\right)}{u'\left(x_1\right)}\right]$$

$$\text{where } h\left(\bar{T}\right) = \sum_{\tau=2}^{\bar{T}}\beta^{\tau-2}.$$

With $\tilde{x}_2 = (1+\tilde{g}_2)\,x_1$, $u'(x_t) = x_t^{-1}$, after some rearrangements and after using $\ln(1+\tilde{g}_2) \approx \tilde{g}_2$ (for $\tilde{g}_2$ small), the last equation can be written as

$$DR_{1,2}^{\bar{T}>2} \approx -\ln\beta - \ln E_1\left[\frac{\exp\left(\left(-kh\left(\bar{T}\right)-1\right)\tilde{g}_2\right)}{E_1\left[\exp\left(-kh\left(\bar{T}\right)\tilde{g}_2\right)\right]}\right].$$

Now assume in addition that $\tilde{g}_t \sim N\left(\mu_t, \sigma_t^2\right) \ \forall \ t \geq 2$. The moment generating function $M_{\tilde{y}}(a) \equiv E\left[\exp\left(a\tilde{y}\right)\right]$ of a normally distributed random variable $\tilde{y} \sim N\left(\mu, \sigma^2\right)$ is $M_{\tilde{y}}(a) = \exp\left(a\mu + \frac{\sigma^2}{2}a^2\right)$. Using the moment generating function of $\tilde{g}_2$ in the last equation, we can write

$$\begin{aligned} DR_{1,2}^{\bar{T}>2} &\approx -\ln\beta - \ln\frac{\exp\left(\left(-kh\left(\bar{T}\right)-1\right)\mu_2 + \frac{\sigma_2^2}{2}\left(-kh\left(\bar{T}\right)-1\right)^2\right)}{\exp\left(-kh\left(\bar{T}\right)\mu_2 + \frac{\sigma_2^2}{2}\left(kh\left(\bar{T}\right)\right)^2\right)} \\ &\approx -\ln\beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}2kh\left(\bar{T}\right) \end{aligned}$$

or equivalently

$$DR_{1,2}^{\bar{T}>2} \approx -\ln\beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}2k - \frac{\sigma_2^2}{2}2k\sum_{\tau=3}^{\bar{T}}\beta^{\tau-2}.$$

Substituting $\delta = -\ln\beta$ and slightly rewriting the sum yields the Extended Ramsey Equation for Risk-Sensitive preferences:

$$DR_{1,2}^{\bar{T}>2} \approx \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}2k - \frac{\sigma_2^2}{2}2k\sum_{\tau=0}^{\bar{T}-3}\exp(-\delta)^{\tau+1}.$$

If $\bar{T} = 2$, then we use $u(x_t) = \ln x_t$ $(t = 1, 2)$, $\tilde{g}_2 \sim N\left(\mu_2, \sigma_2^2\right)$, $\tilde{x}_2 = (1 + \tilde{g}_2)x_1$ and $\ln(1 + \tilde{g}_2) \approx \tilde{g}_2$ in equation (12), i.e. in the discounting function for situation B, which, after substituting $\delta = -\ln\beta$, yields

$$DR_{1,2}^{\bar{T}=2} \approx \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}2k.$$

B.4 No horizon effect under Epstein-Zin preferences

Epstein-Zin preferences with homogeneous CES felicity $u(x_t) = \frac{x_t^\rho}{\rho}$ are representable by a KP recursive utility function of the form

$$U_t = \frac{x_t^\rho}{\rho} + \beta\left(E_t U_{t+1}^{\frac{\alpha}{\rho}}\right)^{\frac{\rho}{\alpha}}. \tag{30}$$

This is the KP recursive EZ preference representation employed by Traeger (2011, 2014) from which (18) can be obtained. The instantaneous discount rate of a decision maker with preferences as in (30) is given by

