UNISEX PRICING OF GERMAN PARTICIPATING LIFE ANNUITIES – BOON OR BANE FOR CUSTOMER AND INSURANCE COMPANY?

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Abstract

This paper explores how the requirement of gender-neutral premium calculation and the new rules of surplus participation for PLA affect insurance company profitability and policyholder's wellbeing. Data for annuity prices in the German market show an implied gender mix of about 70% females and 30% males. In addition, we develop a realistically calibrated asset and liability model for a participating life annuity with stochastic mortality, interest rates, and equity returns. We show that for males the potential disadvantages of unisex pricing derived from premium or benefit comparisons for participating life annuities are substantially lower if we analyze their lifetime utility by taking also into account stochastic mortality and distributed surpluses.

1. INTRODUCTION

Participating payout life annuities (PLAs) are one of the most important products in the German life insurance market. Also called with-profits annuities, PLAs provide the annuitants with a guaranteed lifelong payment stream and an additional non-guaranteed annual surplus, which depends on the insurer's overall experience on mortality and investment returns. Both the pricing of guaranteed benefits as well as the mechanics of sharing surpluses between policyholders and the life insurance company are highly regulated. Recently, important changes in the insurance legislation and regulation have impact on the attractiveness of PLA, both for the annuitants and the life insurance companies. Besides new rules concerning surplus determination and distribution, recorded in the Life Insurance Reform Act (LIRA, LVRG in German), an important challenge emerged in December 2012 with the requirement of gender-neutral (unisex) pricing of annuities offered in the European Union. Unisex pricing imposes on the insurer the risk of gender mix miscalculations. The assumptions about the mix of males and females for pricing guaranteed benefits could be different from the realized male/female composition in the cohort of annuitants. Especially a situation when more than expected females enter the pool could have negative impact on the insurer's stability. Yet, if the insurance company assumes too high fractions of females to calculate premiums, the benefit stream is less attractive for males because of their lower life expectancy. While for an annuity with fixed lifelong benefits these implications are obvious, they are less clear for the system of non-guaranteed surpluses connected with PLA.

This paper explores how the requirement of gender-neutral premium calculation and the new rules of surplus participation for PLA affect both parties of the insurance contract: the insurance company profitability and the utility for the policyholder. We conduct an empirical analysis of implied assumptions on the male/female mix based on data for annuity prices offered in the German market. In addition, we extend prior research by Maurer, Rogalla, and Siegelin (2013) and develop a realistically calibrated asset and liability model for a participating life annuity with reserve and premium calculation using unisex-mortality tables. The model permits stochastic individual and systematic mortality patterns, uncertain interest rates, and equity returns. Such a model allows us to study the impact unisex pricing on insurers profitability and policyholders wellbeing.

Prior research on gender-neutral pricing lives off the idea of adverse selection in insurance markets developed by Rothschild and Stiglitz (1976). They showed that for an insurance pool consisting of high risk and low risk individuals there will be no equilibrium in the market. Rea (1987) theoretically investigates the market reaction to the ban on sex-specific life annuities and shows that such a ban leads to the redistribution of wealth. Finkelstein, Poterba, and Rothschild (2009) apply Rea's analysis empirically on the UK market for compulsory retirement annuities and find that more than 3% of retirement wealth is redistributed from men to women. The authors state that welfare implications, modest in their research for compulsory market research, can amplify for voluntary markets, where individuals may choose not to annuitize at all. This is consistent with earlier work by Finkelstein and Poterba (2002) stating that adverse selection in compulsory markets is considerably lower than in voluntary markets. Gaudecker and Weber

(2006) study the effects of abolishing the gender-specific annuity pricing on the example of German Riester plans. They find considerable decrease in the annuity payouts for men coupled with only slight rise in the payouts for women, and predict strong adverse selection effects. The implementation of gender-neutral pricing is not completely new, shows contrarian development in different countries, but cannot lean on long-term payout experience for products and regulations in their contemporary structure. Surprisingly, it was only in 1986 that the supervisory authority in Germany required the differentiation between males and females in the life insurance calculation.[1] US legislators, in contrast, commanded in the 1980-ies just the opposite: the gender-neutral pricing for both employer-sponsored and non-employer-sponsored insurance plans. Switzerland, the European pioneer in funded employment - linked pensions, always collected gender-neutral contributions, but starts with gender-neutral payouts only in 2014[2]. Sweden adopted gender-neutral pricing for its new statutory and mandatory occupational products; however, the programs started only 1994 and cover generations still far away from retirement (Palmer, 2000). In the UK annuities, bought from the state contribution, have to be priced on the gender-neutral basis.

As to our best knowledge, the utility and the shortfall influence of gender-neutral pricing during annuities' payout phase has not been analyzed. Our contribution fills this gap: In our paper, we focus on immediate participating payout life annuities. We look at the policyholder's utility, insurer's returns and stability. Within this framework, we investigate different assumptions for gender-neutral calculations and their realizations as well as the effects caused by different surplus allocation amounts. Our procedure is as follows: In the next section we discuss the problem of gender-neutral pricing, guaranteed interest rates and longevity in the context the German Life Insurance Reform Act. We provide a descriptive analysis of the observed benefit changes caused by conversion to unisex pricing and introduce our research questions. Section 3 explains our model and calibration. Section 4 describes the results from annuitants' and insurers' perspective. The final section concludes and presents a lookout for future research.

2. Challenges of gender-neutral pricing and the regulatory framework for surplus sharing

Life insurance products are equipped with a series of very essential guarantees. Two most important ones are interest rate and mortality guarantees. The interest rate guarantee is represented by the interest rate (GIR) used to discount the future payments to the policyholders. The mortality guarantee means that the mortality assumptions are fixed at the time the annuity contract is signed and remain the same during the whole lifetime of the contract. These two guarantees substantially influence retirees' utility resulting from an annuity contract. Both guarantees can be valuable for the policyholder on the one hand. On the other hand, however, they are risky for the insurance company.

[1] See Claus, Der Geschäftsplan für die Großlebensversicherung, VerBAV, 1986, 242.
[2] Bundesamt für Sozialversicherungen, Faktenblatt 19, 2010.

During times of serious capital market and economic changes, such as enduring phase of low interest rates, inadequate interest rate guarantees may endanger the stability of the insurer and thus the insured's benefit payments.

Guaranteeing the use of the same mortality table during the whole duration of an insurance contract represents a mortality guarantee and poses a risk for the insurer. The introduction of unisex pricing divides the mortality risks into two groups: longevity risk and gender mix risk. Longevity risk consists in failing to forecast the mortality developments in the pool of insureds correctly, gender mix risk describes the situation if the gender composition assumed for the calculation of the premiums and gender mix realized in the insured pool differ.

The annuity in the payout phase is a product influenced by several risk categories. Besides longevity risk and gender mix risk the impact of the interest rate is very high. The purpose of the regulatory framework for surplus sharing is to adjust the entitlement of policyholders in earnings of the insurance company according to the sources of surplus. The result of an insurance company's calculation and longevity/gender mix risk is reflected in the mortality surplus, while the relationship between profits from capital markets and guarantee undertakings determines the asset return. Due to this risk accumulation effecting the insurer's liabilities participating annuities become an interesting product for scientific research. To this aim we investigate immediate participating life annuities offered at the German market in our further analysis.

2.1 Gender Mix Risk

In accordance with the EU Directive (2004/113) and the Ruling of the European Court of Justice (Case C-236/09) gender-neutral pricing is mandatory in the European Union since December 21th 2012[3]. This applies for life and non-life insurance products obtainable in the public markets.

The implementation of unisex pricing can cause adverse selection: In a population with an identical number of high and low risk individuals more high risk individuals will be in favor of purchasing unisex products, while low risk individuals will abstain from doing so. The immediate participating life annuities (PLAs) consider females to be the high risk individuals due to their higher life expectancy. Usually, all mortality estimations were gender-based, as well as the mortality tables used for pricing insurance products. Obviously, in case of unisex pricing, insurers cannot choose the easy path of using only one of the existing gender-based tables. They have to make assumptions about the gender composition of their pool of insured's and insurance companies might face the possibility of adverse selection when choosing a gender mix. Thus, the introduction of unisex pricing exposes the insurers to an additional risk.

In order to show to what extent unisex pricing may be influenced by the gender mix assumptions, we aim to calculate the gender mix implied in the calculation of PLA providers on the German

[3] EU Directive 2004/113 implements the principle of equal treatment between men and women in the access to and supply of goods and services, Ruling of the European Court of Justice (Case C-236/09, Test Achats) makes it illegal to differentiate by sex in calculating the insurance premiums. This Directive does not apply to the products, where the employer is the contract party. The Directive does not prohibit the collection and storage of information on gender.

annuity market and analyze the effects of alternative gender compositions on the insureds and the insurer.

