Insurance Fraud in a Rothschild-Stiglitz World*

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Abstract

This paper combines two strands of literature in insurance economics: Adverse selection and *Ex post* moral hazard. The paper derives the characteristics of a separating contract à la Rothschild & Stiglitz (1976) – assuming it exists – in a world where agents are endowed with private information regarding the state of the world AND their probability of loss. The model therefore combines costly state verification without commitment with the signalling of risk types. In such a world, it is quite possible for the low risk agents to have full insurance even if they need to signal their risk type to the principal. It is also possible for the low risk agents to pay a higher premium than their his type was known. I show that low-risk individuals are more likely to commit insurance fraud and are less likely to be audited when adverse selection is present than if risk types were known. In other words, there is always more fraud and more successful fraud when adverse selection in risk type exists in an economy. Finally, I show that insurance fraud may be reduced if insurers are restricted to offer a unique contract to all potential policyholders.

JEL classification: D82, G2, C72.

Keywords: Non-commitment, Ex post moral hazard, Separating contracts, Type signalling.

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Abstract. This paper combines two strands of literature in insurance economics: Adverse selection and *Ex post* moral hazard. The paper derives the characteristics of a separating contract à la Rothschild & Stiglitz (1976) – assuming it exists – in a world where agents are endowed with private information regarding the state of the world AND their probability of loss. The model therefore combines costly state verification without commitment with the signalling of risk types. In such a world, it is quite possible for the low risk agents to have full insurance even if they need to signal their risk type to the principal. It is also possible for the low risk agents to pay a higher premium than their his type was known. I show that low-risk individuals are more likely to commit insurance fraud and are less likely to be audited when adverse selection is present than if risk types were known. In other words, there is always more fraud and more successful fraud when adverse selection in risk type exists in an economy. Finally, I show that insurance fraud may be reduced if insurers are restricted to offer a unique contract to all potential policyholders.

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1 Introduction

Another possible title for this paper could be "Optimal contracting in the presence of adverse selection and ex post moral hazard"

Information asymmetry problems, such as adverse selection and moral hazard, have been extensively studied. The literature that examines the compounding impact of different types of information asymmetry is less extensive, however. It is the goal of this paper to fill part of that gap by examining the combination of *ex post* moral hazard in the spirit of Townsend (1979) with adverse selection in the spirit of Rothschild and Stiglitz (1976). The model I develop therefore addresses the problem of insurance fraud when insured agents have access to two sources of private information: 1- They know their probability of accident *ex ante*; and 2- They know what is the true state of the world *ex post*. I will also assume that although the insurer can verify the true state of the world *ex post* by paying an auditing fee (à la Townsend 1979), she is unfortunately unable to commit to any verification strategy *ex ante*. In essence, I combine an agent's adverse selection and *ex post* moral hazard in a world where the principal cannot commit.

The costly state verification approach developed in Townsend (1979) has a natural application in explaining the insurance fraud phenomenon. In a costly state verification setup, the optimal insurance contract between the principal and the agent stipulates a no-auditing region where a fixed payment is made, and an auditing region (where audits are always performed) where the agent has full insurance less a deductible (see also Gale and Hellwig, 1985). Mookherjee and Png (1989) and Bond and Crocker (1997) showed that truth-telling can be achieved using stochastic audits, a Pareto superior approach to Townsend's since less money is wasted on audits. The traditional mechanism design literature has concentrated on situations where it is optimal for all players to tell the truth. Commitment is an important component of any contractual relationship, and particularly so in a multi-period setting since agents are generally better off if they can agree to a long-term full commitment contract. The problem is that when the time comes to implement the contract, both players may find it in their best interests to renegotiate to attain *ex post* efficiency. Another way to present the challenge faced by truth-telling mechanisms is that they do not seem to have been effective in reducing income-tax fraud (see Graetz *et al.*, 1986, for an early contribution) or insurance fraud (see Picard 1996).

It is because fraud is still present in the economy that I assume that the principal is unable to commit (see Khalil, 1997, and Krasa and Villamil, 2000, for a discussion of the commitment versus non-commitment debate). As presented in Boyer (2004), the principal's commitment to a strategy is often unreasonable since the players can be better off *a posteriori* if the principal deviates from the strategy agreed upon *a priori* (see also Picard 2013 for an excellent survey of the insurance fraud literature in such a context).

In the current paper, I assume an economy composed of high risk and of low risk agents (an agent's risk type is thus defined as his probability of loss) in proportions that are known to all market participants. Each agent privately knows his own risk type. After purchasing the insurance contract, an unfortunate event may occur to the agent (i.e., he may suffer a loss). Such information is known only to the agent. The agent must then report to the principal one of the two possible states of the world: Loss or No loss. Upon hearing the agent's report, the principal decides to audit the report or not. Auditing is costly to the principal, but it is perfect in the sense that the true state of the world is revealed to her.

As we will see, even in the presence of insurance fraud there are parameter values such that a separating equilibrium à la Rothschild-Stiglitz exists. This separating equilibrium has the same basic characteristics of the traditional Rothschild-Stiglitz contract in the sense that the high risk type is insured as if his type was known, whereas the low risk type must signal his lower probability of loss by accepting a contract that is less appealing than if his type was known. This signal is achieved mostly by accepting a contract that provides the low-risk agent less protection in the event of a loss that if his risk type was known.

Interestingly, the presence of insurance fraud can cause situations where the low-risk agent signals his type not only by reducing the indemnity payment, but also by increasing the premium he pays. This is a new result in the literature since it is always the case in the classic insurance economics literature that a lower indemnity payment is associated with a lower premium.

A second particular situation that can happen as a result of insurance fraud is for the low risk agents to be fully insured. This result may explain the popularity of replacement-cost new contracts (see Dionne and Gagné 2001) and of low-deductible policies (see Sydnor 2006). This result comes solely from the low risk agents' behavior as they must increase their likelihood of committing fraud to compensate for the lower expected utility they receive from their need to signal their risk type through contracts that offet them lower indemnity payments. The low-risk agent's behavior with respect to fraud partially explains why they often feel justified padding claims (see Tennyson 2002 and Miyazaki 2009). In contrast, when agent types are known, it can be easily shown that the high-risk agents are more prone to commit fraud as the probability of fraud increases with the probability of loss. With adverse selection, the low risk agents' probability of fraud may become higher than the high risk agents' probability of fraud, a situation that would never happen if risk types were known . This is due to the fact that it is easier to hide a fraudulent claim when many policyholders are already filing insurance claims.

One final result of the paper is that, assuming for some reason insurers are forbidden from offering a menu of contracts, then there are parameter values such that the amount of fraud in the economy is reduced when all agents are required to purchase the same contract. In other words, limiting the number of items in the contract menu may be Pareto improving.

The remainder of the paper is structured in the following way. In the next section of the paper I present and solve the claiming game between an agent and the principal assuming that the agent's type is known. The core of the paper is found in Section 3. It presents the signalling game that the two types of agents play with the principal and solve for the menu of contracts that yields an equilibrium. Section 3 also discusses the impact of adverse selection and type signalling on the amount of insurance fraud in the economy. I present in Section 4 the structure of the pooling contract if it existed, and examine the impact on the amount of fraud in the economy. Finally, Section 5 concludes.

2 Ex Post Moral Hazard with Two Types

2.1 Some basic assumptions

Let us start with some basic assumptions; readers will recognize the same setup as in Boyer (2000), Bourgeon and Picard (2014), and in the survey of Picard (2013). The principal/insurer is risk-neutral. Agents/policyholders are risk-averse Neumann-Morgenstern utility maximizer, with $U(W)$ being their utility function over final wealth W (and with $U'(W) > 0$, $U''(W) < 0$). All agents have exogenous wealth given by Y. Agents are exposed to a potential loss of size λ, irrespective of the agent type, with either probability π_L if the agent is a low risk type, or probability π_H if the agent is a high risk type. The probability that an agent's risk-type is low is given by ρ. The distribution of losses given by the quadruplet $\Xi = \{\rho, \pi_L, \pi_H, \lambda\}$ is common knowledge. The payment received by an agent is given by $\beta \in \{\beta_L, \beta_H\}$, where β_L (resp. β_H) is the indemnity payment to the agent that purchased the contract designed for the low risk (resp. high risk) type. When the claiming game is played, the players must take into account not only the value of λ, but also those of β_L (resp. β_H). Finally, a contract-specific premium must be paid. Let α_H be the premium for the high risk type, and α_L be the premium for the low risk type. Table A in the Appendix summarizes the notation.

The sequence of the game is displayed in Figure 1. The claiming game is played in stages 2 to 5. In stage 1 the contract is signed. In stage 0, Nature privately reveals to each agents his risk type: π_L or π_H.

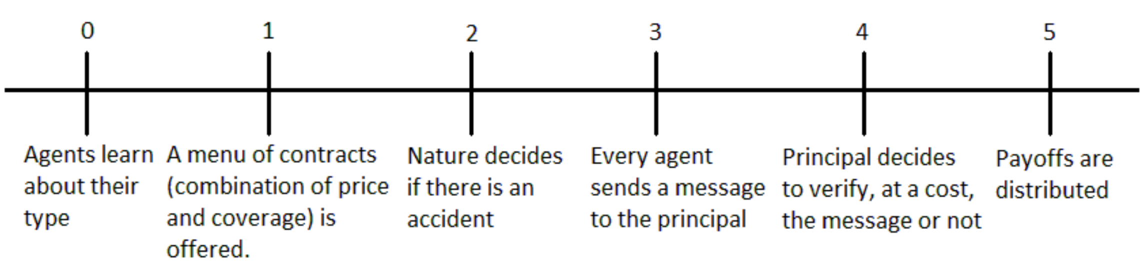

Figure 1: Sequence of play

Nature also plays in stage 2 by revealing to each agent independently and privately whether he suffered a loss or not. All agents then independently send a message to the principal. This message is related to the state of the world that each agent has observed - that is, whether they suffered a loss or not. Messages sent by the agent to the principal are costless, whether the message is truthful or not. The principal does not know what state of the world each agent observed unless she conducts a costly audit (cost of c per audit) in stage 4. Finally, the payoffs are paid in stage 5.

If the principal verifies the state of the world, then she must compensate the audited agent according to the benefit corresponding to his true loss (see Bourgeon and Picard, 2014, for an alternate approach, but with the same basic insurance fraud model derived in Boyer 2004), which is zero in the current setting. An agent caught committing fraud (i.e., reporting the wrong state of the world to the principal and being audited) must incur two types of penalties. First, there is penalty k_1, denominated in utility terms. k_1 is a **deadweight loss** in the sense that it is only incurred by the agent and does not profit the principal. Second, the agent also pays penalty k_2 **to the principal**. Table B in the appendix lists all the possible payoffs to the players contingent on their actions.

Letting η_i be the probability that an agent of type $i \in \{L, H\}$ reports having a loss when he did not have one and letting ν_i be the probability that the insurer audits the report made by an agent of type $i \in \{L, H\}$, the players' expected payoffs, are given by,

$$\begin{aligned} EU &= \pi_i U\left(Y - \alpha_i - \lambda + \beta_i\right) + \left(1 - \pi_i\right)\left(1 - \eta_i\right) U\left(Y - \alpha_i\right) \\ &\quad + \left(1 - \pi_i\right)\eta_i \left\{\nu_i \left[U\left(Y - \alpha_i - k_2\right) - k_1\right] + \left(1 - \nu_i\right) U\left(Y - \alpha_i + \beta_i\right)\right\} \end{aligned} \tag{1}$$

for the agents, and by

$$PI = \alpha_i - \left\{\pi_i \beta_i + \left(1 - \pi_i\right)\eta_i\left(1 - \nu_i\right)\beta_i + c\nu_i\left[\pi_i + (1 - \pi_i)\eta_i\right] + \left(1 - \pi_i\right)\eta_i \nu_i k_2\right\} \tag{2}$$

for the insurer. These objective functions are presented in more detail below. The remainder of this section will solve the fraud game that is played between the players to simplify the maximization problem.

2.2 The Claiming game

It is clear that contract parameters are fixed for the duration of the game once agents have signed the insurance contract. Thus, given that β_L, β_H, α_H and α_L are chosen in stage 1, the solution to the claiming game played in stages 2 through 5 gives us a Perfect Bayesian Nash Equilibrium (PBNE) which, by definition is a sextuplet in this game. Figure 2 is an extensive-form representation of the claiming game.

