Competitive Insurance Markets Under Asymmetric Information
When Agents Make Mistakes

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Abstract: We analyze a competitive insurance market under asymmetric information when consumers can make mistakes. Mistakes by high risks impose a negative externality on low risks as the available coverage is further restricted. Furthermore, they exacerbate the equilibrium non-existence problem. Mistakes by low risks impose a positive externality on high risks because the available coverage is increased. The equilibrium non-existence problem is alleviated. We also show that, besides equilibria with a binding self-selection constraint on high risks, market outcomes where both self-selection constraints do not bind can arise. Furthermore, we study utilitarian welfare and find that zero mistakes are not optimal.

Keywords: private information; adverse selection; mistakes

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1. Introduction

The study of asymmetric information on competitive insurance markets started in the 1970ies and has since then produced a huge body of literature with both theoretical and empirical contributions. Rothschild and Stiglitz (1976) demonstrate that if insurers pursue a pure Cournot-Nash strategy a pooling equilibrium is not possible; the only possible equilibria are separating contracts but even they might not exist if there are not enough high risks in the market. Furthermore, the equilibrium might fail to be second-best efficient.

Researchers extended this setup in various directions. Wilson (1977) proposes an anticipatory equilibrium concept to remedy the non-existence problem and Riley (1979a,b) introduces a reactive equilibrium where firms add new contracts after observing an entrant’s offer. Finally, Miyazaki (1977) and Spence (1978) allow for cross-subsidization between policies which produces a second-best allocation in any case (Crocker and Snow, 1985).

Common to the vast majority of papers is the assumption that consumers know their risk type with certainty. In this article we examine what happens when this assumption is relaxed, i.e., when individuals make mistakes when assessing their risk and hence choose the “wrong” contract. Insurance companies will then have to take mistakes into account when pricing policies due to competitive pressure. We show how this affects the equilibrium contracts and consumer welfare. Apparently, the direct effect of mistakes is to reduce welfare because some agents do not behave in their best interest. Still, indirect effects as the slackening of incentive compatibility or better per-unit prices for consumers who are not mistaken have to be factored into the analysis. As a result net effects remain ambiguous. To shed light on this indeterminacy we provide a series of numerical examples of utilitarian social welfare under different assumptions on mistakes by consumers.

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4 Diamond (1974) studies the incentive effects of negligence standards in a model with random variations in care so that agents might fail to exert the intended level of care. Demougin and Fluet (2001) study the tradeoff between monetary incentives and monitoring in a principal-agent setup where agents can make mistakes. Thistle (2010) studies how mistakes affect efficiency wages. To the best of our knowledge there is no analysis of mistakes on competitive insurance markets under asymmetric information to date.
In experimental economics the notion of mistakes in individual decisions is quite salient. Harless and Camerer (1994) and Hey and Orme (1994) investigate decision making under risk and draw on different implicit assumptions about the “errors” made by their experimental subjects during the course of the experiment. Loomes and Sugden (1995) provide yet a third approach for a stochastic specification of choice under risk. Hey (1995) and Carbone and Hey (2000) survey the different avenues for incorporating mistakes in economic and financial decision making. In the evaluation of experimental data it is standard to incorporate an element of stochastic choice to capture the notion of errors or mistakes in the decision making of individuals.

When it comes to theoretical applications, the role of mistakes is less well understood. In the insurance literature some authors depart from the assumption that risk types are known by assuming that individuals are uninformed but can acquire information about risk e.g. through a genetic test. In the full information equilibrium there is no incentive to acquire information due to classification risk (Doherty and Thistle, 1996). Polborn et al. (2000) find similar results in life insurance markets where exclusivity may not hold and other authors extend this setup by introducing (clinical) productivity of the information in the form of primary or secondary prevention (Doherty and Posey, 1998; Barigozzi and Henriet, 2009; Peter et al., 2013). Still, individuals do not make mistakes when choosing contracts in these set-ups.