To start with, we collect market prices for immediate PLAs before and after conversion to unisex tariffs. Our data are generated by the comparison platforms for policy brokers provided by Morgen&Morgen[4] and Fond Finanz[5]. These comparison platforms calculate premiums or benefits for given gender and age as well as product characteristics. The vast majority of the tariffs currently in use by annuity providers in German insurance market are represented in our data.

We gather market premiums for 67 year old males and females purchasing an immediate PLA in December 2012 and in January 2013. In both cases the contract starts in January 2013 and a single premium of 100,000€ is paid. Table 1 presents the minimum, median and maximum market PLA benefits without rebates for male, female, and unisex tariffs based on 42 gender-specific and 11 gender-neutral observations.

Table 1 here.

Before conversion to unisex pricing, female annuitants received on average 47€ less than male annuitants for the same single premium. Since the introduction of unisex tariffs both genders receive same monthly benefits with a median of 390.24€. On average, the introduction of unisex tariffs led to a benefit reduction of 7.1% for males and benefit increase of 4.5% for females.

Transition to unisex pricing lowered the spread of the offers: The differences in benefit of about 4% between the median and minimum/maximum offers are relatively small, resulting in absolute differences of less than 20€ per month.

To derive the gender mix implied by the observed market offers we compare market PLA benefits to the actuarially fair benefits which we calculate for both genders according to formula (A1) in the appendix.

Table 2 here.

Table 2 compares the median market PLA benefits for males and females to the actuarially fair benefit. The actuarially fair monthly benefit for an immediate PLA with an initial single premium of 100,000€ for a 67 year old male and female, both born in 1947, is 449.22€ and 400.56€, respectively.

The comparison to the actuarially fair benefits shows a relative difference of 6.5% for males and 6.8% for females, which should account for respective market cost loads. The statistics of the German Insurance Association (GDV)[6] reports that administrative and sales costs in 2011 are 6.6% of the premium paid, which is very close to the cost loads implied in the market PLA benefits.

Thus, for gender-neutral tariffs we derive the implied gender mix assuming market consistent cost loadings of 6.6%. To obtain the actuarially fair market unisex offer for 100,000€ premium, we

[4] See https://mumoffice.morgenundmorgen.de
[5] See http://portal.softfair.de/softfair/lv_lotse.html
[6] See GDV, Life insurance in Numbers, 2012, pages 7 and 26.

adjust the median unisex market benefits of 390.24€ from the Table 1 by cost loadings and receive a unisex market benefits without costs of 416€. For the same premium amount of 100,000€, the absence of cost loadings results in the benefit increase of almost 26€.

Next, we calculate the actuarially fair unisex benefits for alternative female/male mixes. We shift the gender mix in 10 percentage point steps starting with 0% females, which means a gender-specific calculated annuity for men, up to 100% females implying a gender-based calculation for women, see Table 3.

Table 3 here.

Finally, we compare the results of the calculated alternative gender mixes to the market PLA benefit without costs, 416€, and find that a gender mix of 70% females and 30% males (corresponding to the actuarially fair benefits of 413.87€) has the lowest deviation to the actuarially fair market offer of only 51 bps.

The derived gender mix does not correspond with the reported empirical data. In the portfolio of policies of UK life insurances the inverted gender mix is observed: Here, a gender split of approximately 65% males and 35% females is reported.[7] These numbers are similar to the reported gender mix of Germans with corporate pensions. According to the German Federal Ministry for Labor and social affairs (2011) 64% of individuals with occupational pensions in form of direct insurance are males. Therefore, the assumed gender mix used for pricing unisex annuities in Germany, and the empirically feasible gender mix do not match. In the following chapters we aim to analyze the impact of this new risk category, the risk of miscalculation of the gender mix, on the insured's utility and the return and shortfall risk of annuity providers.

2.2 GIR and Longevity Risk within the Context of Surplus Sharing Regulations

A traditional participating annuity locks a certain interest rate for the whole duration of the contract. This rate is called guaranteed interest rate (GIR) and is used to calculate the annuity premium. In Germany the determination of the GIR by the life insurer is restricted: The limit is set by German Ministry of Finance, and is usually determined as 60% of the average yield of government bonds over the last 10 years. Therefore, the development of the current yield of government bonds can be taken as an prediction for a forthcoming reduction or an increase of the GIR.

Figure 1 shows the development of the guaranteed interest rate from 1994 to 2013 as well as the net invest return of German insurance companies and the current yield of government bonds. The net investment return of an insurance company depends on the periods' coupon payments for bonds, dividends for stocks and other income streams as well as on the profit or loss from selling assets reported by amortized costs.

Figure 1 here.

[7] Anderson, D. and Grimshaw, Consultation, 2012, page 8.

From 1994, the first year after the deregulation of the German insurance market, until now, the GIR was reduced stepwise from 4% to 1.75%. The net investment return of German insurance companies follows a similar trend, although it exceeds the GIR for the whole observation period. The difference between the net investment return and the GIR declined.

The current yield of government bonds experienced a sharp decline since 1994. Taking a closer look on the development of the current yield on government bonds and the GIR, we notice that each time the current yield drew near or even fell below the GIR, the latter was reduced shortly afterwards. This situation occurred for example in 2010: The GIR was reduced in January 2012 from 2.25% to 1.75%. Except since this last reduction, the current yield of government bonds never reached the level of the GIR since 1999. Consequently, there was a further decrease of the GIR in January 2015 to a guaranteed interest rate of 1.25%.

In such case, the guaranteed benefits for an immediate participating life annuity for the same single premium will drop considerably: A reduction in the GIR of 0.5% will reduce the benefits by about 6%. Such reduction will be quite noticeable to the customers.

Another risk connected with PLAs is longevity risk which is incorporated in the mortality tables used. For life annuity products in the payout phase the hazardous side of the mortality risk from the insurer's perspective is increased life expectancy. Longevity risk can be separated into two components, systematic and idiosyncratic longevity. Idiosyncratic longevity describes the variation risk of a single individual which the insurer handles by pooling annuity contracts. Systematic longevity cannot be reduced by pooling. Here, the insurer is exposed to a change in the average life expectancy. To deal with this kind of risk, insurers modify mortalities by adding risk loadings, and create risk specific mortality tables. Currently, the "DAV 2004 R" mortality table is used by the vast majority of the German annuity providers for pricing annuities. For the product where the hazardous side of the mortality risk is the reduced life expectancy, the term life insurance, the mortality table "DAV 2008 T" is used for pricing. Both tables are derived by the German Actuarial Society. In the annuity table "DAV 2004 R", a male (female) aged 67 in 2010 has a life expectancy of 22.7 years (26.3 years), which is more than 6 years (7 years) longer than the population average of 16 years (19 years), see Table 4. According the term life table "DAV 2008 T", a male (female) individual aged 67 has a remaining lifetime of 14.9 years (18.1 years), 1.1 years (0.9 years) less than the average population mortality.

Table 4 here.

To predict mortality developments, mortality rates are forecasted and the methods improve over time: While the former annuity tables "DAV 1994 R" used an age shift to forecast future mortality and depended on age and gender, the annuity tables "DAV 2004 R" use a trend function to forecast the development of mortality rates by also using the year of birth.

From the insurer's perspective long term guarantees like interest rate and mortality guarantees are risky and costly. Risky, as an improper calculation of guarantees reduces the insurer's return and may endanger its stability. Costly, because Solvency II capital requirements put additional loadings on the insurer as long term guarantees embedded in life insurance products have to be valued. In contrast, from the annuitant's perspective interest and mortality guarantees are valuable as they increase their utility. Whereas seen from the annuitant's perspective interest and mortality guarantees are valuable as they increase in their utility.

The new regulations adopted with Life Insurance Reform Act (LIRA/LVRG) indicate the essential relevance of the abovementioned issues. These new regulations apply to the participation in the unrealized reserves, distribution of dividends, specification of a profitability indicator, accounting of acquisition costs, reduction of the guaranteed interest rate and profit distribution. We focus on the latter two items which have a big potential of interesting statements and comparisons. As mentioned above the guaranteed interest rate is reduced from 1.75% in 2014 to 1.25% in 2015. Concerning surplus sharing the amount of minimum allocation to the policyholders is determined by percentages of the three sources of return: Asset returns, mortality returns and other returns.

Figure 2 here.

Figure 2 gives and overview of the Minimum Surplus Allocation Decree before and after the LIRA implementation. For mortality return, the minimum profit sharing requirement increased from 75% to 90%. Moreover, the new act offers the possibility of offsetting negative asset returns by positive mortality or other returns. Summing up all three positions delivers the minimal surplus allocation in the current year. The change in this legal act can be seen as a balancing act between customer's requirement and insurance company's risk-bearing capacity.

In what follows, we extend the existing research to the payout phase from the perspective of the annuity purchaser in terms of annuitant's utility and the insurer providing the annuity by examining the internal rate of return and the shortfall probability.

To study the effects of deviations in the actual and assumed gender mix as well as the effects of changes in legal surplus allocation rules, we develop a full-fledged, realistically calibrated stochastic asset and liability model for a German life insurer. This stylized company sells only one product, a single-premium participating life annuity or a fixed life annuity, to a specific cohort of equal individuals that are exposed to capital market and longevity risks.