Let η_i represent the probability that the agent announces a loss λ when he did not suffer a loss. ν_i then represents the probability that an agent who reported a loss of λ is audited. Lemma 1 provides the unique PBNE in mixed equilibrium of this claiming game.

Lemma 1 *For each agent type $i \in \{L, H\}$, the unique PBNE in mixed strategy of this game is such that: 1- the agent who suffered a loss always tells the truth; 2- the agent who suffered no loss reports a loss*

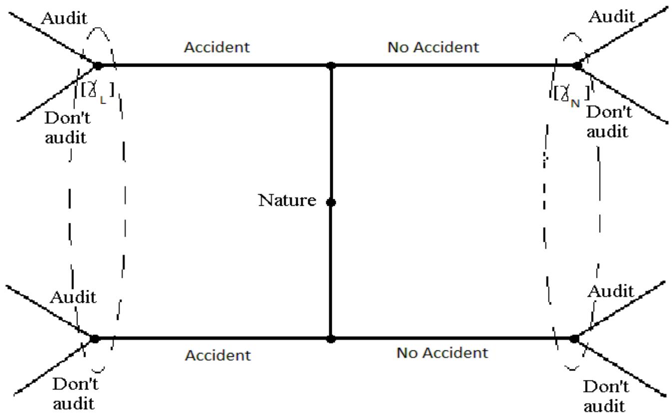

Figure 2: Extensive form of the claiming game when the agent type is known.

with probability η_i; 3- the principal never audits a no loss report; and 4- the principal audits a claim with probability ν_i. In the current setting, η_i and ν_i are given by

$$\eta_i = \left(\frac{c}{\beta_i + k_2 - c}\right)\left(\frac{\pi_i}{1-\pi_i}\right) \tag{3}$$

$$\nu_i = \frac{U\left(Y - \alpha_i + \beta_i\right) - U\left(Y - \alpha_i\right)}{U\left(Y - \alpha_i + \beta_i\right) - \left[U\left(Y - \alpha_i - k_2\right) - k_1\right]} \tag{4}$$

Proof:[1] *See Appendix 7.2* •

Lemma 1 defines the optimal behavior of each player as a function of the state of the world. Say full insurance is offered to all so that $\beta_L = \beta_H = \lambda$. Clearly the high risk type would commit more fraud ($\eta_H > \eta_L$) because $\frac{\partial \eta_i}{\partial \pi_i} > 0$. Of course, the probability of auditing does not depend on the type of the agent when $\beta_L = \beta_H$ since that information would be unknown to the inusrer.

The principal thus derives two important benefits from screening risk types through the use of a menu of contracts. First, screening allows her to know each agent's expected loss as in any Rothschild-Stiglitz equilibrium in insurance markets. Second, and surely of greater importance when examining the insurance fraud phenomenon, screening allows the principal to tailor her audit strategy to each agent's risk type, which determines each agent's fraudulent claiming behavior.

[1] For the PBNE to make any economic sense, we need to have that $\eta_i \in [0,1]$ and $\nu_i \in [0,1]$. For $\nu_i < 1$, a sufficient condition is that $k_1 > 0$ and $k_2 > 0$ and which means that there is a penalty for getting caught reporting the wrong state of the world. A sufficient condition for $\nu_i > 0$ is that $\beta_i > 0$, so that the insurer will audit with positive probability only if the indemnity payment is positive. This condition does not appear to be too restrictive. For η_i to be positive, we need to have $\beta_i > c - k_2$. Finally, to have $\eta_i < 1$, we need

$$\beta_i > \frac{c}{(1-\pi)} - k_2 > c - k_2$$

2.3 The Contract for each agent type

There are four variables that must be specified in the menu of contracts: The coverage in case of a loss by the high risk type (β_H), the coverage in case of a loss by the low risk type (β_L), the premium for the high risk type (α_H), and the premium for the low risk type (α_L).

2.3.1 The principal's (participation) zero-profit function

Given the players' optimal strategies, it is possible to find the insurance premium that gives zero expected profits to the principal for each agent type. For an agent of type $i \in \{L, H\}$, this premium is implicitly given by

$$\alpha_i = \underbrace{\pi_i \beta_i}_{\text{Indemnity when loss}} + \underbrace{(1-\pi_i)\,\eta_i\,(1-\nu_i)\,\beta_i}_{\text{Rent}} + \underbrace{c\nu_i\,[\pi_i + (1-\pi_i)\eta_i]}_{\text{Cost of audit}} + \underbrace{(1-\pi_i)\,\eta_i \nu_i k_2}_{\text{Monetary penalty}} \tag{5}$$

On the right, $\pi_i\beta_i$ represents the expected benefits paid when a type-i agent tells the truth. The second term, $(1-\pi_i)\,\eta_i\,(1-\nu_i)\,\beta_i$, is the rent an agent can expect to extract from the principal by lying about the state of the world with probability η_i and not being audited with probability $(1-\nu_i)$. The third term, $c\nu_i\,[\pi_i + (1-\pi_t)\eta_i]$, is the expected cost of the auditing strategy that the principal must pass on to agents. Finally, $(1-\pi_i)\,\eta_i\nu_i k_2$, is the expected amount collected by the principal when the agent is caught committing fraud.

By substituting in (5) for the equilibrium value of η_i given in (3), the price of the contract becomes

$$\alpha_i = \pi_i\beta_i + \pi_i c\left(\frac{\beta_i}{\beta_i + k_2 - c}\right) = \pi_i\beta_i\left[1 + \frac{c}{\beta_i + k_2 - c}\right] \tag{6}$$

This zero-profit constraint has the nice characteristic that it is not related directly to the principal's auditing strategy, which also means that it is not related to the agent's utility function, the agent's initial wealth (Y) or the deadweight penalty (k_1). The term $\left(\frac{c}{\beta_i + k_2 - c}\right)$ represents the implicit loading factor of the insurance contract. This loading factor increases in the cost of auditing c, but decreases in the indemnity payment β_i. From (6), we find that unless $c = k_2$, the zero-profit functions associated with the high risk type crosses the zero-profit function of the low risk type at the points where the indemnity are $\beta = 0$ and $\beta = -k_2$, so that $\alpha = 0$ in both cases. The low risks' zero-profit condition lies below the high risks' for all values of $\beta > 0$, as it should be expected.

In contrast to the classic linear zero-profit conditions in Rothschild and Stiglitz (1976), the zero profit conditions when ex-post moral hazard is present is not necessarily linear. It can also be convex or concave, and even non-monotonic. The shape of the principal's zero-profit function depends on the relationship between the cost of verifying, c, and the penalty that is paid to the principal in case the agent is found guilty of having reported the wrong state of the world, k_2 (in other words, is $k_2 - c$ positive, negative or zero).

Although there are technically three possible cases (depending on $c \lesseqgtr k_2$), the most interesting case occurs when $c > k_2$. It includes the case where the principal receives nothing from catching the agent telling a lie (i.e., $k_2 = 0$),[2] which is more in line with what is observed in reality. This is the case I will concentrate on in this paper. The other two cases are relegated to Appendix 7.3 and provided only for the sake of completeness.

Assuming then that $c > k_2$, the function α_i in (6) reaches an optimum in terms of β_i when $\frac{\partial \alpha_i}{\partial \beta_i} = \pi_i + \pi_i c \frac{k_2 - c}{(\beta_i + k_2 - c)^2} = 0$. There are two real zeros to the function $\frac{\partial \alpha_i}{\partial \beta_i} = 0$: $\beta_i = (c - k_2) + \sqrt{c\,(c - k_2)} > 0$ and $\beta_i = (c - k_2) - \sqrt{c\,(c - k_2)} < 0$. The first zero (when $\beta_i > 0$) is a minimum of the function α since we then have $\frac{\partial^2 \alpha_i}{\partial \beta_i^2} = -2\pi_i c \frac{(k_2 - c)}{(\beta_i + k_2 - c)^3} > 0$ when $c > k_2$ and $\beta_i > c - k_2$.[3] The present exercise allows us to see that the convexity of the premium function increases with the probability of loss π_i. Figure 3 illustrates the zero-profit premium for the high risk and the low risk agents when $c - k_2 > 0$. The function

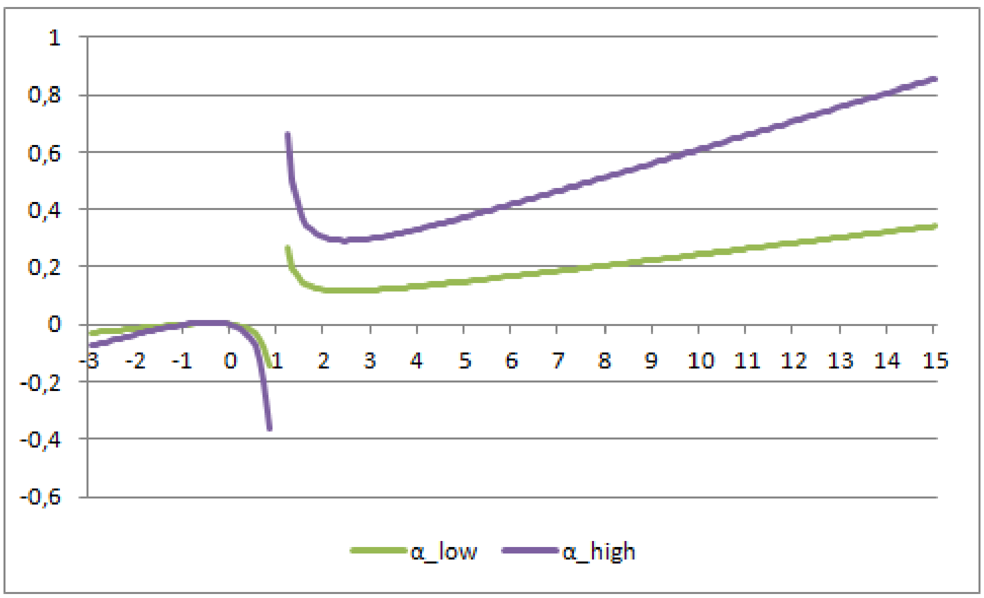

Figure 3: Premium functions $\alpha_i = \pi_i \beta_i \left[1 + \frac{c}{\beta_i + k_2 - c}\right]$ that give the principal zero profit for the high risk agent ($\pi_i = \pi_H$) and the low risk agent ($\pi_i = \pi_L$) when $c > k_2$. The graph is drawn using the following parameters: $Y = 20$, $\pi_L = 2\%$, $\pi_H = 5\%$, $\lambda = 10$, $c = 2$, and $k_2 = 1$. The premium is on the vertical axis and the coverage is on the horizontal axis.

$\alpha_i = \pi_i \beta_i + \pi_i c$ acts as an asymptote that bounds the insurer zero-profit function below for β_i very large and positive (Figure 4), and as an asymptote that bounds the same function above for β_i largely negative. The vertical asymptote, whatever the risk type is, is defined by $\beta = c - k_2$.

As stated earlier, I shall concentrate only on the situation where $c > k_2 \geq 0$ and $\beta_i > c - k_2 > 0$. The reason is mainly that it is the only situation for which the PBNE found in Lemma 1 has a non-degenerate

[2] See Bourgeon and Picard (2012) for a more lengthy discussion of the situation where $k_2 > 0$.