Recently, Huang et al. (2013) investigate ambiguity and how it relates it to competitive insurance markets under asymmetric information. When the risk of loss becomes ambiguous, consumers no longer know their probability of loss with certainty. However, they decompose the risk into general and specific risk with ambiguity on the general risk and private information about the specific risk, which they assume to be perfect. So also in their paper consumers do not choose contracts that were not designed for them which is the focus of our study.

Finally, also differences in risk perception and implications for the Rothschild-Stiglitz model have been discussed. Opp (2005) analyses biased beliefs and demonstrates

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5 Consent Laws can provide an incentive to become tested although a sequential Nash equilibrium might not exist depending on the cost of undertaking the test.
that it depends much on whether high or low risks exhibit optimism. This parallels our result that the role of mistakes by high risks and by low risks is different. Furthermore, Huang et al. (2010) demonstrate that hidden overconfidence can give rise to equilibria with advantageous\(^6\) rather than adverse selection. The reason is that the non-overconfident individual invests in prevention to decrease the probability of loss but also values coverage more than the overconfident type. Again, consumers always choose contracts designed for them contrary to our setup. As a result little is known about how errors or mistakes by consumers affect the functioning of competitive insurance markets under adverse selection.

The remainder of this paper is structured as follows. The next section outlines the model. Section 3 presents our findings. We distinguish between cases where only high risks, only low risks or both risk types are prone to make mistakes. In section 4 we outline the directions in which we intend to expand our analysis. The last section concludes the paper.

2. The Model

As in the classical model by Rothschild and Stiglitz (RS) we consider risk-averse consumers with initial wealth \(W\) who face a random loss of \(L\). Preferences are characterized by an increasing and concave vNM utility function \(U(\cdot)\) over final wealth, \(U'(\cdot) > 0\), \(U''(\cdot) < 0\). We assume that losses occur with probability \(p\) but consumers differ in the probability of loss: it is \(p^L\) for low risks and \(p^H\) for high risks with \(0 < p^L < p^H < 1\). A fraction \(\lambda^L\) of the population is low risk whereas a fraction \(\lambda^H\) of the population is high risk, so \(\lambda^L + \lambda^H = 1\). Insurers know the distribution of types but cannot observe individual risk types, i.e. risk type is private information. The average or pooled probability of loss is \(\bar{p} = \lambda^L p^L + \lambda^H p^H\).

The insureds, however, make mistakes when assessing their risk exposure. This means that some high-risk individuals will erroneously think they are low risk and con-

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\(^6\) De Meza and Webb (2001) were the first to point out the possibility of a negative correlation between risk and coverage in competitive insurance markets under asymmetric information.
versely that some low-risk individuals will erroneously think they are high risk. We denote by \( \varepsilon_i, i \in \{L, H\} \), the fraction of type \( i \) individuals who make mistakes and think they are the other type. We make the assumption that \( 0 \leq \varepsilon_L, \varepsilon_H \leq 0.5 \), i.e., the worst scenarios is that consumers are absolutely clueless about their true risk type and guess randomly whether they are low or high risk. Note that our set-up nests the classical Rothschild and Stiglitz (1976) model by assuming \( \varepsilon_L = \varepsilon_H = 0 \).

With regards to the supply side of the market we follow Rothschild and Stiglitz and assume that insurers compete for profits. We allow for price- and quantity competition between companies. Due to perfect competition in equilibrium insurers must break even so that contract-specific breakeven constraints are binding. As such insurance pricing anticipates that some consumers will make mistakes in their choice so that breakeven contracts have to be based on the following parameters:

\[
\tilde{p}_L = \frac{\lambda_H \varepsilon_H p^H + \lambda_L (1 - \varepsilon_L) p_L}{\lambda_H \varepsilon_H + \lambda_L (1 - \varepsilon_L)}, \quad \tilde{p}_H = \frac{\lambda_H (1 - \varepsilon_H) p^H + \lambda_L \varepsilon_L p_L}{\lambda_H (1 - \varepsilon_H) + \lambda_L \varepsilon_L}.
\]

