Crossover Risks and Their Interconnectedness in the Korean Reverse Mortgage Program

- Evidence from VAR and VARX Analysis -

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Abstract

To reflect the close relationship between interest rates and house prices, I employ a VAR model in predicting corporate bond yield rates and nationwide house price index and a VARX model in predicting CD yield rates and risk-free rates which are related with corporate bond yield rates. I predicted house price indices by house sizes and regions to reflect more closely the prices of individual houses which are collateralized for the reverse mortgage program. I employ ARIMA to forecast the differences of appreciation rates between nationwide house price indices and the house price indices by house sizes and regions. Although the simulation results using the nationwide house price index show that the Korean reverse mortgage program is sustainable at a 95% confidence level, the simulation results using house price indices by sizes and regions conclude that the Korean reverse mortgage program is not sustainable. The different results are because the nationwide house price index does not reflect the price fluctuations of individual houses and most of the reverse mortgages are concentrated in Seoul and its satellite cities, where house price appreciation rates are much lower than in other regions. This finding implies that a nationwide house price index is not a good indicator of house prices to test the sustainability of the Korean reverse mortgage program.

This paper also concludes that policies of longer duration are more exposed to interest rate risk and house price risk. Therefore, longevity risk may inflate the effect of high interest rates and low house prices. Longevity risk may not be a big concern per se in the reverse mortgage program as long as interest rates are low and house price appreciation rates are high because more mortgage insurance premiums are collected and loan balances may be fully covered by house values. However, longevity risks inflate the negative effects of high interest rates and low house prices on the sustainability of the reverse mortgage program.

Keywords: reverse mortgage, mortality rate, mobility rate, longevity risk, house price index, mortgage insurance premium

JEL Classification: E37, G21, G22, G23, J11

1. Introduction

South Korea has the most significant aging population problem in the world. Korea has been classified as an “aging society,” with 7.2% of the population over 65 since 2000, and is expected to be an “aged society” with 14% of the population over 65 in 2018. It will take only 18 years for South Korea to shift from an aging society to an aged society, which is the fastest aging rate in the world. As a result, South Korea has some of the biggest longevity risks.

In addition to problems related to the aging population, retirement income is another social issue because the aging population and retirement income are very closely related. Retirement income plans designed by the Korean government include the National Pension Plan, which is similar to Social Security benefits in the US, and the Retirement Pension Plan. The National Pension Plan was introduced for full-time employees in 1988 and has been mandated for the entire nation since 1999. However, replacement rates have been decreasing from 70% in 1988 to a projected 40% in 2028. With the trend of aging population, the replacement rate may go below 40% after 2028. The Retirement Pension Plan was introduced in 2005. However, the effectiveness of the plan in each company depends on agreements between employer and employees. Moreover, because the employment market is becoming more unstable, the benefits from the Retirement Pension Plan for full-time employees do not seem to be enough for retirement incomes.

The reverse mortgage program was introduced in 2007 as a solution to the problems of the aging population and lack of retirement income. It was modified in 2012 to reflect the economic environment, such as housing price fluctuations and interest rates. The Housing and Finance Corporation (HFC) operates the program and the government guarantees the payments. Under this program, even if loan balances exceed house values, borrowers are not responsible for the excesses of the loan values because of the non-recourse provision included in the program. In addition to a tenure payment option which was first introduced in 2007, HFC introduced a term payment option in 2013 to give borrowers more choices. All payment options are monthly; there is no lump-sum option in the Korean reverse mortgage program. Because borrowers are paid over their whole life with the tenure payment option but paid over a fixed period with the term payment option, lenders may be more exposed to longevity risk by the tenure payment option than by the term payment option.

In addition to longevity risk, housing price risks are bigger in South Korea relative to other countries. It has been common for Koreans to purchase houses for the purpose of investment rather than residence. Hence, housing price volatilities have been large depending on economic environments, and the housing market has been depressed due to the economic recession in recent years. House price indices have been decreasing since 2006 in Seoul and satellite cities where house prices are higher than other regions.

Therefore, the Korean reverse mortgage program may be riskier than the program in other countries because the longevity risk and house price risk are bigger in Korea. In this study, I will model and forecast mortality rates reflecting the trend of the aging population, examine whether the reverse mortgage program is sustainable from the perspective of crossover risks, and compare the tenure payment option with the term payment option. Because the reverse mortgage program is a type of joint life annuity, I will apply separate male and female mortality rates in the model and examine the results of the separate mortality rates. I will also examine how the results differ by annuity types, such as single male, single female, and joint life annuity types.