[3] The function α_i is concave when $\beta_i < c - k_2$ so that a maximum is reached at the second zero (when $\beta_i = (c - k_2) + c\sqrt{c\,(c - k_2)} < 0$) since $\frac{\partial^2 \alpha_i}{\partial \beta_i^2} = -2\pi_i c \frac{(k_2 - c)}{(\beta_i + k_2 - c)^3} < 0$ when $c > k_2$ and $\beta_i < c - k_2$

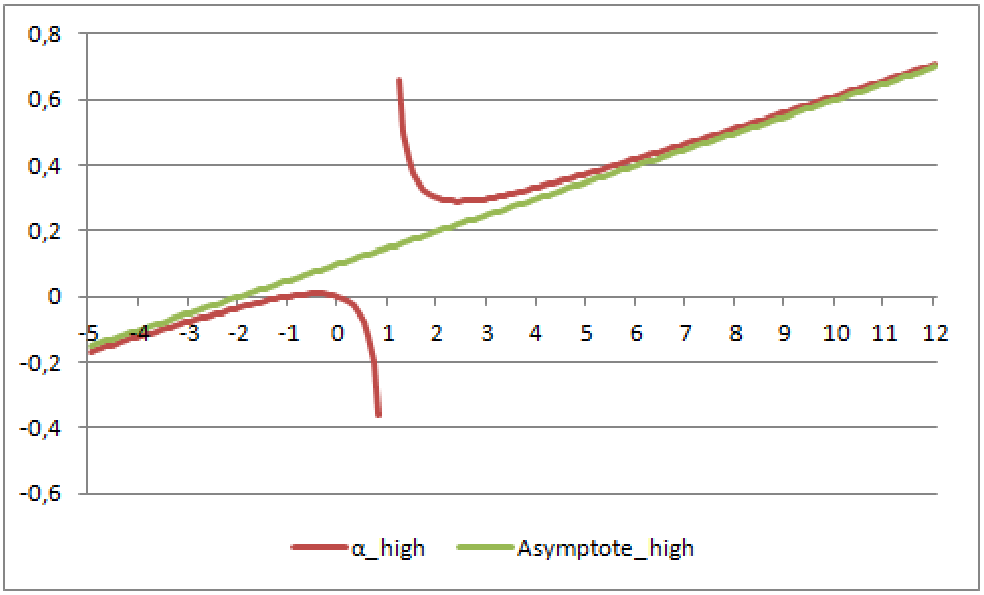

Figure 4: For the high risk agent, the premium function $\alpha_H = \pi_H \beta_H \left[1 + \frac{c}{\beta_H + k_2 - c}\right]$ that gives the principal zero profit when $c > k_2$, and the asymptote that has equation $\alpha_H = \pi_H (\beta_H + c)$. The vertical asymptote at $\beta_H = c - k_2$ is not displayed but can easily be inferred from the graph. The graph is drawn using the following parameters: $Y = 20$, $\pi_H = 5\%$, $\lambda = 10$, $c = 2$, and $k_2 = 1$. The premium is on the vertical axis and the coverage is on the horizontal axis.

solution (i.e., the probability of committing fraud, η_i, and the probability of auditing, ν_i, are comprised between 0 and 1).

2.3.2 The maximization problem

When types are known, the problem for the principal is to choose a coverage and premium pair that maximizes each agent's expected utility over final wealth,

$$\begin{aligned} \max_{\alpha_i, \beta_i, \nu_i, \eta_i} \; & \pi_i U (Y - \alpha_i - \lambda + \beta_i) + (1 - \pi_i)(1 - \eta_i) U (Y - \alpha_i) \\ & + (1 - \pi_i) \eta_i \{\nu_i [U (Y - \alpha_i - k_2) - k_1] + (1 - \nu_i) U (Y - \alpha_i + \beta_i)\} \end{aligned} \tag{MP}$$

subject to the principal's break-even constraint (6), and the equilibrium probability of filing a fraudulent claim (3) and of auditing a claim (4). Assuming that the agents' participation constraint holds (otherwise there wouldn't be any insurance fraud problem since there wouldn't be any insurance), and substituting (3), (4) and (6) into MP, the maximization problem simplifies to

$$\max_{\beta_i} V = \pi_i U \left(Y - \pi_i \left[\beta_i + c \left(\frac{\beta_i}{\beta_i + k_2 - c}\right)\right] - \lambda + \beta_i\right) + (1 - \pi_i) U \left(Y - \pi_i \left[\beta_i + c \left(\frac{\beta_i}{\beta_i + k_2 - c}\right)\right]\right) \tag{7}$$

The deadweight penalty (k_1) is not a factor in the design of the optimal contract. The reason is that the decision to audit comes last in the sequence of play, so that the deadweight penalty is internalized in the principal's audit probability. In contrast, the penalty paid to the principal does not have an impact on the structure of the contract (see also Bourgeon and Picard, 2012). As we can see, the penalty paid to the principal by the agent in case he is found to have committed fraud (k_2) reduces the insurance premium paid.

The solution to the ex post moral hazard game without commitment can then be easily solved by finding the first order condition of problem (7) with respect to β_i. The first order condition can be written as

$$\frac{U'\left(Y-\pi_i\left[\beta_i+c\left(\frac{\beta_i}{\beta_i+k_2-c}\right)\right]-\lambda+\beta_i\right)}{\begin{array}{c}\pi_i U'\left(Y-\pi_i\left[\beta_i+c\left(\frac{\beta_i}{\beta_i+k_2-c}\right)\right]-\lambda+\beta_i\right)\\ +(1-\pi_i)U'\left(Y-\pi_i\left[\beta_i+c\left(\frac{\beta_i}{\beta_i+k_2-c}\right)\right]\right)\end{array}}=\left(1+c\frac{k_2-c}{(\beta_i+k_2-c)^2}\right) \tag{8}$$

We see that if $\beta_i=\lambda$, so that the agent is fully insured, then the condition for an optimum becomes

$$c\frac{k_2-c}{(\beta_i+k_2-c)^2}=0, \tag{9}$$

which can occur only if $k_2=c$. In other words, provided that insurance is purchased at all, the optimal contract for the two agents entails full insurance if and only if the penalty paid to the principal is equal to her auditing cost. I state this result formally in the following proposition.

Proposition 1 *In the presence of ex post moral hazard that characterizes insurance fraud, full insurance is optimal if and only if the penalty paid to the principal in the event the agent is found to have committed insurance fraud is exactly equal to the principal's cost of auditing. If the penalty is less (more) than the cost of auditing, then the optimal indemnity is greater (smaller) than the loss.*

Proof. *Follows from the previous discussion.*•

This proposition tells us that if $k_2<c$ (the case that is most likely in the economy as it encompasses the situation where $k_2=0$) so that $c\frac{k_2-c}{(\beta_i+k_2-c)^2}<0$, then more-that-full insurance is optimal (i.e., $\beta_i>\lambda$), provided that risk-types are known. Figure 5 presents graphically the solution to this game where risk-types are known to all players.

In contrast to Boyer (2004), but in line with Bourgeon and Picard (2014), overcompensation of losses is not solely due to the fact that the principal cannot commit. Partial insurance could occur even in the presence of principal non-commitment provided that the agent pays a part of his penalty directly to the principal (i.e., $k_2\neq 0$). The contracts are structured to increase the principal's incentives to verify the agents' report about the state of the world. The impact of the other model parameters on the indemnity payment is provided in the following corollary.

Corollary 1 *In the presence of ex post moral hazard that characterizes insurance fraud, the indemnity payment increases as the probability of loss increases ($\frac{d\beta_i}{d\pi}>0$), the cost of auditing increases ($\frac{d\beta_i}{dc}>0$) and the penalty paid to the principal decreases ($\frac{d\beta_i}{dk_2}<0$).*

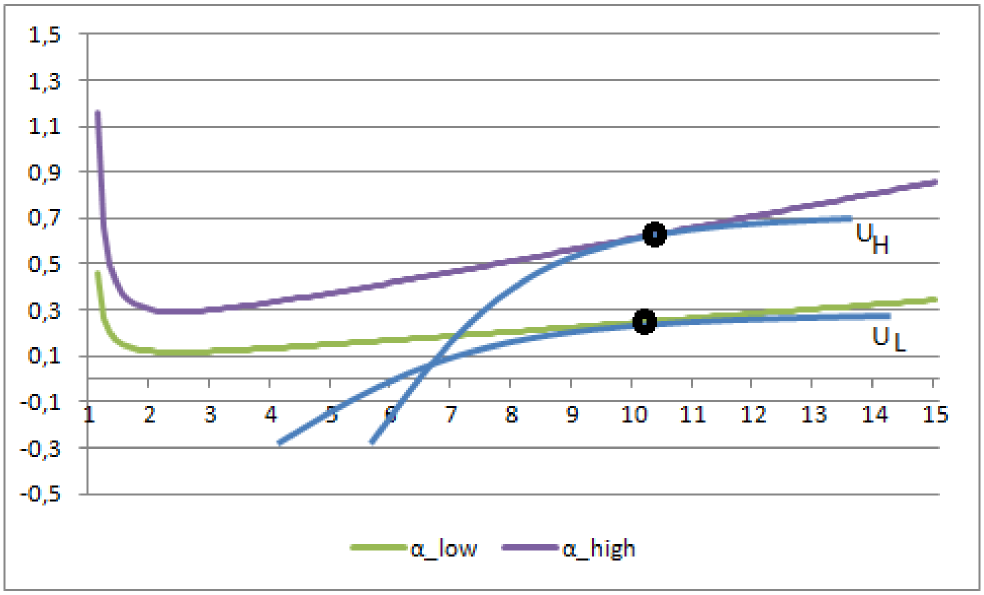

Figure 5: Solution to the game when agent types are known. The indemnity payment is on the horizontal axis and the premium is on the vertical axis. Only the part of the graph where $\beta_i > c - k_2$ is represented here.

Proposition 2 *Proof. Writing the first order condition of problem (7) with respect to β_i as*

$$\begin{aligned}\Omega_i &= 0 = U'\left(Y - \pi_i\left[\beta_i + c\left(\frac{\beta_i}{\beta_i + k_2 - c}\right)\right] - \lambda + \beta_i\right)\left(1 - \pi_i - \pi_i c\left(\frac{k_2 - c}{(\beta_i + k_2 - c)^2}\right)\right) \\ &\quad -(1-\pi_i)U'\left(Y - \pi_i\left[\beta_i + c\left(\frac{\beta_i}{\beta_i + k_2 - c}\right)\right]\right)\left(1 + c\left(\frac{k_2 - c}{(\beta_i + k_2 - c)^2}\right)\right)\end{aligned}$$

and finding the total derivative of

$$d\Omega_i = 0 = \frac{\partial\Omega_i}{\partial\beta_i}d\beta_i + \frac{\partial\Omega_i}{\partial\pi}d\pi + \frac{\partial\Omega_i}{\partial c}dc + \frac{\partial\Omega_i}{\partial k_2}dk_2$$

gives us that sign $\left(\frac{d\beta_i}{dx}\right) = sign\left(\frac{\partial\Omega_i}{\partial x}\right)$ *since* $\frac{\partial\Omega_i}{\partial\beta_i} < 0$ *for* $x \in \{\pi, c, k_2\}$. *Solving provides the appropriate result.*●

The principal is induced to audit with greater probability by increasing what she has to lose by not auditing (that is, the indemnity payment β_i and the penalty paid to her if she finds that the agent has committed fraud k_2). When both the indemnity payment and the penalty paid to the principal are larger, then the incentives for the principal to audit are also larger, which then results in a lower likelihood of committing fraud. This is even more obvious when we realize that the probability of fraud, η_i, decreases when both k_2 and β_i are larger. Picard (2013) and Bourgeon and Picard (2014) provide more thorough discussions of this phenomenon.

3 Adverse Selection with Non Linear Zero Profit Constraints

Following the Rothschild-Stiglitz tradition, the principal's objective is to choose a menu of contracts[4] $(\alpha_L^s, \alpha_H^s, \beta_L^s, \beta_H^s)$ such that

$$\max_{\alpha_L^s, \alpha_H^s, \beta_L^s, \beta_H^s} V = \pi_L U\left(Y - \alpha_L^s - \lambda + \beta_L^s\right) + (1-\pi_L) U\left(Y - \alpha_L^s\right) \tag{AS}$$

subject to

$$\pi_L U\left(Y - \alpha_L^s - \lambda + \beta_L^s\right) + (1-\pi_L) U\left(Y - \alpha_L^s\right) \geq \pi_L U\left(Y - \alpha_H^s - \lambda + \beta_H^s\right) + (1-\pi_L) U\left(Y - \alpha_H^s\right) \tag{IC$_L$}$$

$$\pi_H U\left(Y - \alpha_H^s - \lambda + \beta_H^s\right) + (1-\pi_H) U\left(Y - \alpha_H^s\right) \geq \pi_H U\left(Y - \alpha_L^s - \lambda + \beta_L^s\right) + (1-\pi_H) U\left(Y - \alpha_L^s\right) \tag{IC$_H$}$$

$$\alpha_L^s \geq \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right] \tag{ZC$_L$}$$

$$\alpha_H^s \geq \pi_H \left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right] \tag{ZC$_H$}$$

$$\pi_L U\left(Y - \alpha_L^s - \lambda + \beta_L^s\right) + (1-\pi_L) U\left(Y - \alpha_L^s\right) \geq \pi_L U\left(Y - \lambda\right) + (1-\pi_L) U\left(Y\right) \tag{PC$_L$}$$

$$\pi_H U\left(Y - \alpha_H^s - \lambda + \beta_H^s\right) + (1-\pi_H) U\left(Y - \alpha_H^s\right) \geq \pi_H U\left(Y - \lambda\right) + (1-\pi_H) U\left(Y\right) \tag{PC$_H$}$$

The two incentive compatibility constraints (IC_L and IC_H) stipulate that agents should be better off purchasing the contract that is designed for their own risk type rather than purchasing the contract that is desinged for the other risk type. The two zero-profit constraints (ZC_L and ZC_H) are the principal's zero-profit constraints for each agent type (see equation 6). Finally the two participations constraints (PC_L and PC_H) stipulate that agents must be better off being insured than in autarchy.