It is straightforward to show that \( \tilde{p}_L \) is increasing in mistakes by low and high risks, that \( \tilde{p}_L(0,0) = p^L \), and that \( \tilde{p}_L(0.5,0.5) = \bar{p} \). The reason is that if more low risks make mistakes, there are fewer low risks who buy the policy designed for them such that the probability of loss in the group of individuals who purchase this contract increases. Likewise if more high risks make mistakes, there are more high risks who purchase the low risk contract erroneously such that the probability of loss in the group of individuals who purchase this contract increases. Apparently, in the absence of mistakes only low risks buy the policy designed for them so that the probability of loss coincides with \( p^L \); conversely, if all individuals assess their risk type randomly, the group of purchasers of the low risk policy will on average exhibit the characteristics of the pooled population. Similar reasoning establishes that \( \tilde{p}_H \) is decreasing in mistakes by low and high risks, that \( \tilde{p}_H(0,0) = p^H \), and that \( \tilde{p}_H(0.5,0.5) = \bar{p} \). The mathematical derivation can be found in Appendix A.

In terms of equilibrium analysis, we will utilize the RS equilibrium concept, i.e. (i) no contract in the equilibrium set makes negative expected profits and (ii) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. To develop some intuition we will analyze polar cases in which only low risks or only high risks make mistakes. Then, we will also investigate the nature of equilibrium more in detail.
3. Results

3.1 Mistakes by H-types only

Let us first analyze a situation in which some high risks erroneously think they are low risks but low risks do not make mistakes. Formally, this corresponds to the assumption that $\varepsilon_H > 0$ and $\varepsilon_L = 0$. Consequently, there will be some high risks among individuals purchasing the policy designed for low risks which has to be taken into account when pricing the contract.

Graphically, the low-risk fair-odds line becomes flatter due to the fact that also high risks buy the contract designed for the low risks, see Figure 1. On the $x$-axis we have wealth in the no-loss state ($W_N$) whereas we have wealth in the accident-state on the $y$-axis ($W_A$). In our case, the contract designed for high risks, $H^*$, is unaffected whereas the L-type contract, $L'$, moves along the high-risk indifference curve so that it breaks even under the new fair-odds line. Due to the single-crossing property this implies a lower indifference curve for the low risks.

Consequently, high-risk expected utility is unaffected by the mistakes made by some high risks. Those who correctly assess their risk type still receive contract $H^*$ as if there were no mistakes, those who erroneously think they are low risk receive contract $L'$ and are equally well off due to incentive compatibility. However, low risks no longer receive contract $L$ as in the absence of mistakes but contract $L'$ which constitutes a decrease.
in expected utility for them. In this sense, mistakes by high risks constitute an externality due to the fact that they lower L-type expected utility whereas H-type expected utility remains unaffected. Hence, an increase in $\epsilon_H$ represents a Pareto-deterioration and social utilitarian welfare is lower. Also note that due to the fact that $L'$ lies on a strictly lower indifference curve the Nash equilibrium non-existence problem is aggravated. Consequently, if the equilibrium exists under no mistakes by high risks it might fail to exist when increasing the high-risk propensity of mistakes. Said differently, the critical fraction of high risks necessary to ensure that a Nash equilibrium in separating policies exists is larger, the higher the propensity of high risks to make mistakes.

3.2 Mistakes by L-types Only

Let us now examine what happens when some low risks erroneously think they are high risk whereas high risks do not make mistakes. Technically, this corresponds to the assumption that $\epsilon_H = 0$ and $\epsilon_L > 0$. If some low risks buy the policy designed for the high risks, this has to be taken into account when pricing contracts. Consequently, the high-risk fair-odds line becomes steeper due to the presence of low risks in that contract, see Figure 2.