The extent literature regarding reverse mortgage program used only house price indices that are not reflecting different house price appreciation rates of different house sizes in different regions. In the study, I include house price appreciation rates in different regions and examine whether the results of the test using nationwide house price indices are different from the results of the test using house price indices of different house sizes in different regions.

Wang, Valdez, and Piggott (2008) demonstrated that longevity risk is not as large in terms of the price of the reverse mortgage securitization as interest rate risk or housing price risk. Because the effect of mortality rate improvement is not large in the reverse mortgage program, I will examine the effect of longeviry risk in the perspective of policy periods rather than mortaliy rate improvement. If policyholder is younger, the policy is more exposed to longevity risk because the poilicy period is longer. In the complete model, I will examine the effect of interest rate risk, house price risk and longevity risk on the reverse mortgage program.

In section 2, I will discuss crossover risks in the reverse mortgage program. In section 3, I will explain the process from the activation to the termination of a policy and how the model tests the sustainability of the program, including the present value of the mortgage insurance premium (MIP) and the present value of losses. The sustainability of the program can be tested by comparing the present values of the MIP and the losses. In section 4, I will model and forecast male and female mortality rates and in section 5, I will discuss interest rate and house price appreciation rate based on real-world economic data. Section 6 will cover the design of the simulation and I will show the results of the simulation in section 7. In section 8, I will offer concluding remarks.

2. Risks in the Reverse Mortgage Program

According to Wang, Valdez, and Piggott (2008) and Chen, Cox, and Wang (2010), the reverse mortgage program includes longevity risks, interest rate risks, and housing price risks. If borrowers are paid as long as they live in their houses and they live longer than expected, it is more likely that the loan value will exceed the house value. If interest rates are higher than expected, the loan value will increase faster and will be more likely to exceed the house value. If house prices are lower than expected, the remaining value of the house will be more likely to be zero or negative.

Because mortality rates have improved faster in South Korea than in other developed countries, the longevity risk may be greater in South Korea. A reverse mortgage program that does not take into account continuously improving mortality rates may be more exposed to longevity risk. Furthermore, because house prices have been depressed recently, reverse mortgage benefits may have been over-estimated. Table 1 shows the monthly payments for the Korean reverse mortgage program by ages and payment options when the house price is assumed to be 300 million Korean Won (KRW) as of January 1, 2015.[1, 2, 3] Monthly payments are determined higher for older applicants than for younger applicants and for term payment options than tenure payment options. The monthly payment amounts are determined by the age of the younger spouse if both spouses are alive when the policy is activated, but the monthly payment amounts are not affected by whether one spouse is dead or both are alive.

<Table 1 is here>

Figure 1-1 illustrates the crossover risks in the reverse mortgage program by ages and payment options assuming that the interest rate is 3.23%, which is the reverse mortgage loan accumulation rate as of January, 2015.[4] House price appreciation rate is assumed to be 2.23%

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[1] Because condominium-type houses (referred to as “apartments” in Korea) are traded more frequently than single-dwelling or other types of houses, price evaluation is relatively easier. Therefore, the house price index means the apartment price index in this study.

[2] Exchange rate is approximately US$1 = 1,090KRW as of January, 2015.

[3] HFC changed the mortgage insurance premium on February 1, 2015. This study does not reflect the changes.

[4] Interest rate applied to loan value is determined by CD (Certificate of Deposit) yield rate (91 days) plus 1.1%. CD yield rate applied to reverse mortgage loans is 2.13% in January, 2015.

annually, lower than interest rate. The loan value of the tenure option for a person age 60 exceeds the house value at year 38, but the crossover risk seems to be low because the life expectancies for male and female of age 60 are 28.13 and 31.25 years respectively according to the mortality rates that will be forecasted in section 4. However, tenure payment options cross over house price faster than term payment options and may be riskier than term payment options.

<Figure 1-1 is here>

As Figure 1-2 and Figure 1-3 show, when the interest rate is higher or house price appreciation rate is lower, loan values cross over house values faster, and the reverse mortgage program is more likely to experience losses. Because the monthly benefit of the term payment option is more than the monthly benefit of the tenure payment option, the term payment option has a higher loan value for the payment periods and may be riskier than the tenure payment option at a high interest rate.

