Managerial Motivation and Higher-Order Risk

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July 9, 2015

Abstract

This paper provides the comparative statics for a precautionary effort decision with respect to a change in compensation schedule. Using a framework suggested by Eeckhoudt, Huang, and Tzeng (2012), we extend Kocabiyikoglu and Popescu (2007) by considering the existence of an independent background risk in the future and a higher-order risk improvement in productivity. We provide the necessary and sufficient conditions on preferences for unambiguous comparative statics of compensation in the precautionary effort. Some useful sufficient conditions are further discussed.

Keywords: stochastic dominance; decision analysis; risk aversion ; risk apportionment

JEL classification: D81, D86, G30

1 Introduction

Managerial motivation has long been an important issue in economics and finance. Many papers have studied whether a change in CEO compensation causes CEOs to expend more effort.[1] For example, Kocabiyikoglu and Popescu (2007) assumed that the cost of effort is non-monetary and that the production level of a manager is strictly increasing in her effort. They found that increased pay is not necessarily a factor motivating an increase in her efforts. Kvaloy and Olsen (2015) show that the uncertainty of unpaid or non-credible bonuses will give rise to a negative relationship between effort and performance pay. In spite of the ingenious discussions in the literature, few articles consider the presence of background risk. Our paper intends to fill this gap.

Since Kimball (1990), the literature has analyzed alternative decisions under a framework with the existence of background risk. For example, Eeckhoudt and Schlesinger (2008) provide the necessary and sufficient condition on preferences for a change in an independent background risk to increase saving. Courbage and Rey (2012), Eeckhoudt, Huang and Tzeng (2012) and Wang and Li (2015) examine the impact of background risk on self-protection decisions. Hofmann and Peter (2014) investigate how the optimal risk management and saving portfolio reacts to changes in the size of a background risk.

Unlike most papers in the above literature that examine the effect of a change in background risk on the effort or self-protection decision, our paper analyzes the impact of changes in the payment on the precaution effort decision in the presence of an independent background risk. We analyze the effect by assuming a commonly observed compensation scheme containing a fixed wage and a bonus payment.

We employ a two-period model as in Eeckhoudt, Huang, and Tzeng (2012) and Wang and Li (2015). The CEOs make a decision on efforts in period one, which could result in an improvement in the production level in the second period, by knowing that they will face an independent background risk in the second period. We respectively study the necessary and sufficient condition on preferences regrading a change in the fixed wage and the bonus pay to increase effort.

[1] Some researchers have analyzed whether a change in the CEO's compensation makes her take more risk. For example, Ross (2004) provided the necessary and sufficient conditions for a change in compensation to cause CEOs to take less risk.

Our paper is close to Kocabiyikoglu and Popescu (2007) in the sense that they also examine the impact of compensation on the CEO's motivation under a given compensation structure.[2] Our paper, however, differs from their paper in two ways. First, Kocabiyikoglu and Popescu (2007) do not consider the existence of a background risk in the future. Second, they assume that the manager's effort can result in a first-degree stochastic dominance (FSD) improvement in the production level, whereas higher-order generalizations are made in our paper. The impact of higher-order risk has received a noticeable increase in attention since Eeckhoudt and Schlesinger (2006).[3] Thus, it is important to generalize Kocabiyikoglu and Popescu (2007) by employing higher-order effects.

We first find that, given that an effort could result in an improvement in terms of Nth-degree stochastic dominance (NSD), an increase in the fixed wage will decrease the optimal level of effort if and only if the derived utility of the manager exhibits mixed-risk aversion up to degree $N+1$. Second, we find that an increase in the variable wage increases the optimal level if and only if the manager's derived utility satisfies the condition that the partial nth degree risk aversion, as defined by Chiu, Eeckhoudt and Rey (2012), is not higher than n, where $n = 1, 2, ..., N$. Furthermore, we provide the rationale of our findings by applying the theorems proposed by Eeckhoudt, Schlesinger and Tsetlin (2009) as well as those by Chiu, Eeckhoudt and Rey (2012), which may shed some light on how these theorems with intuitive explanations of higher order risk can be applied to economic or management issues.