3. MODEL AND CALIBRATION

3.1. CAPITAL MARKET AND MORTALITY MODEL

Following the framework developed by Maurer, Rogalla, and Siegelin (2013) the asset portfolio of our life insurance company consists of stocks and bonds. The bond prices depend on the development of the term structure of interest rates which in turn is driven by a multi-factor CIR model as described in Chen and Scott (1993).[8] The bond in our model is a coupon bond as the insurance company has to earn at least the guaranteed interest rate each year. Therefore, the bond price B_t^T at time t with fixed maturity T is calculated as

$$B_t^T = B_0 \cdot \left[\sum_{k=t+1}^{T} c_t^T \cdot \exp\big(-R(t,k-t)\big) + \exp\big(-R(t,T-t)\big) \right], \tag{1}$$

[8] For further details see Appendix B of Maurer et al. (2014).

where B_0 is the face value, c_t^T is the constant coupon rate over T and $R(t,\tau)$ is the τ-period spot rate at time t. The coupon rate is set at the issuance of the bond; it depends on the current term structure and is given by

$$c_t^T = \frac{1 - \exp\left(-R(t, T-t)\right)}{\sum_{k=t+1}^{T} \exp\left(-R(t, k-t)\right)}, \quad (2)$$

with $B_0^T = B_T^T = B_0$.

Alongside with bonds, the insurance company invests in dividend paying stocks. Stock prices S_t and dividends D_t evolve according to the stochastic process described in Appendix B of Maurer, Mitchell, Rogalla and Siegelin (2014).

The calibration of the term structure model depends on historical spot rates of German Federal Securities with 1 to 10 year maturity. We use data provided by Deutsche Bundesbank over the period January 1988 to September 2013.[9] Setting $i = 3$ in the i-factor CIR model provides the best fit to the data and the calibration approach produces the following parameter estimates:

Table 5 here.

The development of the stock prices and dividend rates relies on DAX Total Return Index and DAX Price Index over the same time interval as the term structure calibration. This results in the following estimates: the expected risk premium (μ^{RP}) is equal to 0.9%, the volatility parameter (σ^{RP}) is equal to 26%, and the fixed dividend (μ^{D}) is 2.5%.

As in Maurer, Rogalla and Siegelin (2013) the asset allocation of bonds and stocks follows a constant mix strategy. Hence, every year the asset portfolio of the insurer is adjusted. The reallocation of portfolio occurs when assets are sold to pay benefits to the annuitants. The insurer sells a higher bond percentage than specified in the constant mix strategy if the bonds' fraction of the total asset portfolio is bigger than the targeted share.

To forecast annuitants' actual mortality rates q_x^P we follow the approach of Cairns, Blake and Dowd (CBD, 2006).[10] We use German mortality data from the Human Mortality Database to calibrate the CBD model which leads to point estimates:[11]

Table 6 here.

3.2. PLA PRODUCT DESIGN AND COMPANY MODEL

The business of our stylized life insurance company is selling immediate PLAs which provide benefit payments for the remaining lifetime of the annuitant. The total annual benefits paid to the annuitant include guaranteed benefit payments as well as additional non-guaranteed surpluses.

[9] Specifically, we use the following time series: WZ9808, WZ9810, WZ9812, WZ9814, WZ9816, WZ9818, WZ9820, WZ9822, WZ9824, WZ9826. Available at: http://www.bundesbank.de.
[10] For further details see Appendix B of Maurer et al. (2014).
[11] Specifically, we use the German Life Tables (period 1×1) for Males and Females; last modified: July 12, 2013, version MPv5 for the period 1990–2011. See http://www.mortality.org.

The premium P of an annuitant of age x depends on the initially guaranteed benefit payments BP_0 and an annuity factor (expression in parentheses), that is given by

$$P = BP_0 \cdot \left(\sum_{k=0}^{\omega-(x+1)} \frac{{}_k p_x^A}{\prod_{j=0}^{k}(1+i_j)} \right), \quad (3)$$

where ${}_k p_x^A = \prod_{j=0}^{k-1}\left(1 - q_{x+j}^A\right)$ is the k-period survival probability at age x, q_x^A are the actuarial mortality rates taken from "DAV 2004 R" tables recommended by the German Actuarial Society, ω is the terminal age of this mortality table and i_j is the interest rate guaranteed at year j. We do not consider explicit costs in terms of loading in our model.

The insurer's initial actuarial reserve V_0 corresponds to the total premium income $V_0 = P \cdot I_0$. Initially, we assume I_0 annuitants of the same age purchasing the annuity in the same year with identical interest rate and mortality assumptions. In the following years the guaranteed benefit, the annuity factor as well as the number of annuitants change. The actual guaranteed benefit BP_t can increase over time as we adopt surplus annuitization as participation scheme. Here, the additional surplus SP_t becomes part of the guaranteed benefits for the subsequent years. Therefore, the actual guaranteed is calculated as follows:

$$BP_t = BP_{t-1} + SP_t. \quad (4)$$

The insurer's actuarial reserve V_t develops according to

$$V_t = I_t \cdot BP_t \cdot \left(\sum_{k=0}^{\omega-(x+1+t)} \frac{{}_k p_{x+t}^A}{\prod_{j=0}^{k}(1+i_j)} \right), \quad (5)$$

where I_t represents the number of annuitants which are alive at time t and is given by

$$I_t = \sum_{i=1}^{n} I_t^i. \quad (6)$$

I_t^i is an indicator variable; it is equal to 1 if the annuitant i ($i = 1, \dots, n;\ n = I_0$) is alive at time t and it is 0 if the annuitant is dead. The sequence of indicator variables I_t^i forms a Markov chain for each annuitant i with

$$\begin{aligned} P\left(I_{t+1}^i = 1 \middle| I_t^i = 1\right) &= 1 - q_{x+t}^P = p_{x+t}^P, \\ P\left(I_{t+1}^i = 0 \middle| I_t^i = 1\right) &= q_{x+t}^P, \\ P\left(I_{t+1}^i = 0 \middle| I_t^i = 0\right) &= 1, \end{aligned} \quad (7)$$

where q_{x+t}^P is the actual mortality rate of annuitants of age x at time t.

To determine the additional surplus SP_t we apply the procedure described in Maurer et al. (2013). First, we calculate the company's total annual surplus (*surplus determination*). Subsequently, surpluses are allocated to policyholders and shareholders *(surplus allocation*). Finally, to smooth the policyholders' annual surplus payout, allocated surpluses are distributed to the committed and uncommitted provision for premium refunds *(surplus distribution)*.

Surplus Determination: The total annual surplus TS_t of the insurance company is determined as

$$TS_t = MR_t + AR_t - IR_t, \tag{8}$$

where AR_t is the asset return, MR_t is the mortality return and IR_t is the interest applied to the actuarial reserve. The annual mortality return MR_t which results from the difference of actual and assumed mortalities is calculated as

$$MR_t = V_{t+1} \cdot \left(\frac{l_t - l_{t+1}}{l_t} - q_{x+t}^{A} \right). \tag{9}$$

The asset return AR_t is given by

$$AR_t = (V_t - l_t \cdot BP_t) \cdot i_t^{AR} \tag{10}$$

and the interest on the actuarial reserve develops as follows:

$$IR_t = (V_t - l_t \cdot BP_t) \cdot i_t. \tag{11}$$

Here, i_t is the guaranteed interest rate at year t and i_t^{AR} is the realized net investment return of the bond-stock portfolio. The net investment return is calculated as the sum of all dividend payments, coupon payments, and realized gains/losses due to sale of assets. This sum is divided by the book value of invested assets at the beginning of the year. Formally, the net investment return is given by

$$i_t^{AR} = \frac{\alpha_{S,t} D_t + \beta_{S,t}(S_t - S_0) + \alpha_{B,t} c_t^T B_0 + \beta_{B,t}(B_t^T - B_0)}{\alpha_{S,t}\, S_0 + \alpha_{B,t}\, B_0}. \tag{12}$$

In the upper formula, $\alpha_{S,t}$ ($\alpha_{B,t}$) is the number of stocks (bonds) held in year t and D_t ($c_t^T B_0$) is the dividend (coupon) payment for each stock (bond). $\beta_{S,t}$ denotes the number of stocks sold at market price S_t resulting in a realized gain or loss with respect to the purchase price S_0 of $\beta_{S,t}(S_t - S_0)$. The realized gain or loss from selling $\beta_{B,t}$ bonds at market price B_t^T relative to a book value of B_0 is given by $\beta_{B,t}(B_t^T - B_0)$. In case the company holds stocks/bonds with different book values, assets are sold according to the FIFO rule.

Surplus Allocation: After the total annual surplus TS_t is determined it has to be distributed between policyholders and shareholders. This surplus allocation depends on several constraints to equity and solvency capital as well as on mortality and asset return which we present in the following.