Assuming the principal makes no profit on each contract on average (i.e., equations ZC_L and ZC_H hold with equality), that the two types of agents prefer insurance to autarchy (i.e., equations PC_L and PC_H are not binding), and the appropriate single-crossing property for the incentive compatible constraints (IC_L is not binding, but IC_H is), the problem simplifies to

$$\begin{aligned} \max_{\beta_L^s, \beta_H^s} V \quad = \quad & \pi_L U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right] - \lambda + \beta_L^s\right) \\ & + (1-\pi_L) U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right]\right) \end{aligned} \tag{AS$'$}$$

subject to IC_H.

The solution to this maximization problem[5] is such that

$$\frac{U'\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right] - \lambda + \beta_H^s\right)}{\left[\begin{array}{c} \pi_H U'\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right] - \lambda + \beta_H^s\right) \\ + (1-\pi_H) U'\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right]\right) \end{array}\right]} = \left(1 + \frac{(k_2 - c)\, c}{\left(\beta_H^s + k_2 - c\right)^2}\right)$$

[4] Note that I already substituted the players' Nash strategies in the objective function and in the participation constraints. I shall use superscript s to refer to the case where agent types can be seperated.

[5] See the appendix.

which is the same solution as when the high risk agent's type was known (see equation 8). Note that when $k_2 = 0$, the right hand side becomes $\frac{\beta_H^s(\beta_H^s - 2c)}{\left(\beta_H^s - c\right)^2}$, which means that $\beta_H^s > 2c$ for there to be a solution.

As in the original Rothschild and Stiglitz (1976) paper, the contract designed for the high risk agents is not affected by the presence of low risk agents. Rather, it is the low-risk agents who must signal their type by adapting their behavior and accepting a contract that reflects their lower risk. It is then appropriate to use the binding IC_H constraint to find the value of β_L^s as a function of β_H^s

$$\begin{aligned} 0 \;=\; & \pi_H U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right] - \lambda + \beta_L^s\right) + (1-\pi_H)U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right]\right) \\ & - \pi_H U\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right] - \lambda + \beta_H^s\right) - (1-\pi_H)U\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)\right]\right) \end{aligned} \tag{10}$$

Figure 6 illustrates the contracts under adverse selection with a non-linear zero-profit constraint for each risk type; the horizontal axis represents the indemnity, whereas the premium can be found on the vertical axis. As we see in the figure, the high risk agent's utility function, U_H, is such that he purchases a contract (point A) that is more expensive than the contract offered to the low risk agents (point B) along their utility function U_L. That is obvious from the fact highlighted earlier that the high risks' zero profit function (α_high) always lies above the low risks' (α_low). From the incentive compatibility constraints, we know that the low risks will signal their type by purchasing a contract (point B') that provides a lower coverage than what they would have chosen had their type been known (point B). The low risk agent's expected utility in autarchy is given by the dashed line labelled $\overline{U_L}$.

An interesting difference between the menu of contracts when ex-post moral hazard and principal non-commitment are present and the classical Rothschild-Stiglitz menu of contracts is that low risk agents may have to signal their type by paying a higher premium for this lower level of coverage. And even though this result depends on the value of the parameters, it represents a new result in the economics of insurance.

The reason why the low-risk type may end up paying a higher premium under adverse selection when insurance fraud is a possibility is two-fold: First there is the convex nature of the insurer's zero-profit constraint, and second there is the decreasing relationship between premium and indemnity. In the classical Rothschild-Stiglitz world, the insurers' zero profit condition is a straight line, with $\alpha_L = \pi_L \beta_L$ in the case of the low risk. In the presence of insurance fraud, the zero profit condition becomes $\pi_L\left[\beta_L + c\left(\frac{\beta_L}{\beta_L + k_2 - c}\right)\right]$, so that the derivative with respect to the indemnity is equal to $\pi_L\left[1 + c\frac{k_2 - c}{(\beta_L + k_2 - c)^2}\right]$. The premium function reaches a minimum when $\beta_L^{\min} = -(k_2 - c) + \sqrt{-c\,(k_2 - c)}$, so that the relationship between indemnity and premium is negative when $\beta < -(k_2 - c) + \sqrt{-c\,(k_2 - c)}$.[6] Note that the premium function reaches a minimum at a poin that is independent of the agent's risk of accident, which means that for a given β the zero-profit function is more convex the higher is the risk of the loss (i.e., $\frac{\partial}{\pi}\left(\frac{\partial^2 \alpha}{\partial \beta^2}\right) > 0$) – see Figure 3 for a

[6] In other words, $1 + c\frac{k_2 - c}{(\beta + k_2 - c)^2} < 0$ when $\beta < -(k_2 - c) + \sqrt{-c\,(k_2 - c)}$.

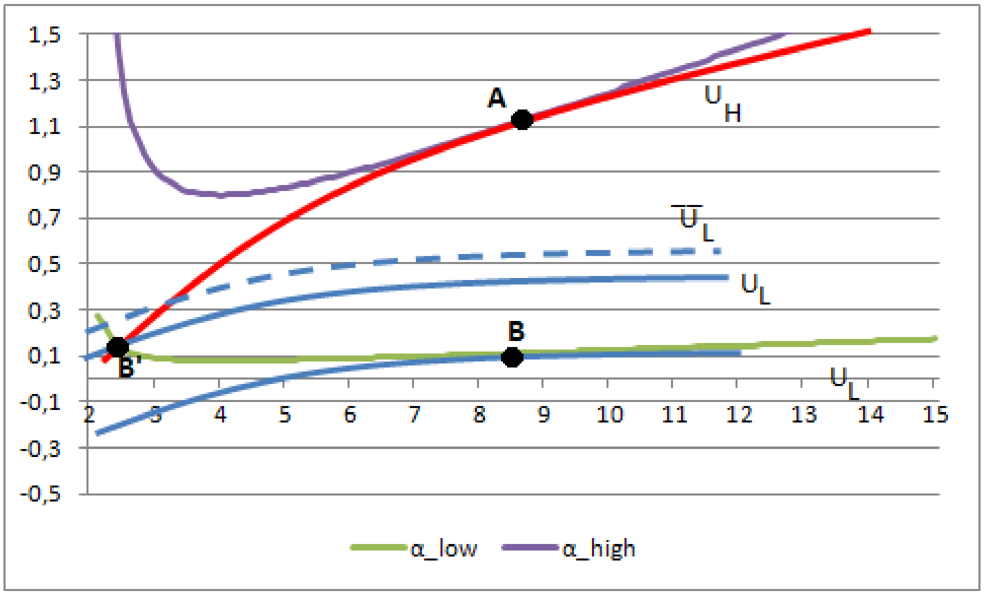

Figure 6: Solution to the game when agent types are unknown. The indemnity payment is on the horizontal axis and the premium is on the vertical axis. The low risk signals his type (function U_L) by accepting a lower converage, but a higher premium than when types are knows. The low risk agent's expected utility in autarchy is given by the dashed line labelled $\overline{U_L}$. The parameter values used for this example are $Y = 20$, $\pi_L = 1\%$, $\pi_H = 10\%$, $c = 2$, and $k_2 = 0$. The utility function is $U(W) = \ln(W)$.

better illustration of this relationship between an agent's risk of a loss and the convexity of the zero-profit function for that agent.

The question that we may raise at this point is to find the conditions under which the premium paid by the agent in the presence of adverse selection is greater than the premium he pays in the absence of adverse selection? To answer this question, it is sufficient to compare $\alpha_L^s = \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s - c}\right)\right]$ with $\alpha_L = \pi_L \left[\beta_L + c\left(\frac{\beta_L}{\beta_L - c}\right)\right]$. Let $k_2 = 0$ for tractability reasons (so that $\beta_L^{\min} = 2c$). Clearly the premium under adverse selection and ex post moral hazard will be greater than the premium under ex post moral hazard (i.e., $\alpha_L^s > \alpha_L$) if and only if

$$\beta_L^s \left(\frac{\beta_L^s}{\beta_L^s - c}\right) > \beta_L \left(\frac{\beta_L}{\beta_L - c}\right)$$

The function $\frac{\beta^2}{\beta - c}$ reaches a minimum at $\beta = 2c$. We also know that β_L^s must be smaller than β_L^* for the signal to be credible. For $\beta_L^s < \beta_L^*$ at the same time as $\beta_L^s \left(\frac{\beta_L^s}{\beta_L^s - c}\right) > \beta_L \left(\frac{\beta_L}{\beta_L - c}\right)$, it must be that $\beta_L^s < 2c$. Another way to look at this is to apply the fact that $\beta_L^s - \beta_L < 0$ since low-risk agents need to signal their type by choosing a lower indemnity payment. It then follows that $\alpha_L^s > \alpha_L$ if and only if

$$\beta_L^s < \beta_L < \frac{c\beta_L^s}{\beta_L^s - c}$$

Whether this is true depends on the value of the parameters. For instance, if β_L^s is "close" to c, then it is obviously true that the low-risk agent's premium under adverse selection will be greater than under full information. The following proposition illustrates the situation.

Proposition 3 *In an economy plagued with adverse selection of risk types and with ex post moral hazard, there exists a difference between each agent's probability of having an accident (say $\Delta\pi$) such that for all differences greater than $\Delta\pi$ (or alternatively for all $\pi_H > \pi_L + \Delta\pi$), the low risk agents ends up paying a greater premium than if his type was known. This is true even if the indemnity payment he receives is lower under adverse selection than when his risk type is known.*

Proof. I want to show the conditions under which $\alpha_L^s > \alpha_L$. This occurs when $\pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s - c}\right)\right] > \pi_L \left[\beta_L + c\left(\frac{\beta_L}{\beta_L - c}\right)\right]$, which is the same as finding $\beta_L < \frac{c\beta_L^s}{\beta_L^s - c}$.

We know that $\pi_H - \pi_L$ is bounded above at $\pi_H - \pi_L = 1$ and below at $\pi_H - \pi_L = 0$. We know that when $\pi_H - \pi_L \to 0$, so that there is no difference between the different agents' risk types, then $\alpha_L^s = \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s - c}\right)\right] = \pi_L \left[\beta_L + c\left(\frac{\beta_L}{\beta_L - c}\right)\right] = \alpha_L$. This means that low risk agent's premium is the same under adverse selection as under full information. When $\pi_H - \pi_L \to 1$ (so that $\pi_H \to 1$ at the same time as $\pi_L \to 0$) then from Equation 10 it follows that

$$0 = U\left(Y - \lambda + \beta_L^s\right) - U\left(Y - c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right) - \lambda\right)$$

so that

$$\beta_L^s = c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)$$

Substituting for $\beta_L^s = c\left(\frac{\beta_H^s}{\beta_H^s + k_2 - c}\right)$ in $\beta_L < \frac{c\beta_L^s}{\beta_L^s - c}$, we find that $\alpha_L^s > \alpha_L$ if and only if

$$\beta_L < \frac{c\beta_H^s}{-k_2 + c}$$

which is obviously true since $c - k_2 > 0$. It follows that $\left(\frac{c}{c - k_2}\right)\beta_H^s > \beta_H^s = \beta_H > \beta_L > \beta_L^s$. Consequently there is a value of $\Delta\pi = \pi_H - \pi_L \in (0, 1)$ such that $\alpha_L^s = \alpha_L$. Therefore the premium paid by the low risk agent in the separating contract is greater than the premium he would have paid had his type been known for all $\pi_H > \pi_L + \Delta\pi$. •

The proposition essentially shows what Figure 3 alluded to. It shows that when risk types are sufficiently different, then because of the convex nature of the zero-profit constraints for each agent type in an economy where agents are able to misrepresent the true state of nature and where the principal cannot commit to an auditing strategy, the separating equilibrium may be such that the low risk agent's contract lies in a region where less coverage is met with a greater premium. This region where greater coverage is met with a lower premium (that is, $\frac{\partial\alpha}{\partial\beta} < 0$) is observed when the low risk agent's coverage is between the principal's cost of auditing and twice her cost of auditing (i.e., $c < \beta < 2c$).