<Figure 1-2 is here>

<Figure 1-3 is here>

Wang, Valdez, and Piggott (2008) and Chen, Cox, and Wang (2010) studied crossover risks in reverse mortgage securitization and sustainability of the Home Equity Conversion Mortgage (HECM) program in the US. Wang, Valdez, and Piggott demonstrated that interest rate risk and house price risk are more severe than longevity risk, while Chen, Cox, and Wang showed that the US HECM program is sustainable using dynamic simulation models with total population mortality rates that combine male and female mortality rates and assume a single life annuity type. However, the reverse mortgage program is a type of joint life annuity, so the separate application of male and female mortality rates to the model may be more appropriate.

Because a borrower is either paid a monthly payment or does not need to pay back the loan as long as one of the borrowing spouses is alive and lives in the house, the policy is likely to continue longer if both spouses are alive when the policy is activated. However, the monthly payment amount is not different whether both spouses are alive or one spouse is dead when the policy is activated. Therefore, if both spouses are alive when the first payment is initiated, the policy has more opportunities for loss.

3. Mortgage Insurance Premium (MIP) and Loan Balance

Because of the non-recourse provision, the reverse mortgage program charges borrowers a mortgage insurance premium (MIP) which is 2% of the house price up front and 0.5% of the loan balance annually as a continuing premium.[5] This premium is the same as in the US HECM program. Because a continuing premium is charged monthly, I applied 0.04167% (or 0.5% / 12) each month to the loan balance in the model.

Loan balance at the end of time tth month is

$$L_t = 0.02 \times H_0 \times \prod_{i=1}^{t}(1 + r_i)^{1/12} + \sum_{i=1}^{t}(B_i + P_i)\prod_{j=i}^{t}(1 + r_j)^{1/12} \qquad (1)$$

where $H_0 =$ house price at time 0

$r_i =$ annual interest rate at ith month

B_i= monthly benefit at ith month

$P_i =$ MIP at ith month,

and MIP at ith month is

$$P_t = \begin{array}{ll} (0.02 \times H_0 + B_t) \times 0.04167\%, & t = 1 \\ (L_{t-1} + B_t) \times 0.04167\%, & t > 1 \end{array} . \qquad (2)$$

In terms of the monthly benefit at ith month, a fixed amount is paid until the reverse mortgage is terminated in the tenure payment option, while a fixed amount is paid until the predetermined

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[5] AS of February 1, 2015, the up-front premium decreased to 1.5% and annual continuing premium increased to 0.75%. As I mentioned above, I will not reflect the premium changes in this study.

term ends in the term payment option.

The loan balance can be divided into pure loan balance and MIP balance. The MIP balance at the end of t^{th} month is

$$\mathrm{MIP_t} = 0.02 \times \mathrm{H_0} \times \prod_{i=1}^{t}(1 + r_i)^{1/12} + \sum_{i=1}^{t} P_i \prod_{j=i}^{t}(1 + r_j)^{1/12} \quad (3)$$

If the loan balance (L_T) is larger than the house value (H_T) when the borrower leaves the house because of death or other reasons (t = T), the difference between the loan balance and the house value is considered a loss and the MIP balance (MIP_T) will be used to recover it.

$$\mathrm{Loss_T} = \max(L_T - H_T, 0) \quad (4)$$

The interest rate charged on loan balance including MIP balance and pure loan balance is CD rate plus 1.1% as long as the policy is in force. However, the MIP balance should be discounted by the risk-free rate (r_f) to calculate the present value because the government guarantees the monthly payments and the MIP balance belongs to the government. Therefore, the loss ($Loss_T$) also should be discounted by the risk-free rate rather than the interest rate:

$$\mathrm{Loss_0} = \mathrm{Loss_T} \prod_{i=1}^{T} \frac{1}{(1+r_{fi})^{1/12}} \quad (5)$$

and

$$\mathrm{MIP_0} = \mathrm{MIP_T} \prod_{i=1}^{T} \frac{1}{(1+r_{fi})^{1/12}} \quad (6)$$

where $r_{fi} =$ risk-free rate at i^{th} month.

If the sum of MIP_0 is larger than the sum of $Loss_0$, the reverse mortgage program will be sustainable.

$$\mathrm{Profit} = \mathrm{MIP_0} - \mathrm{Loss_0} \quad (7)$$

4. Modeling and Forecasting Termination Rate

4-1 Modeling Mortality using the Lee-Carter Model

I employed the Lee-Carter model (Lee & Carter, 1992) for mortality forecasts. This model controls for the time varying components in mortality rates, which are the most important component of longevity risks in an aging society. Chen, Cox, and Wang (2010) and Chen and Cummins (2010) also employed the Lee-Carter model for mortality forecasts in their studies of HECM program sustainability and longevity bond pricing.