Some useful sufficient conditions regarding the original utility of the manager are further discussed. We find that if the original utility of the manager exhibits mixed-risk aversion up to degree $N+1$ then the derived utility of the manager exhibits mixed-risk aversion up to degree $N+1$. On the other hand, if the manager's original utility has the property that the partial nth degree risk aversion is not higher than n, where $n = 1, 2, ..., N$, then the manager's derived utility also has the property that the partial nth degree risk aversion is not higher than n, where $n = 1, 2, ..., N$. In addition, in the absence of background risk, the conditions on the derived utility function are exactly the same as those on the original utility

[2] Neither Kocabiyikoglu and Popescu (2007) nor our paper adopt a principal-agent framework. As mentioned by Kocabiyikoglu and Popescu (2007), this type of analysis can help firms better understand "how employees react to existing or proposed compensation plans, and how their motivation is affected by various factors".

[3] The impact of higher-order risk has been studied in many problems, such as precautionary saving (Eeckhoudt and Schlesinger, 2008), effort (Jindapon and Neilson, 2007), precautionary effort (Eeckhoudt, Huang, and Tzeng, 2012) and financial decisions (Noussair, Trautmann and van de Kuilen, 2014).

function.

The remainder of the paper is organized as follows. Section 2 describes our model's setting, and Section 3 examines the effect of an increase in the fixed or variable wage on optimal precautionary effort. Section 4 concludes the paper.

2 The Model

We construct a two-period model similar in form to that in Eeckhoudt, Huang, and Tzeng (2012). Let $\tilde{x}$ denote the observable profits generated by the manager in period 1, which follows a distribution $G(x)$ with support $[\underline{x}, \bar{x}]$. Assume that the manager could make an effort $e \in [0, 1]$ in period 0 to improve the distribution of $\tilde{x}$. We assume that the cost of effort $c(e)$ is monetary with $c(0) = 0$, $c^{(1)}(0) = 0$, $c^{(1)}(e) > 0$, and $c^{(2)}(e) > 0$, where the superscript (n) denotes the nth derivative, $n = 1, 2$.[4]

As in Jindapon and Neilson (2007) and Eeckhoudt, Huang, and Tzeng (2012), let the distribution of period 1 production $\tilde{x}$, denoted by $H(x)$, be a combination of $G(x)$ and a preferred distribution $F(x)$, i.e., $H(x) = (1-e)G(x)+eF(x)$. In other words, if the maximum level of effort is invested ($e = 1$), then the final distribution becomes $F(x)$. If no effort is made, then the final distribution remains as $G(x)$. Define $F^{(N)}(x) = \int_{\underline{x}}^{\bar{x}} F^{(N-1)}(t)dt$ and $F^{(1)}(x) = F(x)$, and define $G^{(N)}(x)$ similarly. Assume that $F(x)$ dominates $G(x)$ in terms of NSD, i.e.,

$$F^{(N)}(x) \leq G^{(N)}(x), \forall x,$$

$$F^{(n)}(\bar{x}) \leq G^{(n)}(\bar{x}), n = 1, 2, ..., N-1,$$

Similar to Kocabiyikoglu and Popescu (2007), we assume that the compensation scheme is $a + b\tilde{x}$, where a and b are positive constants.[5] That is, the manager would receive a fixed payoff a and a bonus $b\tilde{x}$ that depends on her production level. Furthermore, let $k(\cdot)$ and $u(\cdot)$ respectively denote the utility function of the manager in period 0 and period 1. Assume that $k^{(1)} \geq 0$, $k^{(2)} \leq 0$, $u^{(1)} \geq 0$ and $u^{(2)} \leq 0$, where $k^{(n)}$ and $u^{(n)}$ each denote the nth

[4] Our main theories are unaffected by whether the cost is monetary or non-monetary.

[5] Note that Kocabiyikoglu and Popescu (2007) assumed that the variable wage is $\phi(x)$ with $\phi' > 0$. We will discuss this case later in Section 3.

derivative of the utility functions. Let w denote the wealth of the manager in period 0, and assume that, in period 1, the manager is facing an independent background risk $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ with $E(\varepsilon) = 0$ and following the distribution $T(\varepsilon)$.

Furthermore, assume that the manager is an expected utility maximizer. Then, the objective function is:

$$\max_{e} \quad EU = k(w - c(e)) + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{x}}^{\bar{x}} u(a + bx + \varepsilon) \left[(1-e)dG(x) + edF(x)\right] dT(\varepsilon).$$

Define the derived utility for the manager as:

$$v(a + bx) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} u(a + bx + \varepsilon) dT(\varepsilon).$$

Thus the objective function can be expressed as:

$$\max_{e} \quad EU = k(w - c(e)) + \int_{\underline{x}}^{\bar{x}} v(a + bx) \left[(1-e)dG(x) + edF(x)\right],$$

or, equivalently,

$$\max_{e} \quad EU = k(w - c(e)) + \left[(1-e)Ev(a + b\tilde{x}_G) + eEv(a + b\tilde{x}_F)\right], \tag{1}$$

where E is the expectation operator, and $\tilde{x}_F$ and $\tilde{x}_G$ respectively denote the random variable $\tilde{x}$ following the cumulative distribution functions $F(x)$ and $G(x)$.