3.1 Illustration

To illustrate the situation, suppose agents have a CRRA utility function (i.e., $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$) with $\gamma = 3$. Let also $Y = 11$, $\lambda = 10$, $c = 5$, $k_1 = 0.2$, $k_2 = 0$, $\pi_L = 5\%$ and $\pi_H = 10\%$. In autarchy the low risk agents' expected utility is -0.0289 whereas the high risk agents' expected utility is -0.0537. The minimum of the premium function is reached when $\beta_i = 2c = 10$. When risk types are known, the solution is such that the high risks' expected utility is maximized with contract $\{\beta_H = 12.30; \alpha_H = 2.0725\}$. The low risks' expected utility is maximized with contract $\{\beta_L = 12.35; \alpha_L = 1.0376\}$. The low risk agents' expected utility is then -0.0050 whereas the high risk agents' expected utility is -0.0060.

When the principal knows the agents' risk type, then she knows that high risk agents will opt for a contract that has a less generous coverage. This result contrasts with the one I presented when risk types and states of the world are public knowledge. Under perfect information the premiums are equal to $\alpha_L = 0.5$ and $\alpha_H = 1.0$; as the high risk types are twice as likely to have an accident, their premium is twice as expensive. When the state of the world is private information (i.e., insurance fraud is possible), then the high risk agents pay a bit less than twice as much as the low risk agents because they purchase a contact that has smaller indemnity payments than the contract purchased by the low risks.

When risk types are not known, however, the low risk agents will opt for contract $\{\beta_L^s = 6.70; \alpha_L^s = 1.3203\}$ whereas the high risk agents will obtain the same contract $\{\beta_H^s = \beta_H = 12.30;\ \alpha_H^s = \alpha_H = 2.0725\}$, as when risk types were known. The low risk agents' expected utility is then -0.0057, which is worse than their expected utility in the case of a full type information, but still better than in autarchy.[7]

It is easy to calculate how much fraud there is in this economy contingent on the scenarios of full type information and of adverse selection. Let us start with the case where the principal has perfect information as to the risk type of the agents. Using the equilibrium values of η_i given in (3), fraud probabilities are equal to $\eta_H = 7.61\%$ for the high risk agents and to $\eta_L = 3.58\%$ for the low risk agents. From the equilibrium value of ν_i given in (4), the probability that the principal audits each type of agents is given by $\nu_H = 4.91\%$ (in case of the high risk agents) and $\nu_L = 3.58\%$ (in the case of the low risk agents). We see that the high risk types are more likely to commit fraud than the low risk types. We also see that the high risk agents are more likely to be audited than the low risk agents. Under adverse selection the high risk's probability of fraud and probability of being audited remain the same so that $\eta_H^s = \eta_H$ and $\nu_H^s = \nu_H$, whereas the low risk's probability of fraud increases to $\eta_L^s = 15.59\% > 3.58\% = \eta_L$ at the same time as their probability of being audited decreases to $\nu_L^s = 3.36\% < 3.58\% = \nu_L$. Given that the high risk agents do not change their

[7] It is easy to calculate how much fraud there is in this economy contingent on the scenarios of full type information and of adverse selection. Let us start with the case where the principal has perfect information as to the risk type of the agents. Using the equilibrium values of η_i given in (3), fraud probabilities are equal to $\eta_H = 7.61\%$ for the high risk agents and to $\eta_L = 3.58\%$ for the low risk agents. From the equilibrium value of ν_i given in (4), the probability that the principal audits each type of agents is given by $\nu_H = 4.91\%$ (in case of the high risk agents) and $\nu_L = 3.88\%$ (in the case of the low risk agents). We see that the high risk types are more likely to commit fraud than the low risk types. We also see that the high risk agents are more likely to be audited than the low risk agents.

equilibrium behavior when adverse selection is present (because the parameters of their insurance contract do not change), the only difference in the amount of fraud in the economy muast come from the low risk agents' behavior only.

In this example the incentive compatible contract offered to the low risk agents is such that the low risk agents end up paying a higher premium than when the principal knew the agents' type. Consequently, when adverse selection is present in a costly verification context without commitment, then type signalling can occur through a lower indemnity payment, AND a higher premium.

3.2 Separation and Fraud

When adverse selection is present in the sense that only the agents know their own risk type, then the high risk type's probability of fraud and the principal's probability of auditing a high risk agent remain the same since the contract for the high risk agent in the Rothschild-Stiglitz economy is the same as when his risk type is known. This means that if adverse selection has any impact on the amount of fraud in the economy, it must come from its impact on the low-risk agents' behavior.

It is interesting to see that the amount of fraud in the economy increases because of adverse selection. The total amount of fraud in the economy when types are known is given by

$$\begin{aligned} Fraud &= \rho\left(1-\pi_L\right)\eta_L + \left(1-\rho\right)\left(1-\pi_H\right)\eta_H \\ &= \rho\pi_L\left(\frac{c}{\beta_L+k_2-c}\right) + \left(1-\rho\right)\pi_H\left(\frac{c}{\beta_H+k_2-c}\right) \end{aligned}$$

When an agent's type is private information, the total amount of fraud will be given by

$$Fraud_s = \rho\pi_L\left(\frac{c}{\beta_L^s+k_2-c}\right) + \left(1-\rho\right)\pi_H\left(\frac{c}{\beta_H^s+k_2-c}\right)$$

We are then able to conclude with the following proposition.

Proposition 4 *When adverse selection is present in an economy, then insurance fraud, as defined by the filing of a claim when there was no loss, is increased and the probability of a successful fraudulent claim is also increased.*

Proof. The optimal menu of contracts is such that $\beta_H^s = \beta_H$. This means that $Fraud_s > Fraud$ if and only if $\rho\pi_L\left(\frac{c}{\beta_L^s+k_2-c}\right) > \rho\pi_L\left(\frac{c}{\beta_L+k_2-c}\right)$. This occurs when $\beta_L^s < \beta_L$, which we know to be true since the low risk type needs to signal that he is low risk by accepting a contract that offers less coverage. Consequently adverse selection in risk types leads to an economy where insurance fraud is more prevalent. At the same time as the low risk agent's probability of fraud increases when adverse selection is present (i.e., $\eta_L^s > \eta_L$), the probability of auditing a low-risk agent is reduced (i.e., $\nu_L^s < \nu_L$). To see why, note that $\frac{\partial\nu_L}{\partial\beta_L} < 0$ so that as the low risk must signal its type by choosing a contract that provides a smaller coverage than when types are known. •

Because of adverse selection, low-risk agents are more likely to commit fraud AND are less likely to be audited when they file a claim. This means that low-risk agents are more likely to be successful in committing fraud when adverse selection is present than when types are known.

4 The Pooling Contract

One can wonder whether a pooling contract can survive cream skimming. In habitual settings, pooling equilibriums are not stable because of the single crossing property.[8] It is still the case here (see Figure 5 for a more obvious situation). Consequently, no pooling contract is sustainable in a competitive equilibrium because it is always possible to design a contract that would attract only the low risk individuals (see Snow, 2009, and Mimra and Wambach, 2011, and the references[9] therein for a more thorough discussion). This means that there is a potential side payment à la Miyazaki-Spence that could increase the welfare of all the agents in the economy compared to the pooling contract. Instead of entering that debate, I would rather examine what happens to the amount of insurance fraud if, for some reason, insurers are not allowed to offer separating contracts.

First note that the claiming game changes because the principal can no longer separate the high risk from the low risk individual, meaning that she will not be able to condition her auditing strategy on the agent's type. This means that the extensive form of the game changes to become like that of Figure 7. The solution to this PBNE is provided in the appendix. For the purpose of the remainder of the paper, I will limit my discussion to the case where the parameter values give us an interior solution in the sense that $\eta_i \in (0,1)$ and $\nu_i \in (0,1)$.

Given that only one contract is offered in this game, the principal does not know if a given contract is acquired by an agent whose probability of having an accident is low or high. She does know, however, that proportion of low risk agents in the economy is given by ρ. When time comes for the principal to investigate or not, the only thing she knows is whether the agent announced he suffered a loss or not. She does not know if the agent is of a low-risk or of a high-risk category, or if the report is truthful or not. Irrespective of the agent's type, it is clear that if the agent does not claim any loss then the principal's optimal strategy is to not investigate.

Substituting for the PBNE constraints that are available in the appendix, the insurer faces the following maximization problem when a single contract must be offered to all policyholders in the economy:

$$\max_{\alpha_p,\beta_p} EU_{\eta\nu} = \pi_L U\left(Y - \lambda - \alpha_p + \beta_p\right) + (1-\pi_L)U\left(Y - \alpha_p\right)$$

[8] As Mimra and Wambach (2012) put it: "In the classic RS model with two risk types, the high risk indifference curve is always flatter than the lowrisk indifference curve, as the high risk is always willing to pay more for one more unit of further insurance compared to the lowrisk" (p. 3).

[9] Including Smart (2000), Villeneuve (2003) and Wambach (2000).

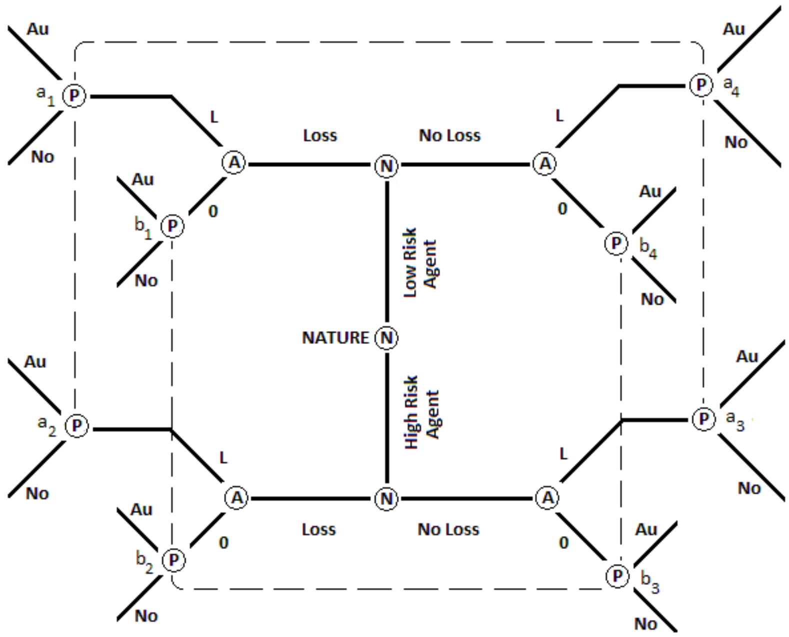

Figure 7: Extensive form of the claiming game when the agent type is not known (pooling contract).

subject to

$$\alpha_p = (\rho\pi_L + (1-\rho)\,\pi_H)\,\beta_p\left(1 + \frac{c}{\beta_p + k_2 - c}\right)$$

This maximization problem is very similar to one when agent types are known (see Equation 6). The solution is of course similar to that presented in Equation 8 and looks like

$$\frac{U'\left(Y - \alpha_p - \lambda + \beta_p\right)}{\pi_L U'\left(Y - \alpha_p - \lambda + \beta_p\right) + (1-\pi_L)\,U'\left(Y - \alpha_p\right)} = \left(1 + c\frac{k_2 - c}{\left(\beta_p + k_2 - c\right)^2}\right) \tag{11}$$

with $\alpha_p\left(\beta_p\right)$ given previously.