To forecast Korean mortality rates, I used the Korean population mortality table from 1970 to 2011 offered by Statistics Korea (KOSTAT), which is a central government organization for statistics.[6] The mortality rate is tabulated by 5-year age intervals except age 0 and age 1.[7] Until 1992, the mortality table assumed that all populations of age 80 die before 85, that is, the mortality rate of age 80 is 1 (${}_5q_{80} = 1$ and $\omega = 84$). The oldest age in the mortality tables was increased to age 100 in 2001 (${}_5q_{100} = 1$ and $\omega = 104$). I used the mortality rate from age 0 to age 75 (q_0 to ${}_5q_{75}$) from 1970 to 2011 to forecast mortality rate because mortality rates from ${}_5q_{80}$ to ${}_5q_{100}$ before 2001 do not have comparative rates.

Lee and Carter (1992) suggest the least squares solution to the equation $\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$ for a given matrix of rates $m_{x,t}$.[8] As Chen, Cox, and Wang (2010) and Chen and Cummins (2010) did, I also applied some restrictions to the model, $a_x = \frac{1}{T}\sum_{t=1}^{T} \ln(m_{x,t})$, $\sum_x b_x = 1$ and $\sum_t k_t = 0$. Lee and Carter also suggest a two-stage procedure in estimating b_x and k_t. In the first stage, the singular value decomposition (SVD) method is suggested to be applied to $\ln(m_{x,t}) - a_x$ to estimate b_x and k_t. In the second stage, k_t is re-estimated for the given a_x and b_x such that the calculated number of deaths from the equation is equal to the actual number of deaths in each year. Because Korean population data published by KOSTAT are

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[6] www.kostat.go.kr

[7] The mortality table shows the probability that the aged person dies before the next tabulated age, i.e., ${}_5q_x$, except age 0 and age group 1-4. The mortality rate of age 0 is q_0, and the mortality rate of age group 1-4 is ${}_4q_1$.

[8] In this study, $m_{x,t}$ means ${}_5q_x$, q_0 or ${}_4q_1$ at time t.

not applicable to mortality data for the calculation of number of deaths, I created mock population data which begin with 100,000 people at age 0 for each year and calculated the number of deaths based on mortality rates at each age group to perform the second stage. The results are shown in Table 2 and Table 3.

<Table 2 is here>

<Table 3 is here>

4-2 Forecasting Mortality Index k_t

To forecast mortality index k_t, I employed a similar method to that used by Lee and Carter (1992), who regressed the differences of k_t and k_{t-1} including an influenza dummy of 1918 in their model.[9] Because there was no comparable mortality shock from 1970 to 2011 in Korea, I did not include a dummy in the model. First, I simply regressed k_t on regressor k_{t-1} using OLS. The results are as follows:

$$\begin{array}{llrcrlll}
\text{Total:} & k_t = & -0.8088 & + & 1.0353 & *k_{t-1} + e_t & R^2 = 0.9995 & \text{Root MSE} = 0.23366 \\
 & & (0.0365) & & (0.0039) & & & \\
\text{Male:} & k_t = & -0.7440 & + & 1.0348 & *k_{t-1} + e_t & R^2 = 0.9993 & \text{Root MSE} = 0.25149 \\
 & & (0.0393) & & (0.0044) & & & \\
\text{Female:} & k_t = & -0.9639 & + & 1.0441 & *k_{t-1} + e_t & R^2 = 0.9994 & \text{Root MSE} = 0.28396. \\
 & & (0.0443) & & (0.0042) & & &
\end{array}$$

The regression results show lower mean squared error than the variance of X_t (= k_t - k_{t-1}) which is used in Lee and Carter (1992). However, the partial autoregression of k_t does not cut off and k_t is not an AR model. Therefore, I fixed the coefficients of k_{t-1} to 1 as Lee and Carter (1992) and Chen and Cummins (2010) did. The averages of X_t (= k_t - k_{t-1}) are -0.7908 (total), -0.7276 (male), and -0.9356 (female). These numbers are much smaller than the -0.365 obtained by Lee

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[9] The result of forecasting k_t in Lee & Carter (1992) is $k_t = k_{t-1} - 0.365 + 5.24\text{flue} + e_t$.

and Carter (1992) and the -0.2172 obtained by Chen and Cummins (2010). This means that the mortality rate improved faster in Korea over the last 40 years than in the US over the last 100 years. The forecasted values of k_t are shown in Table 3.