The first-order condition (FOC) is

$$\frac{\partial EU}{\partial e} = Ev(a + b\tilde{x}_F) - Ev(a + b\tilde{x}_G) - c^{(1)}(e^*)k^{(1)}(w - c(e^*)) = 0, \tag{2}$$

where e^* denotes the optimal level of effort. The second-order condition (SOC) holds under the assumptions $c^{(2)} > 0$, $k^{(1)} > 0$ and $k^{(2)} < 0$ since

$$\frac{\partial^2 EU}{\partial e^2} = -c^{(2)}(e)k^{(1)}(w - c(e)) + \left[c^{(1)}(e)\right]^2 k^{(2)}(w - c(e)) < 0. \tag{3}$$

3 Comparative Statics

In the following, we respectively analyze two cases. The first case is that where the fixed wage increases to A with $A > a$. The second case is that where the variable wage increases to B with $B > b$. The following Proposition provides the comparative statics when the fixed wage increases.

Proposition 1 *Given that $F(x)$ dominates $G(x)$ in terms of NSD, an increase in the fixed wage a will decrease the optimal level of effort e^* in the presence of an independent background risk if and only if the manager's derived utility satisfies $(-1)^{n+1}v^{(n)} \geq 0$, $\forall x$, $n = 1, ..., N+1$.*

Proof. Note that $A > a$ means that A dominates a in terms of first-degree stochastic dominance (FSD). Let e_A^* denote the optimal decision when the fixed wage is A, i.e., e_A^* satisfies

$$Ev(A + b\tilde{x}_F) - Ev(A + b\tilde{x}_G) - k^{(1)}(w - c(e_A^*))c^{(1)}(e_A^*) = 0.$$

Since the SOC holds, $e_A^* \leq e^*$ if and only if

$$Ev(a + b\tilde{x}_F) - Ev(a + b\tilde{x}_G) - k^{(1)}(w - c(e_A^*))c^{(1)}(e_A^*) \geq 0.$$

Thus, the above condition could be rewritten as

$$Ev(A + b\tilde{x}_F) - Ev(A + b\tilde{x}_G) \leq Ev(a + b\tilde{x}_F) - Ev(a + b\tilde{x}_G).$$

Rearranging the above condition and multiplying both sides by $\frac{1}{2}$ yields

$$\frac{1}{2}Ev(A + b\tilde{x}_F) + Ev(a + b\tilde{x}_G) \leq \frac{1}{2}Ev(a + b\tilde{x}_F) + Ev(A + b\tilde{x}_G). \quad (4)$$

The left-hand side of Equation (4) could be viewed as obtaining a lottery which gives $(A+b\tilde{x}_F)$ and $(a+b\tilde{x}_G)$ with equal probability, and the right-hand side of Equation (4) could be viewed as obtaining another lottery which gives $(a+b\tilde{x}_F)$ and $(A+b\tilde{x}_G)$ with equal probability. Since A dominates a in terms of FSD, and $\tilde{x}_F$ dominates $\tilde{x}_G$ in terms of NSD, the left-hand side is a lottery combining good with good and bad with bad, whereas the right-hand side is a lottery combining good with bad. Thus, we could apply Theorem 3 in Eeckhoudt, Schlesinger and Tsetlin (2009) which indicates that Equation (4) holds for all utility functions satisfying

$$(-1)^{(n+1)} v^{(n)} \geq 0, n = 1, ..., N + 1.$$

■

The intuition behind Proposition 1 is as follows. The marginal cost of effort is the same for these two compensation schemes for any given level of effort. However, the marginal benefit of effort under the scheme $a + bx$ is greater than that under the scheme $A + bx$ for all managers with $(-1)^{(n+1)} v^{(n)} \geq 0$, $n = 1, ..., N + 1$. This is because comparing the marginal benefit is the same as comparing two lotteries: obtaining $a + b\tilde{x}_F$ and $A + b\tilde{x}_G$ with equal chance vs. obtaining $A + b\tilde{x}_F$ and $a + b\tilde{x}_G$ with equal chance. According to Eeckhoudt, Schlesinger and Tsetlin (2009), the former is preferred if and only if the agent's utility satisfies: $(-1)^{(n+1)} v^{(n)} \geq 0$, $n = 1, ..., N + 1$.