The amount of fraud in this pooling economy is

$$FRAUD_p = (\rho\pi_L + (1-\rho)\,\pi_H)\,\frac{c}{\beta_p + k_2 - c}$$

whereas it was

$$FRAUD_s = \rho\pi_L\frac{c}{\beta_L^s + k_2 - c} + (1-\rho)\,\pi_H\frac{c}{\beta_H^s + k_2 - c}$$

in the economy that had separating contracts (where β_L^s and β_H^s are independent of ρ).

It is clear that it is only because of the choice of the indemnity payments that the levels of fraud are different in the pooling or the separating economies. There is less fraud in a pooling contract if and only if $FRAUD_p < FRAUD_s$. In terms of β_p only, we then have that there is more fraud in a pooling contract provided that

$$\beta_p > \frac{\rho\pi_L + (1-\rho)\,\pi_H}{\rho\frac{\pi_L}{\beta_L^s + k_2 - c} + (1-\rho)\,\frac{\pi_H}{\beta_H^s + k_2 - c}} - k_2 + c \tag{12}$$

or

$$\beta_p > \frac{\rho\pi_L\left(\frac{1}{\beta_L^s + k_2 - c}\right)\beta_L^s + (1-\rho)\,\pi_H\left(\frac{1}{\beta_H^s + k_2 - c}\right)\beta_H^s}{\rho\pi_L\left(\frac{1}{\beta_L^s + k_2 - c}\right) + (1-\rho)\,\pi_H\left(\frac{1}{\beta_H^s + k_2 - c}\right)} \tag{13}$$

Whether there is more or less fraud in the economy where contracts are mandated to be equal for all participants depends on the proportion of low risk agents (ρ) in the economy. As ρ becomes smaller, the likelihood of having more fraud in the pooling economy increases since it reduces the likelihood that the previous equality holds. I state this results as our last proposition.

Proposition 5 *In an economy where insurers are restricted from offering a separating menu of contracts, the amount of insurance fraud committed will be smaller than in an economy where agents who face different risks self-select into different contracts provided that the proportion of low risk individuals is greater than some proportion ρ^*.*

Proof. We know that as $\rho \to 0$, then the right hand side of Equation 12 converges to β_H^s. We know that $\beta_p < \beta_H^s$. Similarly as $\rho \to 1$, then the right hand side of Equation 12 converges to β_L^s. We know that $\beta_p > \beta_L^s$. So we have that as $\rho \to 0$ there is more fraud with a pooling contract (i.e., $FRAUD_p > FRAUD_s$) whereas as $\rho \to 1$ there is more fraud with a separating contract (i.e., $FRAUD_p < FRAUD_s$). There must therefore exist a $\rho^* \in (0,1)$ such that $FRAUD_p = FRAUD_s$. This is made more clear by the fact that the right hand side of Equation 12 is monotone and decreasing in ρ so that

$$\frac{\partial}{\partial\rho}\left(\frac{\rho\pi_L\left(\frac{1}{\beta_L^s + k_2 - c}\right)\beta_L^s + (1-\rho)\,\pi_H\left(\frac{1}{\beta_H^s + k_2 - c}\right)\beta_H^s}{\rho\pi_L\left(\frac{1}{\beta_L^s + k_2 - c}\right) + (1-\rho)\,\pi_H\left(\frac{1}{\beta_H^s + k_2 - c}\right)}\right) < 0 \tag{14}$$

if and only if

$$-\pi_L\pi_H\left(\frac{1}{\beta_H^s + k_2 - c}\right)\left(\frac{1}{\beta_L^s + k_2 - c}\right)(\beta_H^s - \beta_L^s) < 0$$

which is obviously true since $\beta_H^s > \beta_L^s$. •

Let us illustrate the situation using the same basic example as before. Let there be agents with CRRA utility functions whose coefficient of risk aversion is $\gamma = 3$. Let also be the following parameters $Y = 11$, $\lambda = 10$, $c = 5$, $k_1 = 0.2$, $k_2 = 0$, $\pi_L = 5\%$ and $\pi_H = 10\%$. In the case of the separating equilibrium, the low risk chooses contract ($\beta_L^s = 6.70$; $\alpha_L^s = 1.3203$), whereas the high risk chooses contract ($\beta_H^s = 12.30$; $\alpha_H^s = 2.0725$).

In the pooling of types, with $\rho = 50\%$, both agents receive contract ($\beta_p = 11.59$; $\alpha_p = 1.5290$). Given the parameters of the economy, the condition for having less fraud in the pooling economy is respected since the right hand side of Equation 13 is equal to 8.6621. This is clearly less than $\beta_p = 11.59$. For smaller values of ρ, say $\rho = 5\%$, the inequality in Equation 13 does not hold. The pooling contract is such that ($\beta_p = 11.23$; $\alpha_p = 1.9736$). The right hand side of Equation 13 is itself equal to 11.80. Clearly this is greater than the left hand side, $\beta_p = 11.23$. Consequently, there is more fraud with this pooling contract when ρ is small; and of course there is less fraud under a unique contract when ρ is large.

5 Discussion and Conclusion

This paper re-examines the traditional Rothschild-Stiglitz separating insurance contract in the presence of insurance fraud. I modelled insurance fraud in this paper as a non-cooperative game between a principal (the insurance company), who cannot commit credibly to an auditing strategy, and twice-privately informed agents (the policyholdert) in the sense that they privately know their probability of loss (which can be high or low) AND they privately observe what is the state of the world (i.e., what loss did they suffer). In a two-point distribution of states of the world (loss and no loss), I confirm that the optimal separating contract entails overpayment of the loss for the high risk type, provided that the penalty paid by the agent to the principal in the event he is caught committing fraud is smaller than the principal's cost of auditing.

For the low risk type, the results are more convoluted. The low-risk agents receive a contract that is not as good as the contract that they would have chosen had their risk types been known. Low-risk agents may still receive a contract that provides full insurance in the sense that the indemnity payment is equal to their potential loss. Also, depending on the parameter values, low-risk agents may end up in a situation in which, compared to the situation where their risk type was known, they choose a contract that has a lower indemnity payment AND a higher premium. This higher premium is due of course to the problem of insurance fraud in the economy. To my knowledge, the insurance fraud phenomenon has not been studied in an adverse selection setting à la Rothschild-Stiglitz before. I show that insurance fraud is more prevalent in the presence of adverse selection because low-risk agents must accept a lower indemnity payment in order to signal their risk type. As insurance fraud is negatively linked to the indemnity payment, a lower indemnity payment is associated with more fraud. Moreover, the probability of auditing is decreased in the presence of adverse selection.

I also examined the situation of contract restrictions in the sense that I looked at the situation where insurers are mandated to offer a single contract to every one. This pooling contract is, of course, unstable in the setting as it was shown in many papers. This means that similar to the situation encountered in the Rothschild-Stiglitz setup, there is a wide range of parameter values such that all agents in the economy would prefer the pooling contract situation to the separating contract situation. But once the pooling contract is offered, there are alternate contracts that would attract low-risk agents, but not the high-risk agents and make a positive profit. This discussion about inefficient and second-best contracts notwithstanding, and assuming instead that the government restricts the possibility of offering a menu of contracts, I showed that provided the proportion of high-risk agents is low enough, then the amount of fraud in the economy can be smaller when all agents are forced into a single contract that makes zero profit.

6 References

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7 Appendix

7.1 Tables

Table A. Summary of all the variables used in this paper

Variable name	Definition
Y	Each agent's initial wealth
λ	Size of the potential loss
π_L	Probability of an accident occurring for the low risk agent
π_H	Probability of an accident occurring for the high risk agent
β_L	Indemnity to be paid in the contract designed for the low risk agent
β_H	Indemnity to be paid in the contract designed for the high risk agent
α_L	Premium in the contract designed for the low risk agent
α_H	Premium in the contract designed for the high risk agent
η_L	Low risk agent's probability of committing fraud
η_H	High risk agent's probability of committing fraud
ν_L	Principal's probability of auditing provided the "low risk contract" is purchase
ν_H	Principal's probability of auditing provided the "high risk contract" is purchase
c	Principal's cost of auditing an agent's report
k_1	Deadweight penalty paid by the agent for committing fraud
k_2	Penalty paid by the agent to the principal for committing fraud

Table B. Payoffs to an agent of type i (i.e., an agent who has a probability of loss π_i) and to the principal.

State of the world	Action of Agent i	Action of Principal	Payoff to Agent i	Payoff to Principal
No accident	*Tell truth*	*Audits*	$U(Y-\alpha_i)$	$\alpha_i - c$
No accident	Tell truth	Does not audit	$U(Y-\alpha_i)$	α_i
No accident	Lie	Audits	$U(Y-\alpha_i-k_2)-k_1$	α_i+k_2-c
No accident	Lie	Does not audit	$U(Y-\alpha_i+\beta_i)$	$\alpha_i-\beta_i$
Loss	Tell truth	Audits	$U(Y-\alpha_i-\lambda+\beta_i)$	$\alpha_i-\beta_i-c$
Loss	Tell truth	Does not audit	$U(Y-\alpha_i-\lambda+\beta_i)$	$\alpha_i-\beta_i$
Loss	*Lie*	*Audits*	$U(Y-\alpha_i-\lambda_H+\beta_i-k_2)-k_1$	$\alpha_i-\beta_i+k_2-c$
Loss	*Lie*	*Does not audit*	$U(Y-\alpha_i-\lambda_H)$	α_i

Payoffs are contingent on the state of the world and on each player's action.
States in italics represent actions that are off the equilibrium path.

7.2 Proofs

7.2.1 Solution to the claiming game when the agent type is known (Lemma 1, see Boyer, 2000).

From the left-hand side of figure 2, it is clear that γ_0, the principal's posterior belief that the agent suffered a loss given that the agent sent message $m = 0$, is zero. Suppose Nature chooses there to be an accident. Sending message $m = \lambda$ then always dominates sending message $m = 0$, whatever the principal does. Also, if the principal hears message $m = 0$, then she knows for sure that she is not playing at the upper node of left information set. Consequently the only meaningful strategy for the principal is never to audit since she collects $-c$ if she does, and 0 if she does not. Let's now move to the right side of figure 2.

Let η be the probability (in the mixed-strategy sense) that the agent sends message $m = \lambda$ when Nature chooses there to be a loss. By Bayes' rule it follows that γ_λ, the principal's posterior belief that the reported loss is real is given by

$$\gamma_\lambda = \frac{\rho}{\rho + (1-\rho)\,\eta}$$

Only one strategy on the part of the agent makes the principal indifferent as to whether to audit or not. That strategy must be such that

$$\gamma_\lambda = \frac{\beta_H + k_2 - c}{\beta_H + k_2}$$

Isolating η in the two previous equation gives

$$\eta = \left(\frac{c}{\beta_H + k_2 - c}\right)\left(\frac{\rho}{1-\rho}\right)$$

All that is left to calculate is the principal's strategy at information set 1.H. Her strategy must be such that the agent is indifferent between telling the truth and lying, given that $m = 0$. Let ν be the probability (in a mixed-strategy sense) of auditing a $m = \lambda$ message. ν then solves

$$U(Y-\alpha) = \nu\left[U(Y-\alpha-k_2) - k_1\right] + (1-\nu)\,U(Y-\alpha+\beta_H)$$

which means that

$$\nu = \frac{U(Y-\alpha+\beta) - U(Y-\alpha)}{U(Y-\alpha+\beta) - U(Y-\alpha-k_2) + k_1}$$

All six elements of the PBNE have been found.