Figure 2 shows the fitted and forecasted values of the log of mortality rates from the obtained parameters and the real values of the log of mortality rates in selected ages (20, 40, 60, and 70). In 2011, the fitted values are underestimated in age 20 but properly estimated in ages 40, 60, and 70 from the obtained parameters. Because this research is concerned with the reverse mortgage program, which is used by old-aged populations, I will use the results for a_x, b_x, and k_t to forecast mortality rates using the equation, $\ln\left(\widehat{m}_{x,t}\right) = a_x + b_x * \hat{k}_t$. The forecasted mortality rates for older ages are shown in Table 4.

<Figure 2 is here>

<Table 4 is here>

4-3 Interpolation and Extrapolation

Because the mortality rate forecasted above is $\ln({}_5q_x)$, it needs to be converted to q_x, q_{x+1}, q_{x+2}, q_{x+3} , and q_{x+4} appropriately. I assume that the force of mortality follows the Gompertz function, which is $\mu_x = \mathrm{B}\,\mathrm{C}^x$. Based on the Gompertz assumption of force of mortality,

$$p_x = e^{-\int_x^{x+1} \mathrm{B}\,\mathrm{C}^t\,dt} = e^{-\frac{\mathrm{B}\,\mathrm{C}^x(\mathrm{C}-1)}{\ln \mathrm{C}}}, \tag{8}$$

and

$${}_5p_x = e^{-\int_x^{x+5} \mathrm{B}\,\mathrm{C}^t\,dt} = e^{-\frac{\mathrm{B}\,\mathrm{C}^x(\mathrm{C}^5-1)}{\ln \mathrm{C}}}. \tag{9}$$

Because I am interested in the mortality rates of old-aged populations over 60 who are eligible for the reverse mortgage program, I estimated Gompertz parameters B and C, which minimize the sum of squared differences between actual ${}_5q_x$ and estimated ${}_5q_x$ for 50 and older ages in 2011. According to the equation above, $q_x = 1 - p_x = 1 - e^{-\frac{\mathrm{B}\,\mathrm{C}^x(\mathrm{C}-1)}{\ln \mathrm{C}}}$. The parameters are also used to extrapolate mortality rates over age 79. In 2011, the estimated B is 1.15×10^{-5} and 2.06×10^{-6} respectively for male and female, and the estimated C is 1.114328 and 1.129341 respectively for male and female.

4-4 Modification of Gompertz Parameters for Forecasted Mortality Rates

For the forecasted mortality rates, the Gompertz parameters obtained from the 2011 mortality rates should be modified to reflect improved mortality. Because $\ln({}_5q_{x,t})$ is equal to $\ln({}_5q_{x,t-1}) + b_x(k_t - k_{t-1})$, $\ln({}_5q_{x,t})$ decreases by $b_x(k_t - k_{t-1})$ every year. Therefore, ${}_5q_{x,t}$ decreases by the rate of $e^{b_x(k_t - k_{t-1})}$ every year. Let ${}_5p_{x,t}/{}_5p_{x,t-1}$ be e^{α}, then ${}_5p_{x,t}$ is derived from ${}_5p_{x,t-1}$ as follows:

$$\begin{aligned}
{}_5p_{x,t} &= {}_5p_{x,t-1}\, e^{\alpha} \\
&= e^{-\frac{B\,C^x(C^5-1)}{\ln C}}\, e^{\alpha} \\
&= e^{-\frac{B\,C^x(C^5-1)-\alpha\ln C}{\ln C}} \\
&= e^{-\frac{\left(B-\frac{\alpha\ln C}{C^x(C^5-1)}\right)C^x(C^5-1)}{\ln C}} \\
&= e^{-\frac{B'\,C^x(C^5-1)}{\ln C}}, \qquad (10)
\end{aligned}$$

where B and C are Gompertz parameters for ages from x to x+4 at time t-1 and $B' = \left(B - \frac{\alpha \ln C}{C^x(C^5-1)}\right)$. The parameter C does not change over time, but the parameter B changes to B' in the next year in the Gompertz function. The mortality improvement factor α is not expressed explicitly by $e^{b_x(k_t - k_{t-1})}$, but we can calculate α for each age group and each year from the mortality forecasting based on $\ln({}_5q_{x,t}) = \ln({}_5q_{x,t-1}) + b_x(k_t - k_{t-1})$.

I assume $q_{117} = 1$ for male and $q_{118} = 1$ for female, following the Korean life insurance reference annuity mortality table issued by the Korea Insurance Development Institute (KIDI) in 2012. Figure 3 shows the projected mortality rates for age 60, 70, and 80 from 2011 to 2070 by gender. Figure 4 compares the forecasted population mortality rates with Korean reference annuity mortality rates that are applicable for males and females aged 60 as of 2014. Because the Korean reference annuity mortality table includes risk margin, it is lower than the forecasted population mortality rates.