If the insurer meets the solvency requirements, a fixed percentage ap of the total annual surplus is allocated to policyholders in case this amount exceeds the regulatory minimum. The schematic regulatory minimum requirements before and after validity of the Life Insurance Reform Act are presented in Section 2.2. According to the old regulation policyholders are eligible for at least 75% of mortality surplus and 90% of asset surplus and they do not participate in negative returns. Moreover, offsetting negative returns in one category by positive returns in other categories is prohibited. Introduction of LIRA entitles policyholders to at least 90% of mortality and asset surplus as well as negative asset returns can be offset against positive mortality returns (as we dispense costs). In case the insurer does not meet solvency requirements but still has equity capital only the regulatory minimum surplus is allocated to policyholders. If there is no equity capital left the complete surplus is kept by the annuity provider.

Therefore, the total allocated surplus AS_t to policyholders before validity of LIRA is calculated as

$$AS_t = \begin{cases} \max(\max(0.75 \cdot MR_t, 0) + \max(0.9 \cdot AR_t - IR_t, 0), ap \cdot TS_t), & \text{if } E_t + UCPPR_t > 0.04 \cdot (V_t + CPPR_t) \text{ and } E_t > 0, \\ \max(0.75 \cdot MR_t, 0) + \max(0.9 \cdot AR_t - IR_t, 0), & \text{if } E_t + UCPPR_t \leq 0.04 \cdot (V_t + CPPR_t) \text{ and } E_t > 0, \\ 0, & \text{if } E_t \leq 0, \end{cases} \tag{13}$$

and after introducing LIRA

$$AS_t = \begin{cases} \max(\max(0.75 \cdot MR_t, 0) + \max(0.9 \cdot AR_t - IR_t, \min(AR_t - IR_t, 0)), ap \cdot TS_t), & \text{if } E_t + UCPPR_t > 0.04 \cdot (V_t + CPPR_t) \text{ and } E_t > 0, \\ \max(0.9 \cdot MR_t, 0) + \max(0.9 \cdot AR_t - IR_t, 0), & \text{if } E_t + UCPPR_t \leq 0.04 \cdot (V_t + CPPR_t) \text{ and } E_t > 0, \\ 0, & \text{if } E_t \leq 0, \end{cases} \tag{14}$$

where E_t is equity capital, $UCPPR_t$ is uncommitted provision for premium refunds (PPR) and $CPPR_t$ is committed PPR. As long as solvency requirements are met, shareholders receive an annual dividend of 2.5% of the current equity capital at the end of each period. In our results we will concentrate on the regulations before validity of LIRA and draw a short comparison to a setting after introduction of LIRA for each finding.

Surplus Distribution: In the next step, the total allocated surplus AS_t is distributed to the committed and the uncommitted PPR which are special items in the life insurer's balance sheet. Surpluses assigned to the $CPPR_t$ are distributed to the annuitant in the following year while surpluses assigned to the $UCPPR_t$ are not distributed immediately. The $UCPPR_t$ is a collective buffer account, which is used to store remaining surpluses or withdraw required surpluses.

The distribution is performed by an optimization algorithm that is aiming to smooth surplus distribution over time maintaining a sufficient buffer account. To this end, the following objective function is maximized:[12]

$$\max \; g(UCPPR_{t+1}) + f(CPPR_{t+1}) \tag{15a}$$

[12] In contrast to the smoothing algorithm used in Maurer et al. (2014), the exponents in equation 15b are increased and two additional constraints, equation 15d and 15e, are included in the optimization problem.

$$g(UCPPR_{t+1}) = -\left(\frac{UCPPR_{t+1}}{UCPRR_{aim}}\right)^6 + 6\cdot\left(\frac{UCPPR_{t+1}}{UCPRR_{aim}}\right) - 4 \tag{15b}$$

$$f(CPPR_{t+1}) = -\left(\frac{CPPR_{t+1}}{CPPR_t^{adj}} + \frac{CPPR_t^{adj} - AS_t}{CPPR_t^{adj}}\right)^2 + 2\left(\frac{CPPR_{t+1}}{CPPR_t^{adj}} + \frac{CPPR_t^{adj} - AS_t}{CPPR_t^{adj}}\right) \tag{15c}$$

$$UPPRR_{aim} = \min\left(u_{aim}\cdot\ {}_tV_x\,;\ UCPPR_{reg}\right) \tag{15d}$$

$$UCPPR_{reg} = \max\left(UCPPR_{t+1} + CPPR_{t+1};\ \sum_{i=0}^{2} AS_{t-i}\right) - CPPR_{t+1} \tag{15e}$$

$$\frac{CPPR_{t+1}}{CPPR_t^{adj}} \in \left[\frac{1}{u}\,;u\right], u \geq 1 \tag{15f}$$

$$CPPR_t^{adj} = CPPR_t\cdot\frac{I_t}{I_{t-1}} \tag{15g}$$

$$UCPPR_{t+1} = UCPPR_t + AS_t - CPPR_{t+1} \geq 0. \tag{15h}$$

The algorithm has a prospective and retrospective view. The prospective view is contained in a constraint (equation 15b) which targets to maintain a sufficient buffer account. The function g reaches its maximum if the uncommitted PPR equals a specified percentage u_{aim} of the actuarial reserve ${}_tV_x$ provided that the maximum uncommitted PPR, $UCPPR_{reg}$, is not exceeded (equation 15d). The regulator designates the maximal sum of uncommitted and committed PPR as the sum of the total allocated surplus over the three previous years. Therefore, the $UCPPR_{reg}$ is limited in equation 15e to this maximal amount. Both polynomial functions, f and g, reach their maximum when the expression in the parentheses is equal to one.

The function f, the retrospective constraint (equation 15c), aims to keep the surplus distribution to the annuitants stable regarding the current total allocated surplus. The fraction $CPPR_{t+1}/CPPR_t^{adj}$ is equal to one if the actual distributed surplus is on the same level as in the previous year, where $CPPR_t^{adj}$ is the last period's distributed surplus adjusted by the deceased individuals $\frac{I_t}{I_{t-1}}$. In addition, the fraction $CPPR_{t+1}/CPPR_t^{adj}$ is only allowed to move within a certain range (equation 15f). The second term $\left(CPPR_t^{adj} - AS_t\right)/CPPR_t^{adj}$ penalizes (rewards) withholding surplus by the insurer if the total allocated surplus exceeds (falls behind) the surplus distributed to the annuitants in the previous period.

To determine the amount by which to increase the guaranteed benefits, the additional surplus SP_t, the annuitant's individual share of the committed PPR is annuitized:

$$SP_t = \frac{CPPR_t}{I_t\cdot\left(\sum_{k=0}^{\omega-(x+1+t)}\frac{{}_kp_{x+t}^A}{\prod_{j=0}^{k}(1+i_j)}\right)}. \tag{16}$$

Finally, the remaining profits, $TS_t - AS_t$, are distributed to the company and the insurer's equity capital E_{t+1} develops according to

$$E_{t+1} = \begin{cases} \left(E_t \cdot \left(1 + R(t,1)\right) + TS_t - AS_t\right) \cdot (1 - \mu^D), & \text{if } E_t + UCPPR_t > 0.04 \cdot (V_t + CPPR_t), \\ E_t \cdot \left(1 + R(t,1)\right) + TS_t - AS_t, & \text{else,} \end{cases} \quad (17)$$

where $R(t,1)$ is the 1-year government bond spot rate, and μ^D is a fixed dividend rate paid to the shareholders. The dividend is paid if the insurance company has enough solvency capital.

3.3. Utility Equivalent Fixed Life Annuity

To determine the value a PLA is delivering to annuitants we calculate the utility equivalent fixed annuity using a time additive CRRA utility function. The expected lifetime utility is given by

$$U = E\left(\sum_{t=0}^{\omega-x} \beta^t \;{}_t p_x^P \; \frac{{BP_t}^{(1-\gamma)}}{1-\gamma}\right), \quad (18)$$

where ${}_t p_x^P$ is the unisex survival probability of an individual of age x, γ is the coefficient of relative risk aversion and $\beta < 1$ is the individuals' subjective time preference. To calculate the gender mix survival probability ${}_t p_x^P$ we apply the same method as described in Appendix A. Following Maurer et al. (2013) the expected life-time utility U is transformed into a utility equivalent fixed annuity EA:

$$EA = E\left[\frac{U\,(1-\gamma)}{\sum_{t=0}^{\omega-x} \beta^t \;{}_t p_x^P}\right]^{\frac{1}{(1-\gamma)}}. \quad (19)$$

The EA can be interpreted as the constant guaranteed lifelong income stream the annuitant requires to give up the upside potential of a PLA with uncertain surpluses.

4. Results

4.1. Setup

In this section we first analyze the impact of alternative assumed and realized gender compositions on annuitant's utility as well as the role of surplus participation. Second, we take the insurer's perspective by analyzing the insurance company's risk and return for different pricing constellations and gender compositions. To this end, we simulate 5.000 independent sample paths for a cohort of 10,000 individuals, aged 67 in 2013 who purchase an immediate participating life annuity (PLA) for 100.000€ single premium.