Therefore, high risks receive contract $H'$ and low risks contract $L'$ in the new equilibrium. $H'$ provides a higher expected utility for high risks than $H^*$ so mistakes by L-types represent a positive externality to H-types. Also L-types who do not make mistakes receive a contract that provides a larger expected utility to them than contract $L$. The reason is that the better contract for H-types that becomes affordable due to the presence of L-types who made a mistake alleviates the incentive compatibility constraint. So also L-types who do not make mistakes benefit from L-types who make mistakes. However, the L-types who erroneously take contract $H'$ are worse off than if they selected contract $L'$ and they are also worse off than if they had contract $L$. 
We see that a situation in which some L-types make mistakes is not comparable to a situation without mistakes in the Pareto-sense as some individuals are better off (H-types and L-types who do not make mistakes) and some are worse off (L-types who do make mistakes). This also shows that the welfare implications of mistakes by L-types are ambiguous. Also note that the Nash equilibrium non-existence problem is alleviated because low risks who do not make mistakes achieve a higher indifference curve. So if the equilibrium does not exist under no mistakes by low risks it might exist when increasing the low-risk propensity of mistakes. Said differently, the critical fraction of high risks necessary to ensure that a Nash equilibrium in separating policies exists is lower, the higher the propensity of high risks to make mistakes.

3.3 Alternative Equilibria

The above analysis was under the implicit assumption that the equilibrium is characterized by full insurance for buyers of the high-risk policy and a binding self-selection constraint on them to determine the available coverage for the buyers of the low-risk policy. To show that other types of equilibria might exist consider the extreme case in which $\epsilon_L = \epsilon_H = 0.5$. Then, only contracts on the pooled fair-odds line can be offered but still some consumers think they are low risk and some they are high risk.
Consider Figure 3 which summarizes a possible scenario like this. In this situation, cream skimming low risks out of a pooling contract is no longer possible. Assume that contract $L'$ is offered on the fair-odds line for pooled contracts to maximize low-risk expected utility. In the Rothschild-Stiglitz model this cannot be a Nash equilibrium because under the assumption that everybody offers $L'$, it would be profitable to deviate to $G$ to attract low risks and make a profit on them. In our situation, contract $G$ still attracts low risks but also all those high risks who erroneously think they are low risk. Therefore, $G$ is no longer a profitable deviation from $L'$ because it lies above that fair-odds line for individuals who think they are low risk. But still, $L'$ cannot be sustained as a Nash equilibrium in the market.

Assume that contract $G'$ is offered in addition to $L'$. It attracts individuals who believe they are high risk out of contract $L'$ and is below the fair-odds line for individuals who think they are high risk. Paradoxically, “cream”-skimming here refers to high risks or rather individuals who think they are high risk. Due to the fact that there are enough low risks who erroneously buy the high risk policy this will be a profitable deviation from $L'$ which cannot be sustained as a Nash equilibrium. The solution to this is to offer contract $H'$ in addition. Notice that $\{L', H'\}$ satisfies incentive compatibility, i.e., individuals who
think they are low risk are not attracted by contract $H'$, and individuals who think they are high risk are not attracted by contract $L'$. Note that both self-selection constraints are slack, i.e., high risks are strictly better off with their contract and low risks are strictly better off with theirs, too. Also note that there is no price discrimination but quantity discrimination only. One can show that the locus of mistake propensities where the equilibrium changes from a binding (B) to a non-binding (NB) self-selection constraint on high risks is decreasing in $(\varepsilon_L, \varepsilon_H)$-space, see Appendix B.

In terms of welfare there are various effects involved. First, high risks buying contract $H'$ are better off than in the absence of mistakes because $H'$ offers full coverage at a better rate. High risks who mistakenly think they are low risk and buy contract $L'$ can be better off or worse off than if they receive a full-coverage contract at their fair rate. This depends on the location of $L'$ on the pooling line. Also for low risks the consequences in terms of expected utility when buying contract $L'$ or $H'$ are ambiguous because the contract determined by incentive compatibility in the absence of mistakes might give them more or less expected utility. Consequently, this situation is in general not Pareto-comparable to a scenario without mistakes and the net effect on social utilitarian welfare is ambiguous.