<Figure 3 is here>

<Figure 4 is here>

4-5 Mobility Rate

Mobility rate is the probability that borrowers will move out for any reason except death. When borrowers move to nursing care facilities due to their health conditions or to other family members' houses or other towns, they move out of their houses and their reverse mortgage policies are terminated. Mobility rate has a very similar effect on the reverse mortgage program as the mortality rate. Conducting their research in the US, Wang, Valdez, and Piggott (2008) and Chen, Cox, and Wang (2010) assumed a mobility rate of 30% of mortality rates, which is an actuarial assumption of U.S. Housing and Urban Development (HUD).

However, Chow, Szymanoski, and DiVenti (2000) find that the HUD's assumption is too low for younger aged people (64 to 66) and slightly high for older aged people (84 to 86). For borrowers aged 74 to 76, the HUD's assumption is consistent with the actual rates only for policy year 5 to 8, which is around age 80. They also find that the first-year termination rates are substantially lower than in the next years.

Zhai (2000) explains mobility rates from the perspective of health-related rates and non-health-related rates. He cites the 1997 American Housing Survey by the U.S. Census Bureau to explain the adverse relation between non-health-related mobility rates and ages of borrowers. He also claims that there is a positive relation between health-related mobility rates and ages of borrowers.

Taking into consideration Chow, Szymanoski, and DiVenti (2000) and Zhai (2000), I combine the effects of health-related rates and non-health-related rates into mobility rates by $\alpha \times q_x + \beta \times (1 - q_x)$ where q_x is a mortality rate of age x. It would be reasonable to expect that health-related rates are related to mortality rates and non-health-related rates are related to survival rates.[10] A (alpha) must be less than 0.3, and β should be small enough for mobility rates of younger aged people not to be too high. Although estimated mobility rates cannot be compared with actual mobility rates because termination data for the Korean reverse mortgage program are not available, estimating mobility rates using the equation above would be more appropriate than the HUD's assumption.

For an age group of 65 to 105, I set α and β to 25% and 0.5% initially.[11] Supporting the findings of Chow, Szymanoski, and DiVenti (2000) and accepting that overall mobility rates are not significantly different from the HUD's assumption, α and β are finally chosen as 25.1% and 0.2%. The chi-square statistic of the log-rank test for the equality of survival function (which tests the null hypothesis that the mobility rates using α and β of 25.1% and 0.2% are the same as the mobility rates of the HUD's assumption) is 0.020207, and the *p*-value is 0.89.[12] [13] The

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[10] Non-health-related mobility rates may be affected by financial conditions, family-related issues, or many other personal reasons of borrowers. Investigating the causes of non-health-related move-out requires data which are not publicly available and thus beyond this research. I assume that non-health-related mobility rates are linearly related to survival rates in this research to simplify the model.

[11] The initial rates of 25% and 0.5% were chosen at my discretion. If the initial values are different, the final choices may be different.

[12] The test result is not tabulated in the paper.

[13] For the test, I used a pseudo sample of 100,000 each for male and female starting at age 65. Because this is the test for equal mobility rates, deaths are considered as censored data. The test ends at age 105 and compares the total estimated number of $\alpha \times q_x + \beta \times (1 - q_x)$ with the total number of the HUD's assumption for the sample.

expected mobility rates of the HUD's assumption are 0.00269, 0.012124, and 0.021963 respectively at ages 65, 80, and 85, and the estimated mobility rates using α and β of 25.1% and 0.2% are 0.004233, 0.012079, and 0.020274 respectively at age 65, 80, and 85. The comparison of mobility rates at specific ages shows that the estimated mobility rates are consistent with Chow, Szymanoski, and DiVenti's findings. For the test of the sustainability of the Korean reverse mortgage program, $\alpha \times q_x + \beta \times (1 - q_x)$ is used for the mobility rates in this research when α is 25.1% and β is 0.2%.

5. House Appreciation Rate and Interest Rate

I will investigate apartment price rather than other types of houses because apartments are more frequently traded and their price indices are more reliable than others. Figure 5 shows the apartment price index yield rates by size from 1987 to 2012.[14] The apartment price index yield rates are different by size but they move very similarly except for the last six years. Annual average yield rates are 5.11%, 5.48%, and 6.60% respectively for large, medium, and small sizes.