For each year, guaranteed benefits as well as the actuarial reserve are calculated using a fixed guaranteed interest rate and annuitant mortality tables "DAV 2004 R". In the case of a PLA the insurer pays an annual surplus as described in the previous section. In the first year the surplus paid to the annuitant is set to 2% of the guaranteed benefit. In general, the surplus allocation parameter, which specifies the level of the annuitant surplus participation, is set to $ap = 0.9$ and the boundary parameter u is set to 1.25.

We calculate the asset and liability side of the company's balance sheet. The liability side includes equity capital, actuarial reserve, committed and uncommitted PPR. Our company is initially endowed with an equity capital of 1.9% of the actuarial reserve, which is in line with the German insurance industry. As it is common to distribute surpluses to the annuitants with a PLA in the first year, we set the committed PPR to 2% of the actuarial reserve. Moreover, the uncommitted PPR initial has 50% of its target value. The target value of the uncommitted PPR is double GIR plus 2%. The actuarial reserve contains the premiums collected from the initial cohort of 10,000 annuitants.

The actuarial reserve and the committed PPR add up to the insurer's asset portfolio. For all simulations the portfolio has a fixed asset allocation of 10% stocks and 90% bonds with an initial maturity of 10 years. The uncommitted PPR and the equity capital are invested in a cash account which bears interest of the one year risk free rate. Shareholders receive an annual dividend of 2.5% of the current equity capital at the end of each period.

The equivalent FLA which provides the same lifetime utility as the PLA is calculated based on our simulation results. Hereby we assume a relative risk aversion $\gamma = 5$ and a time preference factor $\beta = 0.96$.

4.2. ANNUITANT'S PERSPECTIVE

We first investigate the miscalculation of the gender mix from the perspective of an annuitant. To this end we analyze the utility equivalent FLA. On the one hand, we vary the gender mix used for pricing the immediate single premium PLA in 10% steps from 0% females to 100% females. On the other hand, we also vary the realized gender mix of the cohort in 10% steps from only males to only females. Combining both variations in our model leads to our findings. Considering the effects from the annuitant's perspective separately for males and females, we discover that the gender-induced deviation of the utility equivalent FLA for all considered cases is about one percent, meaning that the values for males are always approximately one percent higher than for females. For that reason we present and discuss in detail only our results for males. Figure 3 illustrates male's utility equivalent FLA for unisex calculation as well as a gender-based calculation.

Utility is measured as the benefit a fix life annuity must pay to generate the same utility for an annuitant as an immediate PLA including surplus participation.

Figure 3 here.

Figure 3 depicts the amount of annuitant's utility equivalent FLA by lines. Dots represent an equal gender mix for pricing and actual realization. This means utility equivalent FLAs on the left of each dot (except for 0% females) are calculated with a higher percentage of women than realized. To the right of each point the assumed female share is lower than in the actual cohort. The annuitant's utility equivalent FLA is the higher the lower the female percentage used for pricing. Consequently, the utility equivalent FLA for men, which means 0% females in pricing, is the highest. This remains true for any share of females in the actual cohort. The annuitant's utility equivalent FLA, however, depends not only on the initial irreversible pricing percentage of females, but also on the share of females in the actual cohort. For any given female pricing percentage, the increase in the realized female share lowers the annuitant's utility equivalent FLA. For example, if we assume pricing based on 40% females, the annuitant's utility equivalent FLA is higher than 5.8 thousand € in case there are no females in the actual cohort. For a female share of 100% in the actual cohort, the utility equivalent FLA is approximately 5.7 thousand €. Here, we see the effects of total adverse selection, which literature describes as merging two diverse risk categories with no possibility to differentiate between them. As expected, the male utility is slightly lower within this range.

Our analysis also illustrates, that the same utility equivalent FLA can be achieved for two alternative constellations: Pricing a relatively high and realizing a relatively low share of females or pricing a relatively low and realizing a relatively high percentage of females. This means, that it may not always be an outright disadvantage to the annuitant, when a high share of females is assumed in the calculation. In our empirical analysis we show that the market pricing percentage of females is in fact high, on average 70%. A low realized female share compensates, under certain conditions, this disadvantage.

Comparing our result to the utility equivalent FLA after LIRA but with a GIR of 1.75% delivers only negligible utility differences. Clearly, additionally changing the GIR to 1.25% induces higher utility equivalent FLAs (about 4% higher).

As we recognized above unisex calculation is detrimental to men compared to a gender-specific calculation as it generates a lower utility equivalent FLA in all cases. In order to quantify the disadvantage for men and the advantage for women by unisex calculation we calculate the amount a costumer has to pay to reach a certain utility equivalent FLA. On the one hand we are comparing unisex and gender-based calculation, on the other hand the different sexes in a unisex world. Our basis is always a PLA before validity of LIRA with 100,000€ single premium. We solve this numerically by Newton's method.

Figure 4

Figure 4 displays the immediate single premiums a man or a woman has to pay to reach the same utility equivalent FLA in a unisex setting as in a gender-based setting, which is called gender-specific utility equivalent single premium (Panel A). The utility equivalent to reach is calculated with 0% females (males) for pricing and 0%-100% females in the actual cohort. Secondly, the single premiums in order to reach the same utility equivalent FLA in a gender-neutral setting as the opposite sex are shown (Panel B). We call this the gender-neutral utility equivalent single premium. The desirable utility equivalent FLA is calculated for different pricing and realization mixes. An extract of the respective utility equivalent FLAs for men can be found in Figure 3.

Looking at Panel A we find the gender-specific utility equivalent single premium. In other words, we determine the additional amount one has to pay to the annuitant to induce him to accept a unisex calculated annuity instead of a gender-specific annuity for 100,000€ premium while maintaining his or her utility. The three-dimensional graph shows that, in general, males require higher premiums than females, and that, independent of the sex, these premiums decline with the decrease in the female share used for pricing.

The base case for men, that is 100% males are used for pricing, remains constant at 100,000€ for every mix in the actual cohort as in this case the desired utility equivalent FLA is reached. Adding additional females to the pricing cohort causes an increase in the single premium. Considering one pricing mix, the premiums are nearly constant for different actual cohort mix realizations. With every additional 10% females the gender-specific utility equivalent single premium rises by 1% until 10%.

Almost the same magnitudes hold true for women. For the base case here we consider 100% females used for pricing. Again, the single premium stays at 100,000€ for every mix in the actual cohort. Adding more males to the pricing cohort causes a decline in the single premium up to 10% below 100,000€.

The difference between single premiums for men and women is 10% in almost every case. On average a man has to pay 4.8% more than the original premium to enter a unisex calculated PLA and a woman 4.7% less after conversion to unisex in order to reach the same utility equivalent FLA compared to a gender-based calculation. Men's position is worsened by almost the same percentage as females benefit from unisex conversion. In both cases we recognize that varying the actual cohort mix and fixing the pricing mix has the same effect to a gender-based calculation as to a gender-neutral calculation. This becomes clear as the results hardly change for differing female shares in the realized cohort for a certain pricing mix.

Panel B depicts the gender-neutral utility equivalent single premium. The original annuity to determine the utility equivalent FLA for the respective sex is a unisex calculated annuity with a premium of 100,000€. Again, the single premium for men is above and the single premium for woman below 100,000€.

We discover that the difference between the gender-neutral calculated contracts to reach the respective other sex's utility is about 1%. A man has to pay on average 970€ more than a woman to attain the same utility equivalent FLA, whereas a women on average needs 960€ less than a man. The impact on males and females of varying the actual cohort and pricing mix is the same. Due to this the single premiums result in flat planes for different gender mixes. This outcome seems obvious but is complex, as we are considering PLAs including surplus participation. Mortality surpluses have a different influence on the lifetime utility for both sexes depending on the actual cohort.

To summarize, taking the utility equivalent FLA as a basis to report on the one hand the advantages and disadvantages of conversion to unisex calculation and on the other hand the benefits and detriments comparing sexes in a unisex calculation results in an almost symmetric change in opposite directions for males and females in both cases. Examining our empirical market data of benefits for the unisex conversion in Table 1 we find that men's disadvantage considering their lifetime utility, which includes mortality rates and the total annuity with surpluses, is lower than observed in the empirical market data. This means, for a holistic analysis to assess the impact of unisex conversion it is not sufficient to look at guaranteed benefits or single premiums but it is essential to consider the utility that is generated over lifetime.

Table 7 here.

For the sake of completeness, Table 7 reports additional information about the gender-specific utility equivalent single premium according to Figure 4, Panel A. Additionally, we consider the cases after the effective date of LIRA, to enhance comparability, with a GIR of 1.75% and data after introducing LIRA with the actually valid GIR of 1.25%. We recognize that there is only a minimal difference between the values before LIRA and after LIRA with a GIR of 1.75%. This implies that both changes in the regulation, increasing the mortality surplus participation and opening the possibility of cross charging, compensate to some extend for each other. The effect of lowering the GIR is shown by higher single premiums.