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln\frac{E_1\left[\frac{U_2^{\frac{\alpha}{\rho}-1}}{\left(E_1\left[U_2^{\frac{\alpha}{\rho}}\right]\right)^{1-\frac{\rho}{\alpha}}}u'(\tilde{x}_2)\right]}{u'(x_1)}, \tag{31}$$

where uncertain consumption $\tilde{x}_t$ can be written as $\tilde{x}_t = (1 + \tilde{g}_2)x_1\frac{\tilde{x}_t}{(1+\tilde{g}_2)x_1}$. Exploiting the homogeneity of $u(\tilde{x}_t)$ we can write $u(\tilde{x}_t) = \rho\frac{((1+\tilde{g}_2)x_1)^\rho}{\rho}u\left(\frac{\tilde{x}_t}{(1+\tilde{g}_2)x_1}\right) = \rho u(\tilde{x}_2)\,u\left(\frac{\tilde{x}_t}{\tilde{x}_2}\right)$, where the argument of the latter felicity function is statistically independent of $\tilde{g}_2$ since risk on growth is independently distributed. Starting with the $t = \bar{T}$ (terminal period) specification of (30), we can solve recursively for continuation utility U_2. In each recursion, the independent distribution of $\tilde{g}_t$ enables us to factor the term $\rho u(\tilde{x}_2)$ out:

$$\begin{aligned}
U_{\bar{T}} &= \rho u(\tilde{x}_2)\, u(\tilde{x}_{\bar{T}}/\tilde{x}_2) \\
U_{\bar{T}-1} &= \rho u(\tilde{x}_2)\,(u(\tilde{x}_{\bar{T}-1}/\tilde{x}_2) + \beta(E_{\bar{T}-1}[u(\tilde{x}_{\bar{T}}/\tilde{x}_2)]^{\frac{\alpha}{\rho}})^{\frac{\rho}{\alpha}}) \\
U_{\bar{T}-2} &= \rho u(\tilde{x}_2)\,(u(\tilde{x}_{\bar{T}-2}/\tilde{x}_2) + \beta(E_{\bar{T}-2}[u(\tilde{x}_{\bar{T}-1}/\tilde{x}_2) + \beta(E_{\bar{T}-1}[u(\tilde{x}_{\bar{T}}/\tilde{x}_2)]^{\frac{\alpha}{\rho}})^{\frac{\rho}{\alpha}}]^{\frac{\alpha}{\rho}})^{\frac{\rho}{\alpha}}) \\
U_{\bar{T}-3} &= \rho u(\tilde{x}_2)\,(...).
\end{aligned}$$

Finally we arrive at

$$U_2 = \rho u(\tilde{x}_2)\, h\left(\bar{T}\right), \tag{32}$$

where $h\left(\bar{T}\right)$ is a function that depends on the horizon $\bar{T}$ as well as on the (uncertain) growth rates $\tilde{g}_t$ with $t > 2$. Note that the term $h\left(\bar{T}\right)$ is independent of risk on period 2 growth, $\tilde{g}_2$. Plugging (32) into the discounting equation of the EZ decision maker (equation 31) yields

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln \frac{E_1\left[\frac{u(\tilde{x}_2)^{\frac{\alpha}{\rho}-1} h\left(\bar{T}\right)^{\frac{\alpha}{\rho}-1}}{\left(E_1\left[u(\tilde{x}_2)^{\frac{\alpha}{\rho}} h\left(\bar{T}\right)^{\frac{\alpha}{\rho}}\right]\right)^{1-\frac{\rho}{\alpha}}} u'(\tilde{x}_2)\right]}{u'(x_1)}.$$

Since the risk contained in $h\left(\bar{T}\right)$ is independent of the risk in period 2, $h\left(\bar{T}\right)$ can be taken out of the expectation operator and thus cancels out. We are left with

$$DR_{1,2}^{\bar{T}>2} = -\ln\beta - \ln \frac{E_1\left[\frac{u(\tilde{x}_2)^{\frac{\alpha}{\rho}-1}}{\left(E_1\left[u(\tilde{x}_2)^{\frac{\alpha}{\rho}}\right]\right)^{1-\frac{\rho}{\alpha}}} u'(\tilde{x}_2)\right]}{u'(x_1)}, \tag{33}$$

which is independent of the horizon after $t = 2$, hence $DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$. I have thus shown that the instantaneous discount rate $DR_{1,2}^{\bar{T}>2}$ of the KP recursive EZ decision maker with homogeneous felicity $u(\tilde{x}_t)$ and independently distributed growth risk is not subject to a horizon effect.