4. Utilitarian Social Welfare

In our model social utilitarian welfare is given by the following expression:

$$SW(\varepsilon_L, \varepsilon_H) = \lambda_L [\varepsilon_L EU_L(H') + (1 - \varepsilon_L)EU_L(L')] + \lambda_H [\varepsilon_H EU_H(L') + (1 - \varepsilon_H)EU_H(H')]$$

It reflects the fact that individuals who make mistakes choose contracts designed for the other risk type but still their well-being needs to be assessed based on their true underlying risk type. $EU_i$ is shorthand for expected utility of a type $i$ individual, $i \in \{L, H\}$. Increases in the propensity to make mistakes affect welfare in two respects: First, more individuals choose contracts designed for the other risk type (direct effect); second, the equilibrium menu of contracts changes (indirect effect). The following analysis will shed light on the different effects.

Let us start with mistakes by L-types. With a slight abuse of notation we obtain that
\[
\frac{\partial SW(\varepsilon_L, \varepsilon_H)}{\partial \varepsilon_L} = \lambda_L [EU_L(H') - EU_L(L')] + \lambda_L \left[ \varepsilon_L EU'_L(H') \frac{\partial H'}{\partial \varepsilon_L} + (1 - \varepsilon_L) EU'_L(L') \frac{\partial L'}{\partial \varepsilon_L} \right] \\
+ \lambda_H \left[ \varepsilon_H EU'_H(L') \frac{\partial L'}{\partial \varepsilon_L} + (1 - \varepsilon_H) EU'_H(H') \frac{\partial H'}{\partial \varepsilon_L} \right].
\]

The incentive compatibility constraint on low risks is non-binding in any case; therefore, the first term is negative because low risks are better off with their contract than with the high-risk contract. This is a negative direct effect of more low risks making mistakes. The second and third term describe that changes in the propensity to make mistakes are accompanied by adjustments in the equilibrium menu of contracts which in turn alter expected utility. For the high-risk contract we obtain that
\[
\frac{\partial}{\partial \varepsilon_L} (W - \bar{p}H) = -\frac{\partial \bar{p}}{\partial \varepsilon_L} H > 0.
\]

As more L-types make mistakes, fair pricing needs to reflect the lower probability of accident in the group of buyers of the policy \(H\). As a result both high risks and low risks who buy the high-risk policy are better off. The effect on the low-risk contract depends on the expected utility also depends on the underlying regime. Under (B) high risks are equally well off under their contract than under the low-risk contract; therefore, their expected utility increases in \(\varepsilon_L\). For low risks the increase in coverage is accompanied by an increase in the per-unit price of insurance (as long as \(\varepsilon_H > 0\)) so that the net effect is indeterminate. Under (NB) we can exploit the envelope theorem to find that low risks as worse off by the increase in \(\varepsilon_L\) because the per-unit price of coverage increases. High risks who buy the low risk policy are also worse off because the level of coverage is smaller\(^7\) and the per-unit price is larger.

Next we investigate mistakes by H-types. Similar to above we obtain that
\[
\frac{\partial SW(\varepsilon_L, \varepsilon_H)}{\partial \varepsilon_H} = \lambda_H [EU_H(L') - EU_H(H')] + \lambda_L \left[ \varepsilon_L EU'_L(H') \frac{\partial H'}{\partial \varepsilon_H} + (1 - \varepsilon_L) EU'_L(L') \frac{\partial L'}{\partial \varepsilon_H} \right] \\
+ \lambda_H \left[ \varepsilon_H EU'_H(L') \frac{\partial L'}{\partial \varepsilon_H} + (1 - \varepsilon_H) EU'_H(H') \frac{\partial H'}{\partial \varepsilon_H} \right].
\]