4.3. INSURER'S PERSPECTIVE

Having analyzed the effects of unisex calculation on the annuitant we now turn towards the insurance company. The considered parameters from both perspectives are linked as the financial situation of the insurer affects the total pension. We examine insurer's profitability and stability in form of the expected internal rate of return (IRR) and shortfall probability.

The expected IRR in Figure 5 is calculated based on the capital provided by the investors of the insurance company for each simulation run. This initial investment, together with dividend payments at the end of each financial year, as well as the liquidation value remaining when the last annuitant dies, are included in the IRR calculation. The liquidation value consists of the remaining equity capital, the committed PPR, the uncommitted PPR and the actuarial reserve. The shortfall probability in Figure 5 is calculated as the number of events, when the insurer has negative equity relative to all simulations runs. A simulation run is only counted if the number of annuitants in the cohort is positive.

Figure 5.

Panel A illustrates combinations of assumed and realized share of females and their effect on the shortfall probability as well as on the expected IRR of the insurer before validity of LIRA. The circles represent the shortfall probability, the solid lines show the expected IRR. Colors depict different assumed pricing shares. Ceteris paribus, a higher female pricing percentage is associated with a higher expected IRR. Consequently, a gender-based calculated annuity for men has the lowest expected IRR and a gender-specific calculated contract for women the highest one. Our analysis shows, however, that within a realistic IRR range, a given expected IRR can be achieved with different combinations of assumed and realized shares of females. For a given expected IRR, a higher pricing percentage of females can tolerate a higher realized female share in the actual cohort: An IRR of 3.7% can be achieved without females in the realized cohort for a 40% female calculation assumption, and with 60% females in the realized cohort for a 70% female calculation assumption.

For any calculation female percentage, the expected IRR is the higher, the lower the realized share of females. Surprisingly, from the first glance, the expected IRR increases slightly for the case of total adverse selection. The reason is as follows: Adverse selection results in rising number of insurer's shortfalls during the early retirement years. By design of our surplus optimization algorithm, the surplus distribution percentage is reduced to a minimum. This surplus distribution percentage recovers only slowly if the insurer gains profits again with the result that annuitant's surpluses are kept inside the company, and contribute to the observed expected IRR increase.

We already mentioned that it may not always be a disadvantage to the annuitant, when a high share of females is used for pricing, as this percentage influences the financial stability of the insurer. The figure shows that for any actually realized female percentage, the shortfall probability is higher for lower female calculation assumptions. The issue of insurer stability is also of interest to the annuitants as those are concerned about the reliability of their annuity payments.

There is, however another interesting result: The shortfall probability is at minimum level if females and males are relatively fairly balanced resulting in the form of a parabola that opens up. The increasing shortfall probability for higher than calculated female shares has a straightforward explanation: In such case, the calculated annuitant's benefit is too high. At first glance the increase of the shortfall probability for the case of lower than calculated female shares seems surprising. The reason for this development is that in a cohort with considerably less females than assumed excessive mortality surpluses arise and must be distributed quite promptly. This increases the level of the guaranteed benefit payments to the surviving annuitants. To fulfill this obligation the insurer has to sell more assets, which permanently reduces the total investment returns. The coincidence of a bear capital market, high annuitant benefits and low total investment returns can cause a shortfall as it strips the insurer out of equity. These reasons are more relevant for the shape of the results than choosing the same gender mix for pricing as actually realized.

Thus, we show that although a high female pricing percentage lowers the annuitant's utility equivalent FLA, the lowest female pricing percentage should not be the annuitant's selection criterion. Both, the annuitant and the insurer, benefit from a cautious but reasonable gender calculation assumption which corresponds to the actually realized gender composition. This is an important insight which can be used by both annuity providers and prospective annuitants.

Panel B shows the same parameters after introducing LIRA with a GIR of 1.75%. The course of the IRR has almost the same shape as before but is slightly higher. However analyzing the shortfall probability clearly shows a change: For all cases of pricing and realizations it is lower and just above zero. Both developments can be explained by the new possibility of cross charging negative asset returns.

5. CONCLUSION

Providing retirees with a guaranteed income for life and upside potential via surplus sharing is a unique quality of participating payout life annuities. We present a stylized German life insurance company, which offers only this product. Our model considers a stochastic capital market as well as stochastic mortality developments, which are calibrated on German data. We examine the effects of gender-neutral pricing in the context of German surplus regulations on both the annuitant's utility and the insurer's profit and stability.

We start by analyzing the impacts of gender-neutral pricing from annuitant's and insurer's perspective, allowing for a stepwise deviation of the assumed and realized gender mix. The annuitant's utility declines when the share of females increases: This is true for both the assumed pricing cohort and the actually realized pool of annuitants. Quantifying differences between gender-based and unisex calculation for the same utility in form of the required single unisex premium results in a difference of about +/- 5% for men/women. Comparing the single premiums that are charged to reach the utility equivalent fixed life annuity of the respective other sex in a unisex calculated world delivers +/- 1% difference. Comparing our findings with the empirical data delivers a lower disadvantage for men. The crucial point is to consider mortality rates and surplus additional to guaranteed benefits or single premiums as they have an impact on annuitant's lifetime utility.

From the insurer's perspective the higher the calculated share of females in the insured pool, the higher the insurer's return gets, but it also decreases for each assumed female level with the increase in the share of females in the actually realized pool. A significant spread between the assumed and the realized gender mix increases the insurer's shortfall probability. While a high shortfall probability for a high realized share of females is self-explaining, the shortfall increase for a low female level in the actual cohort is not that apparent. In this last case, a lower as assumed female share leads to excessive surpluses, which have to be distributed, as annuitants pass away earlier than expected. The resulting stronger increase in the level of the benefits endangers the insurance company. Higher benefit payments reduce the insurer's assets causing lower investment returns in the following years. The combination of high liabilities towards the annuitants, low investment returns and a reduced amount of assets exhausts the insurer's equity in a bear market. Taking into account changes by LIRA in the surplus allocation results in a minor influence on annuitant's utility and in an improvement of insurer's stability and profitability.

Our findings could be of interest to potential retirees and annuity providers. Miscalculation of the gender mix adds a new risk to German insurance companies, but also for annuitant's, as their lifetime utility is directly linked with financial status of the insurer. Embedded in the quite strict German regulation system for insurers it requires the choice of cautious, but nevertheless realistic assumptions. The present analysis also shows a possible direction for future research: Impacts of incorrect unisex pricing are reflected in nearly all risk categories of the regulation system of Solvency II. In this context the long-term effects of mispricing a PLA on capital requirements and shortfall probability, as defined in Solvency II, are important issues.

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APPENDIX

The actuarial fair monthly guaranteed benefit ${}^{(12)}B$ for a single premium P_0 is calculated as

$$ {}^{(12)}B = \frac{P_0}{12 \cdot {}^{(12)}P_x} \tag{A1} $$

where ${}^{(12)}P_x$ is the premium of an annuity with 1€ monthly benefit payment and it is given by the following formula (Fodor, 2003)

$$ {}^{(12)}P_x = P_x - k^{(12)} \tag{A2} $$

Here, P_x is premium of an annuity with annual benefit payment

$$ P_x = \sum_{k=0}^{\omega-(x+1)} \frac{{}_kp_x^A}{(1+GIR)^k} \tag{A3} $$

where q_x^A, $(p_x^A = (1 - q_x^A))$ is the unisex mortality rate for age x and GIR the guaranteed interest rate. The factor $k^{(12)}$ is given by

$$ k^{(12)} = \frac{1}{12} \sum_{i=0}^{11} \frac{(1+GIR) \cdot i}{12 + i \cdot GIR} \tag{A4} $$