Proposition 1 can be extended to the case where $G(x)$ has more Nth-degree risk than $F(x)$, i.e.,

$$F^{(N)}(x) \leq G^{(N)}(x), \ \forall x,$$

and

$$F^{(n)}(\bar{x}) = G^{(n)}(\bar{x}), \ n = 1, 2, ..., N.$$

In this case, we could apply the Corollary in Eeckhoudt, Schlesinger and Tsetlin (2009) and conclude that an increase in the fixed wage a will decrease the optimal level of effort e^* if and only if the manager's derived utility satisfies: $(-1)^{(N+1)} v^{(N)} \geq 0$.

For the condition of the initial utility u, we provide the following corollary:

Corollary 1 *Given that $F(x)$ dominates $G(x)$ in terms of NSD, an increase in the fixed wage a will decrease the optimal level of effort e^* in the presence of an independent background risk if the manager's utility u satisfies $(-1)^{n+1} u^{(n)} \geq 0$, $\forall x$, $n = 1, ..., N + 1$.*

The proof is trivial since $v^{(n)} \geq 0$ if $u^{(n)} \geq 0, \forall x$.

The following Proposition shows the condition for an increase in effort when the variable wage increases:

Proposition 2 *Assume that* $(-1)^{(n+1)}v^{(n)} \geq 0, n = 1, ..., N+1$*. Given that* $F(x)$ *dominates* $G(x)$ *in terms of NSD, an increase in the variable bonus* b *will increase the optimal level of effort* e^* *in the presence of an independent background risk if and only if the manager's derived utility satisfies* $-\frac{bxv^{(n+1)}(a+bx)}{v^{(n)}(a+bx)} \leq n$*,* $\forall x$*,* $n = 1, ..., N$*.*

Proof. Let e_B^* denote the optimal decision when the proportional wage is B, i.e., e_B^* satisfies:

$$Ev(a + B\tilde{x}_F) - Ev(a + B\tilde{x}_G) - k^{(1)}(w - c(e_B^*))c^{(1)}(e_B^*) = 0.$$

Since the SOC holds, $e_B^* \geq e^*$ if and only if

$$Ev(a + b\tilde{x}_F) - Ev(a + b\tilde{x}_G) - k^{(1)}(w - c(e_B^*))c^{(1)}(e_B^*) \leq 0.$$

Thus, the above condition could be rewritten as

$$Ev(a + B\tilde{x}_F) - Ev(a + B\tilde{x}_G) \geq Ev(a + b\tilde{x}_F) - Ev(a + b\tilde{x}_G).$$

Rearranging the above condition and multiplying both sides by $\frac{1}{2}$ yields

$$\frac{1}{2}Ev(a + B\tilde{x}_F) + Ev(a + b\tilde{x}_G) \geq \frac{1}{2}Ev(a + b\tilde{x}_F) + Ev(a + B\tilde{x}_G). \tag{5}$$

Theorem 2 in Chiu, Eeckhoudt and Rey (2012) indicates that the above condition holds if and only if the derived utility function exhibits

$$-\frac{bxv^{(n+1)}(a + bx)}{v^{(n)}(a + bx)} \leq n, \forall x, n = 1, ..., N.$$

■

The intuition behind Proposition 2 is similar to that of Theorem 2 (ii) in Chiu, Eeckhoudt and Rey (2012). On the one hand, when b is higher, the "risky part" of the manager's payoff becomes higher, which makes a risk-averse manager want to work harder. On the other hand, because b is positive, this means that raising b is raising risk in terms of NSD as well as the manager's expected payoff at the same time, which forms an $(N+1)$th order effect, and makes the manager with $(-1)^{(n+1)}v^{(n)} \geq 0$, $n = 1, ..., N+1$, less willing to work hard. Thus, if the manager's degree of $(n+1)$th risk-aversion is not significantly large enough, i.e., the partial $(n+1)$th order risk aversion is not greater than n, $n = 1, ..., N$, then the former effect will dominate the latter, making her expend more efforts. Similarly, Proposition 2 can be extended to the case where $G(x)$ has more Nth-degree risk than $F(x)$. Assuming that $(-1)^{(n+1)}v^{(n)} \geq 0, n = N, N+1$, Theorem 2 in Chiu, Eeckhoudt and Rey (2012) could help us to predict that an increase in b will increase the optimal level of effort for all managers with $-\frac{bxv^{(N+1)}(a+bx)}{v^{(N)}(a+bx)} \leq N$, $\forall x$.