7.2.2 Solution to the R-S menu of seperating contract.

Rewriting the problem as

$$\begin{aligned}
\max_{\beta_L^s,\beta_H^s} V \quad = \quad & \pi_L U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right] - \lambda + \beta_L^s\right) \\
& +(1-\pi_L)U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right]\right) \\
& -\mu\left[\begin{array}{c}\pi_H\left[U\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right] - \lambda + \beta_H^s\right) - U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right] - \lambda + \beta_L^s\right)\right] \\
+(1-\pi_H)\left[U\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right]\right) - U\left(Y - \pi_L\left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right]\right)\right]\end{array}\right]
\end{aligned}$$

the first order conditions become

$$\frac{\partial}{\partial\beta_H^s} : 0 = -\mu\left[\begin{array}{c}\left((1-\pi_H)\,\pi_H - \pi_H^2\frac{(k_2-c)c}{\left(\beta_H^s+k_2-c\right)^2}\right)U'\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right] - \lambda + \beta_H^s\right) \\
-\left(\pi_H(1-\pi_H) + \pi_H(1-\pi_H)\frac{(k_2-c)c}{\left(\beta_H^s+k_2-c\right)^2}\right)U'\left(Y - \pi_H\left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right]\right)\end{array}\right]$$

$$\frac{\partial}{\partial \beta_L^s} \quad : \quad 0 = \pi_L \left(1 - \pi_L \left[1 + \frac{(k_2 - c)\, c}{(\beta_L^s + k_2 - c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right] - \lambda + \beta_L^s\right)$$
$$-(1-\pi_L)\left(\pi_L \left[1 + \frac{(k_2 - c)\, c}{(\beta_L^s + k_2 - c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s + k_2 - c}\right)\right]\right)$$
$$+\mu \left[\begin{array}{c} \pi_H \left(1 - \pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right] - \lambda + \beta_L^s\right) \\ -(1-\pi_H)\left(\pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) \left[U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right]\right)\right] \end{array}\right]$$

$$\frac{\partial}{\partial \mu} : 0 = \begin{array}{c} \pi_H \left[U\left(Y - \pi_H \left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right] - \lambda + \beta_H^s\right) - U\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right] - \lambda + \beta_L^s\right)\right] \\ +(1-\pi_H)\left[U\left(Y - \pi_H \left[\beta_H^s + c\left(\frac{\beta_H^s}{\beta_H^s+k_2-c}\right)\right]\right) - U\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right]\right)\right] \end{array}$$

so that we find

$$\mu = -\frac{\begin{array}{c} \pi_L \left(1 - \pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right] - \lambda + \beta_L^s\right) \\ -(1-\pi_L)\left(\pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right]\right) \end{array}}{\left[\begin{array}{c} \pi_H \left(1 - \pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right] - \lambda + \beta_L^s\right) \\ -(1-\pi_H)\left(\pi_L \left[1 + \frac{(k_2-c)c}{(\beta_L^s+k_2-c)^2}\right]\right) \left[U'\left(Y - \pi_L \left[\beta_L^s + c\left(\frac{\beta_L^s}{\beta_L^s+k_2-c}\right)\right]\right)\right] \end{array}\right]}$$

7.2.3 Solution to the claiming game when the agent type is not known (pooling contract).

The principal's beliefs has to where she is in the game are given by

$$b_1 = 0$$

$$b_2 = 0$$
$$b_3 = \frac{(1-\rho)(1-\eta_H^p)}{(1-\rho)(1-\eta_H^p) + \rho(1-\eta_L^p)}$$
$$b_4 = \frac{\rho(1-\eta_L^p)}{(1-\rho)(1-\eta_H^p) + \rho(1-\eta_L^p)}$$

When a benefit is requested, her beliefs are given as

$$a_1 = \frac{\rho \pi_L}{\rho(\pi_L + (1-\pi_L)\eta_L^p) + (1-\rho)(\pi_H + (1-\pi_H)\eta_H^p)}$$

$$a_2 = \frac{(1-\rho)\pi_H}{\rho(\pi_L + (1-\pi_L)\eta_L^p) + (1-\rho)(\pi_H + (1-\pi_H)\eta_H^p)}$$
$$a_3 = \frac{(1-\rho)(1-\pi_H)\eta_H^p}{\rho(\pi_L + (1-\pi_L)\eta_L^p) + (1-\rho)(\pi_H + (1-\pi_H)\eta_H^p)}$$
$$a_4 = \frac{\rho(1-\pi_L)\eta_L^p}{\rho(\pi_L + (1-\pi_L)\eta_L^p) + (1-\rho)(\pi_H + (1-\pi_H)\eta_H^p)}$$

For the principal to be indifferent between investigating or not when a loss is claimed, the probability she assigns to a claim being fraudulent $(a_3 + a_4)$ must solve

$$\left(-c - \beta_p\right)(a_1 + a_2) + (k_2 - c)(a_3 + a_4) = -\beta_p \tag{15}$$

where the left hand side represents the principal's expected payoff from investigating (at nodes a_1 and a_2 the report is truthful so the principal has to pay the cost of auditing and the claim), and the right hand side is her payoff from not investigating. Using (15), the belief that fraud is committed is equal to

$$a_3 + a_4 = \left(\frac{c}{\beta_p + k_2}\right)$$

These beliefs are consequent with fraud probabilities that respect the following condition:

$$\eta_L^p = \left(\frac{\rho\pi_L + (1-\rho)\pi_H}{\rho(1-\pi_L)}\right)\frac{c}{\beta_p + k_2 - c} + \frac{(1-\rho)(1-\pi_H)}{\rho(1-\pi_L)}\eta_H^p$$

which is compatible with the fraud probabilities

$$\eta_L^p = \eta_H^p = \left(\frac{\rho\pi_L + (1-\rho)\pi_H}{1-\rho\pi_L - (1-\rho)\pi_H}\right)\frac{c}{\beta_p + k_2 - c}$$

so that all agents commit fraud with the same intensity, which I will note η_p. Note that this pooled probability of fraud is larger (resp. smaller) than the low risk's (resp high risk's) probability of fraud if types were known (i.e., $\eta_L < \eta_p < \eta_H$).

For the agents to be indifferent between committing fraud or not when a loss is not observed, it must be such that

$$\nu_p U\left(Y - \alpha_p + \beta_p\right) + (1-\nu_p)\left[U(Y - \alpha_p - k_2) - k_1\right] = U(Y - \alpha_p)$$

This gives us that the principal investigates with probability

$$\nu = \frac{U\left(Y - \alpha_p + \beta_p\right) - U(Y - \alpha_p)}{U\left(Y - \alpha_p + \beta_p\right) - U(Y - \alpha_p - k_2) + k_1}$$

The principal's beliefs given that a loss was filed are given by

$$\begin{aligned}
a_1 &= \frac{\rho\pi_L}{\rho\left(\pi_L + (1-\pi_L)\eta_p\right) + (1-\rho)\left(\pi_H + (1-\pi_H)\eta_p\right)} \\
&= \frac{\rho\pi_L}{\left(\frac{\rho\pi_L + (1-\rho)\pi_H}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\frac{c}{\beta_p+k_2-c} + \left[\rho\pi_L + (1-\rho)\pi_H\right]\left(1 - \left(\frac{\rho\pi_L+(1-\rho)\pi_H}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\frac{c}{\beta_p+k_2-c}\right)} \\
&= \left(\frac{\rho\pi_L}{\rho\pi_L + (1-\rho)\pi_H}\right)\left(\frac{\beta_p + k_2 - c}{\beta_p + k_2}\right)
\end{aligned}$$

$$\begin{aligned}
a_2 &= \frac{(1-\rho)\pi_H}{\rho(\pi_L + (1-\pi_L)\eta_L) + (1-\rho)(\pi_H + (1-\pi_H)\eta_H)} \\
&= \left(\frac{(1-\rho)\pi_H}{\rho\pi_L + (1-\rho)\pi_H}\right)\left(\frac{\beta_p + k_2 - c}{\beta_p + k_2}\right)
\end{aligned}$$

$$
\begin{aligned}
a_3 &= \frac{(1-\rho)(1-\pi_H)\eta_H}{\rho(\pi_L+(1-\pi_L)\eta_L)+(1-\rho)(\pi_H+(1-\pi_H)\eta_H)} \\
&= \frac{(1-\rho)(1-\pi_H)}{\rho\pi_L+(1-\rho)\pi_H}\left(\left(\frac{\rho\pi_L+(1-\rho)\pi_H}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\frac{c}{\beta_p+k_2-c}\right)\left(\frac{\beta_p+k_2-c}{\beta_p+k_2}\right) \\
&= \left(\frac{(1-\rho)(1-\pi_H)}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\left(\frac{c}{\beta_p+k_2}\right)
\end{aligned}
$$

$$
\begin{aligned}
a_4 &= \frac{\rho(1-\pi_L)\eta_L}{\rho(\pi_L+(1-\pi_L)\eta_L)+(1-\rho)(\pi_H+(1-\pi_H)\eta_H)} \\
&= \left(\frac{\rho(1-\pi_L)}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\left(\frac{c}{\beta_p+k_2}\right)
\end{aligned}
$$

To summarize, the beliefs of the principal in each information node are

$$
a_1 = \left(\frac{\rho\pi_L}{\rho\pi_L+(1-\rho)\pi_H}\right)\left(\frac{\beta_p+k_2-c}{\beta_p+k_2}\right); \qquad a_2 = \left(\frac{(1-\rho)\pi_H}{\rho\pi_L+(1-\rho)\pi_H}\right)\left(\frac{\beta_p+k_2-c}{\beta_p+k_2}\right)
$$

$$
a_3 = \left(\frac{(1-\rho)(1-\pi_H)}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\left(\frac{c}{\beta_p+k_2}\right); \qquad a_4 = \left(\frac{\rho(1-\pi_L)}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\left(\frac{c}{\beta_p+k_2}\right)
$$

$$
b_1 = b_2 = 0; \qquad b_3 = 1-\rho; \qquad b_4 = \rho
$$

7.2.4 The simplification of the maximization problem when a pooling contract exists

The problem faced by the principal is

$$
\begin{aligned}
\max_{\alpha_p,\beta_p,\nu_p,\eta_p} \; & \pi_L U\left(Y-\alpha_p-\lambda+\beta_p\right)+(1-\pi_L)\left(1-\eta_p\right)U(Y-\alpha_p) \\
& +(1-\pi_L)\eta_p\nu_p\left[U(Y-\alpha_p-k_2)-k_1\right]+(1-\pi_L)\eta_p(1-\nu_p)U\left(Y-\alpha_p+\beta_p\right)
\end{aligned}
$$

subject to the principal's zero profit constraint

$$
\begin{aligned}
\alpha_p &= \rho\left[\pi_L\beta_p+(1-\pi_L)\eta_p(1-\nu_p)\beta_p+c\nu_p\left[\pi_L+(1-\pi_L)\eta_p\right]-(1-\pi_L)\eta_p\nu_p k_2\right] \\
&\quad +(1-\rho)\left[\pi_H\beta_p+(1-\pi_H)\eta_p(1-\nu_p)\beta_p+c\nu_p\left[\pi_H+(1-\pi_H)\eta_p\right]-(1-\pi_H)\eta_p\nu_p k_2\right]
\end{aligned}
$$

and the players' PBNE strategies

$$
\eta_p = \left(\frac{\rho\pi_L+(1-\rho)\pi_H}{1-\rho\pi_L-(1-\rho)\pi_H}\right)\frac{c}{\beta_p+k_2-c}
$$

$$
\nu = \frac{U\left(Y-\alpha_p+\beta_p\right)-U(Y-\alpha_p)}{U\left(Y-\alpha_p+\beta_p\right)-U(Y-\alpha_p-k_2)+k_1}
$$

Substituting for the PBNE constraints into the maximization problem and the zero profit constraint yields

$$
\max_{\alpha_p,\beta_p} EU_{\eta\nu} = \pi_L U\left(Y-\lambda-\alpha_p+\beta_p\right)+(1-\pi_L)U(Y-\alpha_p)
$$

subject to

$$
\alpha_p = (\rho\pi_L+(1-\rho)\pi_H)\left(\frac{\beta_p+k_2}{\beta_p+k_2-c}\right)\beta_p
$$

7.3 Relationship between audit cost and penalty.

Suppose that $c = k_2$, so that the penalty paid by the fraudulent agent to the principal compensates her for the cost incurred in conducting such an audit. This penalty does not compensate the principal completely for all audits she conducts, however, since she is compensated only for those audits where she catches an agent committing fraud. In contrast to the total cost of the principal's auditing strategy given by $c\nu_i [\pi_i + (1-\pi_i)\eta_i]$ (see equation 5) the amount she receives is $(1-\pi_i)\eta_i c\nu_i$. The principal's zero-profit function becomes $\alpha_i = \pi_i\beta_i \left[1 + \frac{c}{\beta_i}\right] = \pi_i\beta_i + \pi_i c$, which is nothing more than the premium function when the loading is fixed and equal to $\pi_i c$. The two linear zero-profit functions then cross at point $(\beta, \alpha) = (-c, 0)$. If this is the case, we know that the optimal contract will be full insurance provided any insurance is purchased. Although the economic importance of this situation is not trivial, it does not represent a result that warrants more space in this paper.