The unisex mortality rates q_x^A are determined according to

$$ q_x^A = q_x^{A,M} \cdot u_{x+1} + q_y^{A,F} \cdot (1 - u_{x+1}) \tag{A5} $$

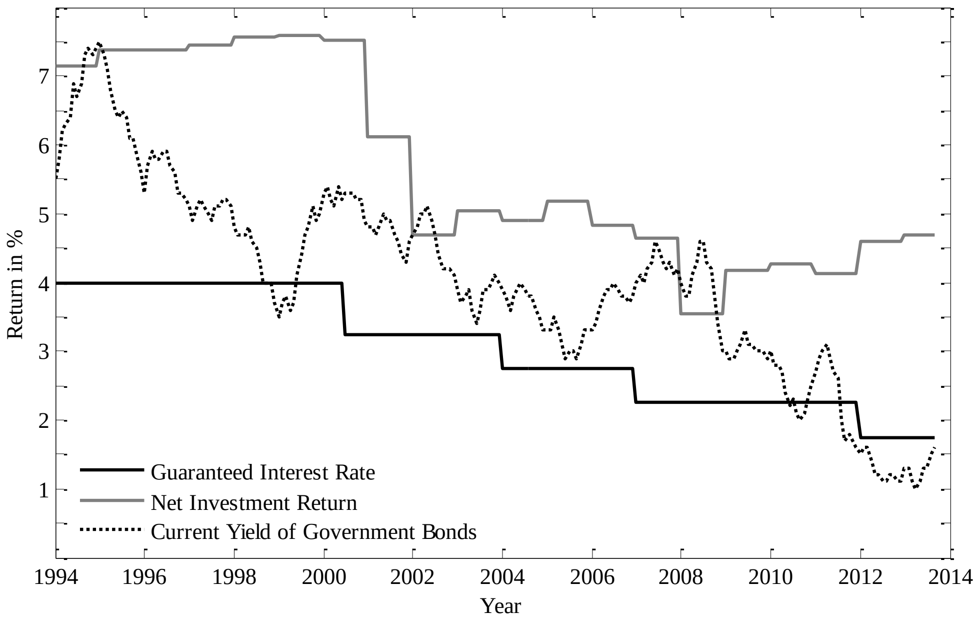
where u_x is a dynamic gender mix factor preventing the natural demix of genders with

$$ u_x = \frac{L_x}{L_y + L_x}, \tag{A6} $$

L_x is the initial number of males, L_y is the initial number of females and they develop by

$$ L_{x+1} = L_x \cdot \left(1 - q_x^{A,M}\right), \qquad L_{y+1} = L_y \cdot \left(1 - q_y^{A,F}\right). \tag{A7} $$

Figure 1: Guaranteed Interest Rate, Net Investment Return and Current Yield of Government Bonds



Notes: Current Yield of German Government Bonds according to Deutsche Bundesbank, WU0115. Maximum possible guaranteed interest rate according to premium refund order. Average net investment return over all German life insurers. Source: Deutsche Bundesbank, DeckRV, GDV (2013).

Figure 2: Minimum Surplus Allocation Decree

Panel A: Minimum Allocation Decree before LIRA

+ max (90% · Asset Return – Interest*;0)
+ max (75% · Mortality Return;0)
+ max (50% · Other Return;0)
Minimal Surplus Allocation in the Current Financial Year

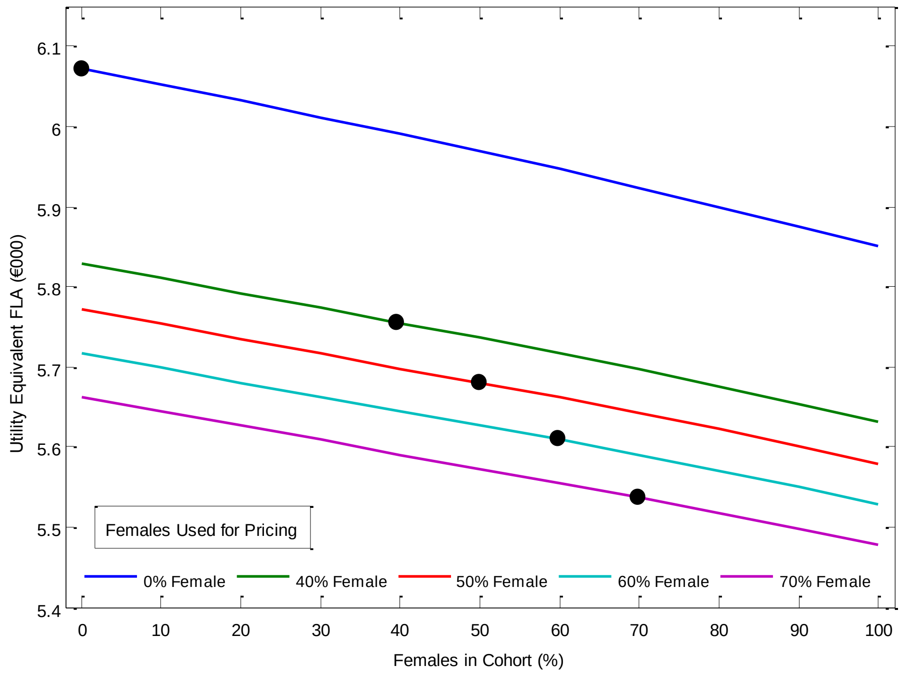
Panel B: Minimum Allocation Decree after LIRA

+ max (90% · Asset Return - Interest*; min(Asset Return-Interest*;0))
+ max (90% · Mortality Return;0)
+ max (50% · Other Return;0)
Minimal Surplus Allocation in the Current Financial Year

* Interest on the Actuarial Reserve

Notes: Minimum Allocation Decree before validity of LIRA (Panel A) and after introducing LIRA (Panel B). Source: Author's illustration.

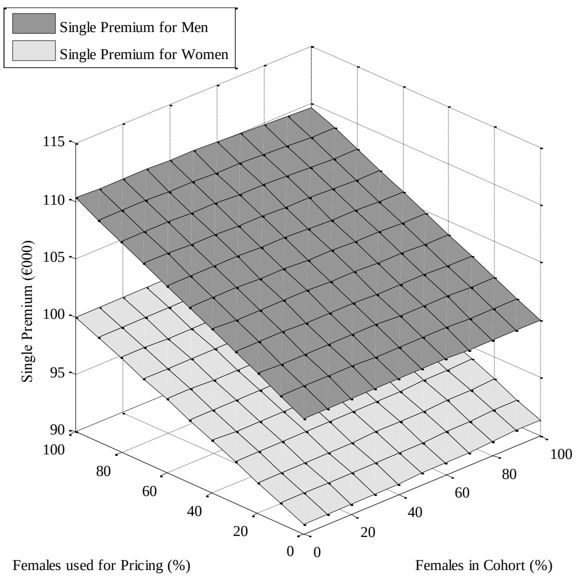
Figure 3: Assumed Gender Mix for Annuity Pricing and Realized Gender Mix in Actual Cohort: Annuitant's Utility Impacts



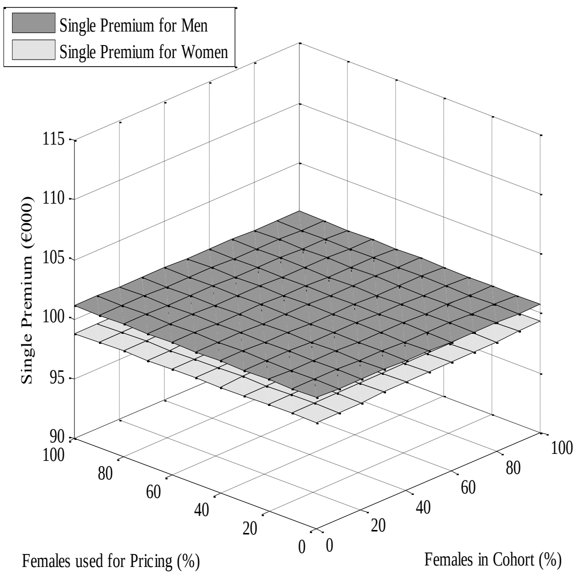
Notes: Figure 3 shows on the vertical axis the utility equivalent fixed life annuity (in €000) of a PLA for males based on a time-additive CRRA utility function. The horizontal axis depicts the realized percentage of females in the cohort of annuitants. Different colors represent the assumed gender mix for pricing guaranteed benefits. The dots on the lines represent an equal gender mix for pricing guaranteed benefits and the realized gender mix in the pool of annuitants. Calibration: Male age 67 in 2013; PLA single premium: 100,000€; time preference: $\beta = 0.96$; relative risk aversion: $\gamma = 5$; GIR: 1.75%; mortality table: "DAV 2004 R"; bond duration: 10 years; surplus allocation to annuitant: 90%; equity capital endowment: 1.9%; solvency limit 4%; initial uncommitted PPR: 2.75%; initial committed PPR: 2%; target uncommitted PPR 5.5%. Source: Authors' calculation.