<Figure 5 is here>

However, if we look by region, yield rates move differently. Figure 6 shows the apartment price index yield rates by size in selected regions from 2008 to 2012. Yield rates move differently in Seoul compared to the rest of the nation and show large differences even between southern Seoul and northern Seoul in some years. We can infer that prices are more volatile for individual units because price indices are averages of individual units.

<Figure 6 is here>

Wang, Valdez, and Piggott (2008) and Ma, Lew, and Synn (2011) employed geometric Brownian motion, and Chen, Cox, and Wang (2010) employed the ARMA-GARCH model to

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[14] small < 62.8㎡ ≤ medium < 95.9㎡ ≤ large

predict dynamic house prices. However, if I run a dynamic simulation using the nationwide house price index, the results may be inaccurate and not reflect house price fluctuation appropriately for individual units that are collateralized for reverse mortgage loans. Therefore, I will use house price indices of different house sizes in different regions to test the sustainability of the Korean reverse mortgage program because price indices by sizes and regions reflect prices of individual units better than a nationwide index does.

Before predicting house appreciation rate by sizes and regions, I forecast nationwide house price appreciation rate using the log of house price appreciation rate ($\ln(\mathrm{HPI}_t/\mathrm{HPI}_{t-1})$) and the log of 3-year corporate bond (AA-) yield rate changes ($\ln(\mathrm{IR}_t/\mathrm{IR}_{t-1})$), employing a VAR (Vector Autoregressive) model. The logs of house price appreciation rates by sizes and regions are predicted using an ARIMA model. CD rates and government bond rates are predicted by a VARX model, which is a VAR model with exogenous variables.

5-1 Forecasting nationwide house price index and corporate bond rate

For the VAR model, I used the monthly data of the natural log of house price index appreciation rate ($\mathrm{ln\_HPI}_t$) and the natural log of 3-year corporate bond (AA-) rate change ($\mathrm{ln\_IR}_t$) from January 1987 to December 2012 (312 time series observations for each variable).[15] Before performing the VAR test, I performed the augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test for $\mathrm{ln\_HPI}_t$ and $\mathrm{ln\_IR}_t$. These two tests are the most commonly used unit-root tests. The results are tabulated in Table 5. They do not have unit roots and the VAR model is employed.[16]

<Table 5 is here>

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[15] Alai et.al. (2014) employed VAR model to forecast the joint dynamics of interest rates, house prices, and rental yields. I referred to Alai et. al. (2014) to predict nationwide house prices and corporate bond rates.

[16] The model may need to employ a vector error correction model (VECM) to resolve a cointegration problem, but VECM is not covered in this paper.

To decide the optimal lag order of the VAR model for $\mathrm{ln\_HPI_t}$ and $\mathrm{ln\_IR_t}$, I compared Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC) lag order selection statistics. The results are tabulated in Table 6. The AIC statistic suggests a lag order of 7, the HQIC statistic suggests a lag order of 5, and the SBIC statistic suggests a lag order of 1. I choose the lag order of 5 based on HQIC, and the appropriate VAR(5) model equation is as follows:

$$\begin{aligned}
\mathrm{ln\_HPI}_t &= \alpha_1 + \beta_{11}\mathrm{ln\_HPI}_{t-1} + \beta_{12}\mathrm{ln\_HPI}_{t-2} + \beta_{13}\mathrm{ln\_HPI}_{t-3} + \beta_{14}\mathrm{ln\_HPI}_{t-4} + \beta_{15}\mathrm{ln\_HPI}_{t-5} \\
&\quad + \gamma_{11}\mathrm{ln\_IR}_{t-1} + \gamma_{12}\mathrm{ln\_IR}_{t-2} + \gamma_{13}\mathrm{ln\_IR}_{t-3} + \gamma_{14}\mathrm{ln\_IR}_{t-4} + \gamma_{15}\mathrm{ln\_IR}_{t-5} + \varepsilon_t \\
\mathrm{ln\_IR}_t &= \alpha_2 + \beta_{21}\mathrm{ln\_HPI}_{t-1} + \beta_{22}\mathrm{ln\_HPI}_{t-2} + \beta_{23}\mathrm{ln\_HPI}_{t-3} + \beta_{24}\mathrm{ln\_HPI}_{t-4} + \beta_{25}\mathrm{ln\_HPI}_{t-5} \\
&\quad + \gamma_{21}\mathrm{ln\_IR}_{t-1} + \gamma_{22}\mathrm{ln\_IR}_{t-2} + \gamma_{23}\mathrm{ln\_IR}_{t-3} + \gamma_{24}\mathrm{ln\_IR}_{t-4} + \gamma_{25}\mathrm{ln\_IR}_{t-5} + e_t \qquad (11)
\end{aligned}$$