\(^7\) This is conditional on the assumption that insurance is not a Giffen good, see Appendix B.
The first term represents the direct effect of mistakes by high risks. Under (B), it is nil due to incentive compatibility whereas under (NB) it is negative because the high-risk self-selection constraint is slack implying that high risks are better off with their contract than with the low-risk contract. The second and third term describe the implications on optimal contracts. For the high risk contract it is
\[
\frac{\partial}{\partial \varepsilon_H} (W - \hat{p}^H L) = - \frac{\partial \hat{p}^H}{\partial \varepsilon_H} L \geq 0,
\]
with a strict inequality if \( \varepsilon_L > 0 \). If more high risks make mistakes, there are fewer high risks in the group of purchasers of the contract \( H' \) resulting in a lower per-unit price of coverage. Hence, everybody who buys the high-risk contract is better off if \( \varepsilon_H \) increases. The effect on the low-risk contract depends again on the equilibrium regime. The reasoning is very similar to before and will be omitted to save space.

5. Numerical Examples

To shed more light on the various effects, we provide a series of numerical examples to illustrate the equilibrium switching locus and the aggregate welfare effects of mistakes. We model preferences drawing on iso-elastic utility,
\[
U(W) = \begin{cases} 
W^{1-\gamma} - 1, & \gamma > 0, \gamma \neq 1, \\
\frac{1 - \gamma}{\log(W)}, & \gamma = 1.
\end{cases}
\]
The parameters for the benchmark case are given as follows:
\( W = 100, L = 50, p^L = 0.1, p^H = 0.3, \lambda^H = \lambda^L = 0.5, \gamma = 2 \).
First we analyze how the locus of mistakes where the equilibrium changes from the (B) to the (NB) case depends on some of the model’s parameters. The impact of risk aversion is illustrated in Figure 4. We can see that an increase in (relative) risk aversion of the underlying utility function shifts the locus to the Northeast. An increase in risk aversion affects both, the incentive compatibility constraint and the optimality condition for low risks. With increased risk aversion the self-selection condition becomes slack indicating that the level of coverage for low risks must increase (see also Crocker and Snow, 2008). An increase in risk aversion also implies that low risks value insurance coverage more on the \( \hat{p}^L \) fair-odds line. Also this indicates an increase in the level of insurance demand. From the
Figure we conclude that in our case this last effect is stronger than the first effect becomes with an increase in risk aversion the (NB) region becomes smaller.

Next let us investigate a change of the proportion of high risks in the market. This is illustrated in Figure 5. We can see that as the share of high risks decreases the (NB) region becomes smaller. Intuitively speaking, when the proportion of high risks is smaller, the loading on insurance coverage for low risks on the pooled fair-odds line becomes smaller so that the utility maximizing level of insurance coverage is higher (excluding Giffen-type behavior).
Finally, we can investigate the probability of loss for high risks (the reasoning for low risks is analogous). Results are illustrated in Figure 6. An increase in the high-risk probability has several effects. First, the per-unit price of insurance for low risks increases implying that less coverage is optimal (under the assumption that insurance is not Giffen). Second, the incentive compatibility constraint is affected. High risks are worse off with their full insurance policy implying that less coverage on the low-risk fair-odds line is needed to make them equally happy. However, they value coverage more due to the increased risk and the per-unit price has increased so that more coverage is needed to compensate this effect. In our case, the effect on low risks prevails because the (NB) region becomes larger as $p^H$ rises.

![Figure 6: The Equilibrium-Switching Locus and the High-Risk Probability of Loss](image)

6. Discussion and Conclusion

In this paper, we investigate the impact of mistakes on competitive insurance markets under asymmetric information. Prior literature has focused on situations, in which consumers choose contracts according to their risk type, i.e., they know their risk type with certainty or they employ rational expectations and choose precisely according to an average type. We relax this assumption by allowing individuals to choose the “wrong” contract, i.e. the contract that was actually designed for the other type of consumers.
We find that it makes a difference whether low risks or high risks make mistakes. In any case, this will have to be anticipated by insurers when setting prices due to perfect competition on the market. If only high risks make mistakes, the fair-odds line for contracts designed for low risks becomes flatter to take errors by some high risks into account. Consequently, the coverage available to low risks is further restricted which constitutes a Pareto-deterioration and lowers social welfare. Also, the equilibrium non-existence problem is aggravated.