Figure 4: Single Premium for Utility Equivalent FLA: Unisex versus Gender-Specification and Male versus Female

Panel A: Gender-Specific Utility Equivalent Single Premium



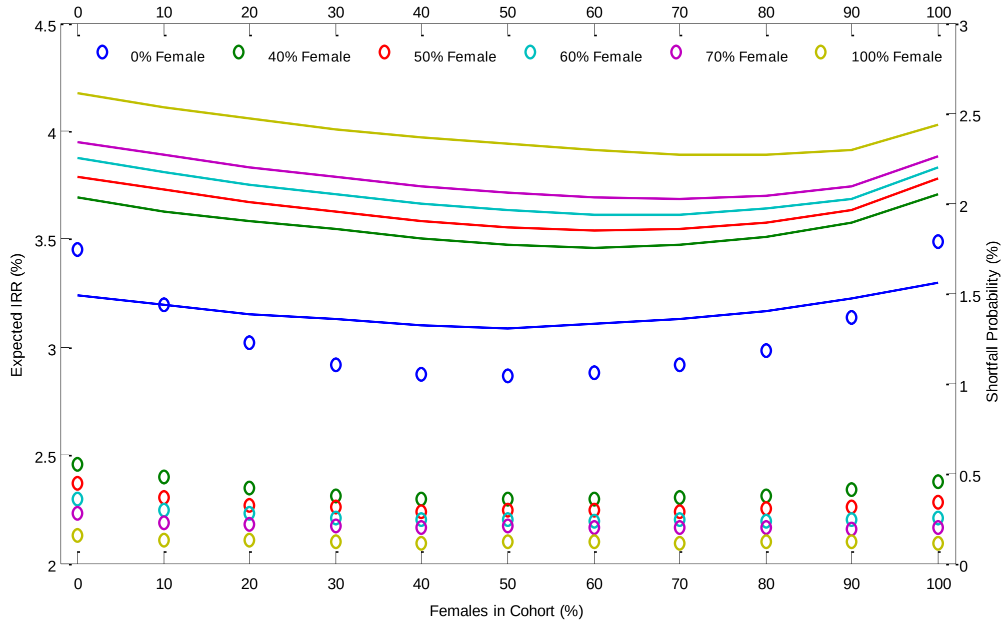
Panel B: Gender-Neutral Utility Equivalent Single Premium



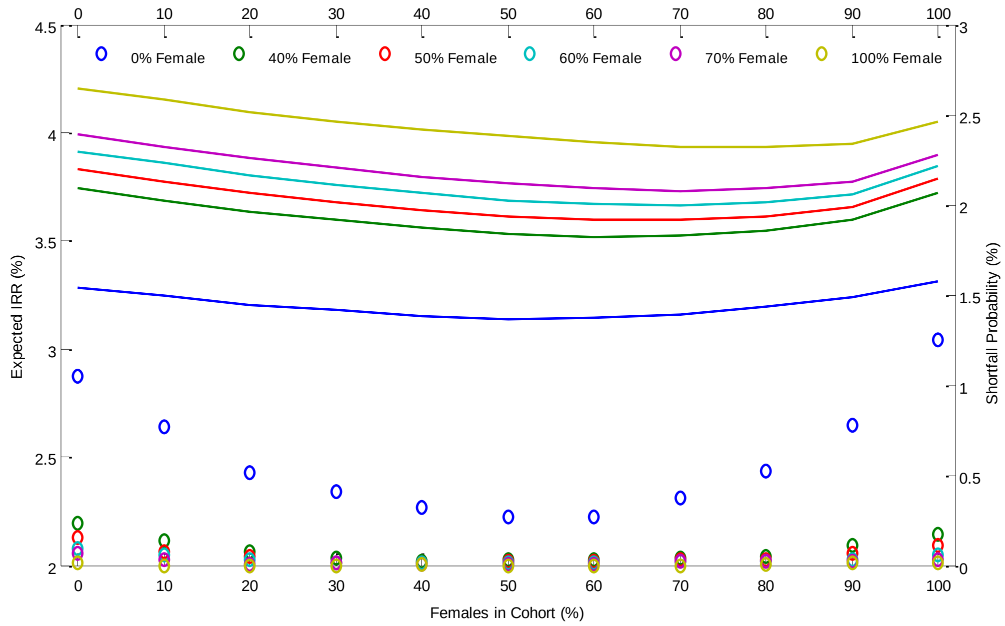
Notes: Figure 5 illustrates the single premium that an annuitant has to pay for a unisex calculated PLA in order to reach the same utility equivalent FLA as with a gender-neutral calculated PLA (Panel A); Panel B depicts the single premium for a unisex calculated contract that is required to achieve the same utility equivalent FLA as with a unisex calculated PLA for the opposite sex. Original PLA Single premium: 100,000€; See also Figure 3. Source: Authors' calculation.

Figure 5: Assumed Gender Mix for Annuity Pricing and Realized Gender Mix in Actual Cohort: Insurer's Profitability and Stability

Panel A: Profitability and Stability before LIRA



Panel B: Profitability and Stability after LIRA



Notes: Expected internal rate of return and shortfall probability for alternative percentages of females used for pricing and in the actual cohort before introducing LIRA (Panel A) and after validity of LIRA (Panel B). Lines represent the expected IRR on the left vertical axis, dots are showing the shortfall probability on the right vertical axis. Different colors represent the assumed gender mix for pricing guaranteed benefits. The horizontal axis depicts the realized percentage of females in the cohort of annuitants. Original PLA single premium: 100,000; See also Figure 3. Source: Authors' calculation.

Table 1: Market PLA Changes Resulting from Gender-Neutral Pricing

	Market PLA Benefits in €			Difference in %	
	Female	Unisex	Male	Female	Male
Minimum	362.81	371.33	388.86	2.3	-4.5
Median	373.49	390.24	420.00	4.5	-7.1
Maximum	406.51	397.60	457.58	-2.2	-13.1

Notes: Minimum, median and maximum gender-specific and gender-neutral monthly benefits of immediate PLAs for 67 year old males and females with a single premium of 100,000€. PLA inception date is January 2013. No cost reduced tariffs. Source: Morgen & Morgen Office, LV Lotse.

Table 2: Costs included in Market PLA Benefits

	Market PLA Benefits in €	Actuarial Fair Benefit in €	Difference in %
Male	420.00	449.22	-6.5
Female	373.49	400.56	-6.8

Note: Median gender-specific and actuarial fair monthly benefits of immediate PLAs for 67 year old individuals with a single premium of 100,000€. PLA purchased in January 2013. Actuarial fair benefits: mortality tables "DAV 2004 R", guaranteed interest rate 1.75%, no costs. Source: Morgen & Morgen Office, LV Lotse, authors' calculation.

Table 3: Actuarial Fair Benefits for Alternative Gender Mixes

Percentage of Females	0	10	20	30	40	50
Actuarial Fair Benefit in €	449,22	443,75	438,43	433,26	428,22	423,32
Percentage of Females	60	70	80	90	100	
Actuarial Fair Benefit in €	418.15	413.87	409.33	404.89	400.56	

Note: Actuarial fair gender-neutral monthly benefit of immediate PLAs for 67 year old individuals for alternative gender mixes with a single premium of 100,000€. PLA purchased in January 2013. Mortality tables "DAV 2004 R", guaranteed interest rate 1.75%, no costs. Source: Authors' calculation.

Table 4: Life Expectancy of Individuals using Alternative Mortality Tables

	Life Expectancy in Years			Difference in %	
Gender	Annuity	Population	Term Life	Annuity	Term Life
Male	22.7	16.0	14.9	0.42	-0.07
Female	26.3	19.0	18.1	0.39	-0.05

Notes: Average life expectancy in years of individuals ages 67 in 2010 for alternative mortality tables. Annuity mortality table: "DAV 2004 R", population mortality table: federal statistical office, term life mortality table: "DAV 2008 T". Source: Authors' Calculation.

Table 5: Estimates of the 3-Factor CIR Model

i	μ_i^{CIR}	α_i	σ_i^{CIR}	λ_i	$r_{i,0}^{CIR}$
1	0.0024	0.3854	0.0949	-0.2670	0.0000
2	0.0035	0.6688	0.0885	-0.7879	0.0001
3	0.0065	0.1318	0.0520	0.2050	0.0159

Note: Estimates of the 3-factor CIR model based on data provided by Datastream. Source: Authors' Calculation.

Table 6: Calibration of the CBD Mortality Model

	Male				Female			
i	K_t	μ^{CBD}	Σ		K_t	μ^{CBD}	Σ	
1	-11.1875	-0.0740	0.0500	0.0000	-13.2525	-0.0758	0.0421	0.0000
2	0.1050	0.0006	- 0.0007	0.0002	0.1251	0.0007	- 0.0006	0.0002

Note: Estimated parameters of the CBD mortality model based on German mortality data for the Human mortality database. $\mathrm{K_t}$ the period mortality index, μ^{CBD} estimated mortality, Σ correlation matrix Source: Authors' calculation.

Table 7: Men's Gender-Specific Utility Equivalent Single Premium for Different Cases (in €)

	Before LIRA			After LIRA, GIR=1.75%			After LIRA, GIR=1.25%		
	Assumed Females for Annuity Pricing								
Females in Cohort	100%	50%	0%	100%	50%	0%	100%	50%	0%
100%	109,678	104,877	100,000	109,682	104,868	100,000	110,890	105,514	100,000
50%	110,108	105,091	100,000	110,106	105,091	100,000	110,965	105,551	100,000
0%	110,275	105,197	100,000	110,271	105,194	100,000	111,135	105,624	100,000

Notes: Single premium a man has to pay for a unisex calculated PLA to reach the same utility equivalent FLA as with a gender-based calculation. Vertical axis realized share of females in the actual cohort, horizontal axis females used for pricing guaranteed benefits. PLA single premium: 100,000€; See also Figure 3. Source: Authors' calculation.