For the condition for the orginal utility u, we provide the following corollary.

Corollary 2 *Assume that $(-1)^{(n+1)}u^{(n)} \geq 0$, $n = 1, ..., N+1$. Given that $F(x)$ dominates $G(x)$ in terms of NSD, an increase in the variable bonus b will increase the optimal level of effort e^* in the presence of an independent background risk if the manager's original utility u satisfies*

$$-\frac{bxu^{(n+1)}(a+bx+\varepsilon)}{u^{(n)}(a+bx+\varepsilon)} \leq n,$$

for all $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$, all x, and $n = 1, 2, ..., N$.

Proof. Consider that n is even. If $(-1)^{(n+1)}u^{(n)} \geq 0$, then v also satisfies $(-1)^{(n+1)}v^{(n)} \geq 0$. If u satisfies the condition in the corollary, then we have

$$u^{(n+1)}(a+bx+\varepsilon) \leq \frac{n}{bx}u^{(n)}(a+bx+\varepsilon),$$

which implies

$$\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} u^{(n+1)}(a+bx+\varepsilon)h(\varepsilon)d\varepsilon \leq \frac{n}{bx}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} u^{(n)}(a+bx+\varepsilon)h(\varepsilon)d\varepsilon.$$

Thus, we have $-\frac{bxv^{(n+1)}(a+bx)}{v^{(n)}(a+bx)} \leq n$. The proof of the case where n is odd is similar and is thus omitted. ■

In the absence of a background risk, our model reduces to a simple two-period model as follows:

$$\max_{e} \quad EU = k(w - c(e)) + \int_{\underline{x}}^{\bar{x}} u(a + bx) \left[(1 - e)dG(x) + edF(x)\right].$$

Or, equivalently,

$$\max_{e} \quad EU = k(w - c(e)) + \left[(1 - e)Eu(a + b\tilde{x}_G) + eEu(a + b\tilde{x}_F)\right]. \tag{6}$$

The objective function (6) is of the same form as the objective function (1) except that the utility function in the second period is the derived utility function v in (1), whereas that is the original utility function u in (6). Thus, the following two Corollaries directly follow from Propositions 1 and 2.

Corollary 3 *Given that $F(x)$ dominates $G(x)$ in terms of NSD, an increase in the fixed wage a will decrease the optimal level of effort in the absence of a background risk if and only if the manager's original utility satisfies $(-1)^{n+1}u^{(n)} \geq 0$, $\forall x$, $n = 1, ..., N + 1$.*

Corollary 4 *Assume that $(-1)^{(n+1)}u^{(n)} \geq 0, n = 1, ..., N + 1$. Given that $F(x)$ dominates $G(x)$ in terms of NSD, an increase in the variable bonus b will increase the optimal level of effort in the absence of a background risk if and only if the manager's orginal utility satisfies $-\frac{bxu^{(n+1)}(a+bx)}{u^{(n)}(a+bx)} \leq n$, $\forall x$, $n = 1, ..., N$.*

Corollary 3 includes Proposition 1 in Kocabiyikoglu and Popescu (2007) as a special case. Knowing that they assume that managerial efforts can improve the production in terms of first-degree stochastic dominance, this is obviously a special case of our Corollary 3 when $N = 1$. Kocabiyikoglu and Popescu (2007) define an agent with utility function u as being aggressive if the degree of relative risk aversion is less than or equal to one, which is $-\frac{(a+bx)u^{(2)}(a+bx)}{u^{(1)}(a+bx)} \leq 1$ in this paper. Obviously, since $u^{(1)} > 0$, $u^{(2)} < 0$ and $a > 0$, $-\frac{(a+bx)u^{(2)}(a+bx)}{u^{(1)}(a+bx)} \leq 1$ implies that $-\frac{bxu^{(2)}(a+bx)}{u^{(1)}(a+bx)} \leq 1$, which is the condition in Corollary 4 when $N = 1$.

4 Conclusion

We have provided the necessary and sufficient conditions for unambiguous comparative statics on the precautionary effort decision with respect to an increase in the fixed or variable wage. We contribute to the literature in that we consider the presence of an independent background risk and take a step forward to study the impact of higher-order risk on the managers' optimal effort level under a commonly seen form of contract.

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