Suppose now that $c < k_2$. Then the function α_i is monotone increasing and concave for all values of $\beta_i > 0 > c - k_2$: $\frac{\partial^2 \alpha_i}{\partial \beta_i^2} = -2\pi_i c \frac{(k_2-c)}{(\beta_i+k_2-c)^3} < 0$. For $\beta_i < c - k_2$, the function is also monotone increasing, but it is convex, as illustrated in Figure 8. Note that the function $\alpha_i = \pi_i\beta_i + \pi_i c$ (the first case examined above)

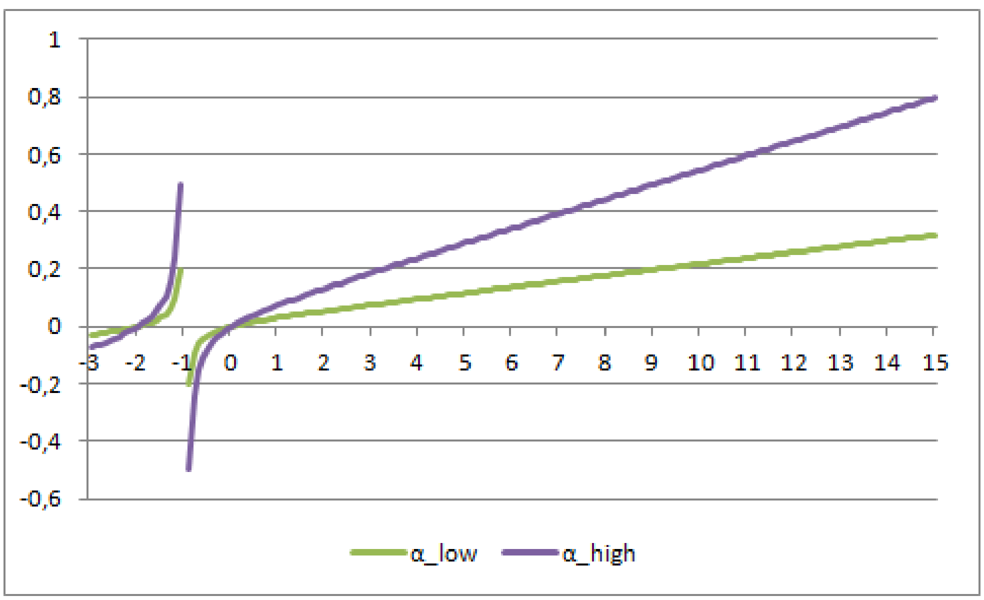

Figure 8: Premium functions $\alpha_i = \pi_i\beta_i \left[1 + \frac{c}{\beta_i + k_2 - c}\right]$ that give the principal zero profit for the high risk agent ($\pi_i = \pi_H$) and the low risk agent ($\pi_i = \pi_L$) when $c < k_2$. The graph is drawn using the following parameters: $Y = 20$, $\pi_L = 2\%$, $\pi_H = 5\%$, $\lambda = 10$, $c = 1$, and $k_2 = 2$. The premium is on the vertical axis and the coverage is on the horizontal axis.

acts as an asymptote that bounds the insurer zero-profit function above for β_i very large and positive, and as an asymptote that bounds the same function below for β_i largely negative. The other asymptote in this case is vertical and delimited as $\beta = c - k_2$. This situation is illustrated in Figure 9 for the case of the low risk agent.

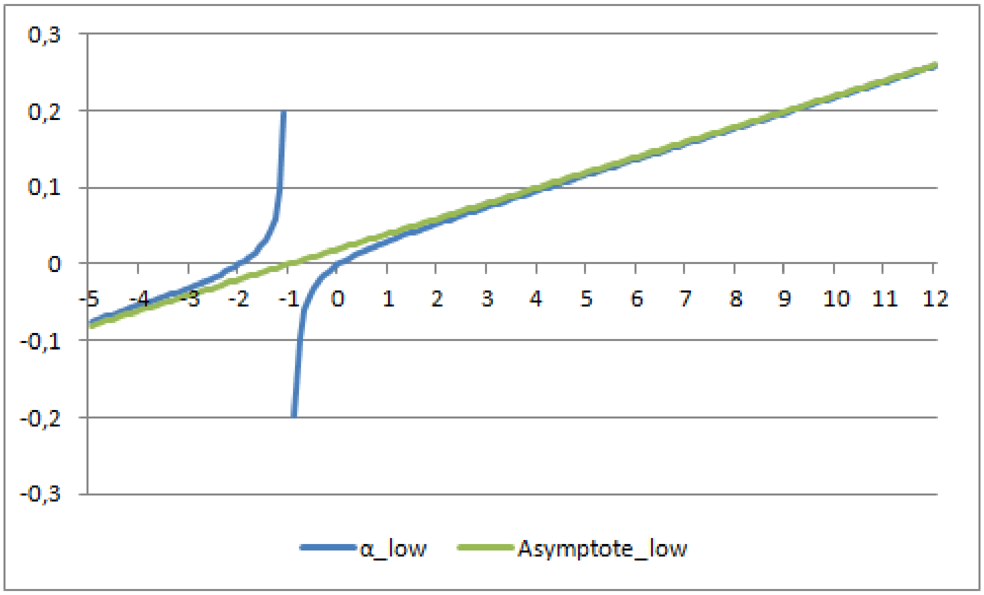

Figure 9: For the low risk agent, the premium function $\alpha_L = \pi_L \beta_L \left[1 + \frac{c}{\beta_L + k_2 - c}\right]$ that gives the principal zero profit when $c < k_2$, and the asymptote that has equation $\alpha_L = \pi_L (\beta_L + c)$. The vertical asymptote at $\beta_L = c - k_2$ is not displayed but can easily be inferred from the graph. The graph is drawn using the following parameters: $Y = 20$, $\pi_L = 2\%$, $\lambda = 10$, $c = 1$, and $k_2 = 2$. The premium is on the vertical axis and the coverage is on the horizontal axis.

We know that $\beta_L^s = \frac{EU_H^* \pm \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{2\pi_L} < 2c$, which can only occur when $\pm\sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c} < 4\pi_L c - EU_H^*$. Obviously, we need $EU_H^* \left(EU_H^* - 4\pi_L c\right) > 0$, which can be achieved when $EU_H^* < 0$, or when $EU_H^* > 4\pi_L c$. We then have

$$\frac{EU_H^* \pm \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{2\pi_L} < \frac{c\beta_L^*}{(\beta_L^* - c)}$$

if and only if

$$\pm\sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c} < 2\pi_L \frac{c\beta_L^*}{(\beta_L^* - c)} - EU_H^* \tag{16}$$

This is true whenever

$$\beta_L^* \left[\pm\sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right] - c\left[\pm\sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right] < 2\pi_L c\beta_L^* - EU_H^*\beta_L^* + cEU_H^*$$

Assume we take the positive root, we then have

$$\left(EU_H^* - 2\pi_L c + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right)\beta_L^* < c\left(EU_H^* + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right)$$

whereas if we take the negative root we have

$$\left(EU_H^* - 2\pi_L c - \sqrt{\left(EU_H^* - 2\pi_L c\right)^2 - 4\left(\pi_L c\right)^2}\right)\beta_L^* < c\left(EU_H^* - \sqrt{EU_H^*\left(EU_H^* - 4\pi_L c\right)}\right)$$

Assume that $EU_H^* < 0$, then

- Taking the positive root always gives us $\left(EU_H^* - 2\pi_L c + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right) < 0$ and $\left(EU_H^* + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L E\right.$ 0 since so that $\alpha_L^s > \alpha_L^*$ if and only if

$$\beta_L^* > c\left(\frac{EU_H^* + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}\right)$$

  which is obviously true since we found that $\beta_L^* > 2c$ when there is no adverse selection and $\left(\frac{EU_H^* + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^*}}\right.$ 1. To see why, note that $EU_H^* + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c} > EU_H^* - 2\pi_L c + \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}$ since $0 > -2\pi_L c$.

- Taking the negative root always gives us $\left(EU_H^* - 2\pi_L c - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}\right) < 0$ and $\left(EU_H^* - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L E\right.$ 0 so that $\alpha_L^s > \alpha_L^*$ if and only if

$$\beta_L^* > c\left(\frac{EU_H^* - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}\right)$$

  which is obviously true since we found that $\beta_L^* > 2c$ when there is no adverse selection and $\left(\frac{EU_H^* - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c - \sqrt{\left(EU_H^*\right)^2 - 4\pi_L EU_H^*}}\right.$ 1.

Assume that $EU_H^* > 0$ so that $EU_H^* > 4\pi_L c$. Then

- Taking the negative root always gives us $\left(EU_H^* - 2\pi_L c - \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}\right) > 0$ and $\left(EU_H^* - \sqrt{(EU_H^*)^2 - 4\pi_L E\right.$ 0 so that $\alpha_L^s > \alpha_L^*$ if and only if

$$\beta_L^* < c\left(\frac{EU_H^* + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}\right)$$

  Given that $\left(\frac{EU_H^* - \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c - \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}\right) > 2$ (to see why, note that $EU_H^* - \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c} > 2EU_H^* - 4\pi_L c - 2\sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}$, so that $0 > -4\pi_L c$

- Taking the positive root always gives us $\left(EU_H^* - 2\pi_L c + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}\right) > 0$ and $\left(EU_H^* + \sqrt{(EU_H^*)^2 - 4\pi_L E\right.$ 0 so that $\alpha_L^s > \alpha_L^*$ if and only if

$$\beta_L^* > c\left(\frac{EU_H^* + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}\right)$$

  which is obviously true since we found that $\beta_L^* > 2c$ when there is no adverse selection and $\left(\frac{EU_H^* + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{EU_H^* - 2\pi_L c + \sqrt{(EU_H^*)^2 - 4\pi_L EU_H\right.$ 1.

- Taking the positive root and squaring on both sides gives us

$$(EU_H^*)^2 - 4\pi_L EU_H^* c < \left(2\pi_L \frac{c\beta_L^*}{(\beta_L^* - c)}\right)^2 - 4\pi_L \frac{c\beta_L^*}{(\beta_L^* - c)} EU_H^* + (EU_H^*)^2 \tag{17}$$

Assume that $EU_H^* < 0$ so that the determinant is always positive.

- Taking the negative root always gives us an equation 16 that holds

- Taking the positive root and squaring on both sides gives us

$$(EU_H^*)^2 - 4\pi_L EU_H^* c < \left(2\pi_L \frac{c\beta_L^*}{(\beta_L^* - c)}\right)^2 - 4\pi_L \frac{c\beta_L^*}{(\beta_L^* - c)} EU_H^* + (EU_H^*)^2 \tag{18}$$

  which becomes

$$EU_H^* < \pi_L \frac{(\beta_L^*)^2}{\beta_L^* - c}$$

  after simplification. This inequality is clearly true since we assumed $EU_H^* < 0$ to start with and $\beta_L^* > c$.

Assume now that $EU_H^* > 0$. We then need $EU_H^* > 4\pi_L c$ for the determinant to make any sense.

- Taking the positive root and squaring (as in equation 18) yields again

$$EU_H^* < \pi_L \frac{(\beta_L^*)^2}{\beta_L^* - c}$$

This is possible if and only if

$$cEU_H^* - EU_H^* \beta_L^* + \pi_L (\beta_L^*)^2 > 0$$

$$\beta_L^* = \frac{EU_H^* \pm \sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{2\pi_L}$$

$$\beta_L^* = \frac{\pm\sqrt{(EU_H^*)^2 - 4\pi_L EU_H^* c}}{} < 2\pi_L c - EU_H^*$$

Given that $EU_H^* > 4\pi_L c$, we would like to find the conditions under which

$$0 > (\beta_L^*)^2 - 4c\beta_L^* + 4c^2 = (\beta_L^* - 2c)^2$$

which is impossible.

- Taking the negative root means that

$$-\sqrt{-EU_H^* (4\pi_L c - EU_H^*)} < 4\pi_L c - EU_H^* < 0$$

if and only if $-EU_H^* (4\pi_L c - EU_H^*) > (4\pi_L c - EU_H^*)^2 > 0$. Dividing everywhere by $4\pi_L c - EU_H^* < 0$ gives us a solution whereby $\frac{\partial \alpha_L}{\partial \beta_L} < 0$ if and only if $0 < 4\pi_L c$, which is obviously always true.