If only low risks make mistakes, the fair-odds line for contracts designed for high risks becomes steeper so that the incentive compatibility constraint is alleviated. In this sense, mistakes by low risks constitute a positive externality to high risks and low risks who assess their risk type correctly. As low risks who make mistakes are strictly worse off, welfare implications are ambiguous. The equilibrium non-existence problem is alleviated, i.e., it is more likely that a Nash equilibrium exists.

Finally, when both types make mistakes, we investigate the case in which the fair-odds lines collapse. This can only happen if they collapse on the fair-odds line for pooled contracts. A single pooling contract is not feasible in equilibrium; however, a no-price discrimination equilibrium with quantity discrimination only emerges. Again, welfare implications are ambiguous as various effects are involved.

Overall, mistakes by economic agents have widely been neglected in prior literature. As we show in this paper, this seems unjustified because there are surprising implications when this assumption is relaxed. Empirical research should clarify whether there are market-specific differences, i.e., whether some lines of insurance are more prone to consumer mistakes than others. This has exciting policy implications which we hope to explore in the future.
References


Appendix A

When applying the quotient rule to $\bar{p}^L = \bar{p}^L(\varepsilon_L, \varepsilon_H)$ and $\bar{p}^H = \bar{p}^H(\varepsilon_L, \varepsilon_H)$ we find that:

$$\frac{\partial \bar{p}^L}{\partial \varepsilon_L} = \frac{\lambda^L \lambda^H \varepsilon_H (p^H - p^L)}{[\lambda^H \varepsilon_H + \lambda^L (1 - \varepsilon_L)]^2} \geq 0,$$

$$\frac{\partial \bar{p}^L}{\partial \varepsilon_H} = \frac{\lambda^L \lambda^H (1 - \varepsilon_L)(p^H - p^L)}{[\lambda^H \varepsilon_H + \lambda^L (1 - \varepsilon_L)]^2} > 0,$$

$$\frac{\partial \bar{p}^H}{\partial \varepsilon_L} = -\frac{\lambda^L \lambda^H (1 - \varepsilon_H)(p^H - p^L)}{[\lambda^H (1 - \varepsilon_H) + \lambda^L \varepsilon_L]^2} < 0,$$

$$\frac{\partial \bar{p}^H}{\partial \varepsilon_H} = -\frac{\lambda^L \lambda^H \varepsilon_L (p^H - p^L)}{[\lambda^H (1 - \varepsilon_H) + \lambda^L \varepsilon_L]^2} \leq 0,$$

with strict inequalities as long as $\varepsilon_L, \varepsilon_H > 0$.

Appendix B

Let us assume that the level of coverage determined by the incentive compatibility constraint coincides with the one that maximizes the expected utility of low risks on the respective fair-odds line. Such a level exists due to continuity. The incentive compatibility constraint is given by

$$u(W_F) - [p^H u(W_A) + (1 - p^H) u(W_N)],$$

with $W_F = W - \bar{p}^H L$ being full coverage for buyers who think they are high risk, $W_A = W - \alpha \bar{p}^L L - (1 - \alpha) L$ being the accident state, and $W_N = W - \alpha \bar{p}^L L$ being the no-accident state for buyers of the partial insurance policy who think they are low risk. Let us denote the incentive compatibility constraint by $F = F(\alpha, \varepsilon_L, \varepsilon_H)$. The contract which maximizes L-type utility on the respective fair-odds line is given by

$$(1 - \bar{p}^L)p^L L u'(W_A) - (1 - p^L)\bar{p}^L L u'(W_N) = 0.$$

Let us denote this first-order condition by $G = G(\alpha, \varepsilon_L, \varepsilon_H)$. Let us inspect the behavior of the level of coverage in the $(\varepsilon_L, \varepsilon_H)$-plane by considering the directional derivative into the direction $v = (v_L, v_H)$. By means of the implicit function theorem, we obtain that
\[ \partial \epsilon = - \left( v_L F_{\epsilon L} + v_H F_{\epsilon H} \right) \]

from incentive compatibility and
\[ \partial \epsilon = - \left( v_L G_{\epsilon L} + v_H G_{\epsilon H} \right) \]

from the utility-maximization for L-types. As both conditions identify the same level of coverage, the directional derivatives coincide resulting in
\[ v_H = \frac{G_{\epsilon L} F_{\epsilon L} - F_{\alpha} G_{\epsilon L}}{F_{\alpha} G_{\epsilon H} - G_{\alpha} F_{\epsilon H}} v_L. \]

The various derivatives are given by
\[ F_{\alpha} = -p^H(1 - p^L)u'(W_A) - (1 - p^H)p^L u'(W_N) < 0, \]
\[ F_{\epsilon L} = -\frac{\partial \tilde{p}^H}{\partial \epsilon L} u'(W_F) + \alpha L \frac{\partial \tilde{p}^L}{\partial \epsilon L} [p^H u'(W_A) + (1 - p^H)u'(W_N)] > 0, \]
\[ F_{\epsilon H} = -\frac{\partial \tilde{p}^H}{\partial \epsilon H} u'(W_F) + \alpha L \frac{\partial \tilde{p}^L}{\partial \epsilon H} [p^H u'(W_A) + (1 - p^H)u'(W_N)] > 0, \]
\[ G_{\epsilon L} = (1 - \tilde{p}^L)^2 p^L u''(W_A) + (1 - p^L)(\tilde{p}^L L)^2 u''(W_N) < 0, \]
\[ G_{\epsilon H} = -\frac{\partial \tilde{p}^L}{\partial \epsilon H} p^L u'(W_A) - (1 - p^L)\frac{\partial \tilde{p}^L}{\partial \epsilon L} u'(W_N) - (1 - \tilde{p}^L)p^L \alpha \frac{\partial \tilde{p}^L}{\partial \epsilon L} u''(W_A) \]
\[ + (1 - p^L)p^L \alpha \frac{\partial \tilde{p}^L}{\partial \epsilon L} u''(W_N), \]
\[ G_{\epsilon H} = -\frac{\partial \tilde{p}^L}{\partial \epsilon H} p^L u'(W_A) - (1 - p^L)\frac{\partial \tilde{p}^L}{\partial \epsilon H} u'(W_N) - (1 - \tilde{p}^L)p^L \alpha \frac{\partial \tilde{p}^L}{\partial \epsilon H} u''(W_A) \]
\[ + (1 - p^L)p^L \alpha \frac{\partial \tilde{p}^L}{\partial \epsilon H} u''(W_N). \]

The first one is negative because an increase in coverage at the L-type rate is beneficial for H-types; together with the minus from the incentive compatibility condition the overall sign is negative. The second is positive because when more low risks make mistakes the rate for H-types decreases whereas the rate for L-types increases. The reasoning for the third term is analogous: When more H-types make mistakes the rate for H-types gets cheaper whereas the one for L-types becomes more expensive. The fourth derivative is negative as from the second-order condition for optimal insurance demand. The fifth and sixth expression are ambiguous; the reason is that insurance might be a Giffen good, i.e., its demand might increase when the per-unit price of insurance coverage increases (see Hoy and Robson, 1981; Briys et al., 1989). Under the reasonable assumption that such be-
havior does not arise, both derivatives are negative indicating that an increase of the per-unit price of coverage, either via an increase in L-type mistakes or H-type mistakes, implies that insurance demand decreases. Taken together we obtain that

\[
\frac{G_\alpha F_{\varepsilon L} - F_\alpha G_{\varepsilon L}}{F_\alpha G_{\varepsilon H} - G_\alpha F_{\varepsilon H}} < 0,
\]

so that a marginal change of H-type mistakes must be accompanied by a marginal change in L-type mistakes of opposite sign to keep both incentive compatibility and optimality for L-types satisfied. As a result the equilibrium switching locus is decreasing in the \((\varepsilon_L, \varepsilon_H)\)-plane.