<Table 6 is here>

Table 7-1 shows that the coefficients of the equations, β_{12}, β_{21}, β_{22}, β_{23}, β_{24}, β_{25}, γ_{11}, γ_{12}, γ_{14}, γ_{23} γ_{24}, γ_{25}, and the constant of the ln_HPI equation are insignificant. If insignificant variables are included in forecasting house price appreciation rate and interest rate, the insignificant coefficient may make the forecast inaccurate. Therefore, I constrained the insignificant coefficients to zero and re-estimated the model. β_{21} became significant in the second model, and coefficients are estimated once more. Table 7-2 shows the results.

<Table 7-1 is here>

<Table 7-2 is here>

5-2 Forecasting house price indices by sizes and regions

Because house price indices are different by sizes and regions and nationwide house price indices are the averages of the various indices, the consideration of various house price indices would result in more accurate predictions. The four region categories for house price indices are

1) northern Seoul, 2) southern Seoul, 3) 6 other major cities (Busan, Daegu, Daejeon, Ulsan, Gwangju, and Sejong), and 4) Gyong-gi province, which surrounds Seoul and includes many satellite cities. The three size categories are small, medium, and large.[17] Multiplying each region by the three sizes leads to 12 subcategories. Therefore, I will perform the test with house price indices of the 12 subcategories. Because more than 95% of the reverse mortgage policies in Korea are for homes in these four regions, it would be appropriate to exclude other regions to simplify the model.

Because the logs of house price index appreciation rates by categories are autoregressive (AR) and also very closely related with the nationwide house price index appreciation rates, an ARMAX model may be considered with the natural log of nationwide house price index appreciation rate as an exogenous variable.[18] However, nationwide house price index appreciation rate is not an exogenous variable because it is an average of different house price indices by sizes and regions. To avoid this endogenous problem, I create a new variable, diff_HPI^c, which is the difference between ln_HPI and ln_HPI^c.

$$\text{diff\_HPI}_t^c = \text{ln\_HPI}_t - \text{ln\_HPI}_t^c \qquad (12)$$

where ln_HPI^c is the log of house price index appreciation rate by sizes and regions.

Difference of house price indices (diff_HPI^c) is represented by different ARIMA models for regions and sizes. Some variables have ARMA or ARIMA models while other variables are white noises in differences. The ARMA or ARIMA models are as follows:

$\text{diff\_HPI}_t^c = \alpha + u_t$ (ARMA) or

$\text{D.diff\_HPI}_t^c = \alpha + u_t$ (ARIMA with one unit root)

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[17] The size criteria are explained above (small < 62.8㎡ ≤ medium < 95.9㎡ ≤ large).

[18] In ARMA model analysis, the natural logs of the house price index appreciation rates by sizes and regions have AR(1), AR(2) or AR(3), without an exception. I did not tabulate the AR models in the paper.

$$u_t = \sum_{\mathrm{i}=1}^{p} \rho_i \, u_{t-i} + \sum_{\mathrm{i}=1}^{q} \theta_i \, \varepsilon_{t-i} + \varepsilon_t \tag{12}$$

where $\mathrm{D.diff\_HPI}_t^c = \mathrm{diff\_HPI}_t^c - \mathrm{diff\_HPI}_{t-1}^c$.

Table 8 shows the results to predict the house appreciation rate by sizes and regions.[19]

<Table 8 is here>

5-3 Forecasting CD rates and risk-free rates

CD (91 days) rates ($\mathrm{CD_t}$) and 3-year government bond rates ($\mathrm{gov\_bond_t}$) are also predicted using the VAR model. Because CD rates and government bond rates are related with 3-year corporate bond (AA-) rates, I employ a VARX model with 3-year corporate bond rate as an exogenous variable. All variables are the logs of appreciation rates.[20]

I performed the augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test for $\mathrm{ln\_CD_t}$ and $\mathrm{ln\_gov\_bond_t}$ as in section 5.1. The results of the two tests show that the variables do not have unit roots at 1% significant levels. All of Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC) lag order selection statistics suggest the lag order of 1. The model is as follows:

$$\mathrm{ln\_CD}_t = \alpha_1 + \beta_{11}\mathrm{ln\_CD}_{t-1} + \gamma_{11}\mathrm{ln\_gov\_bond}_{t-1} + \delta_1\mathrm{ln\_bond}_t + \varepsilon_t$$

$$\mathrm{ln\_gov\_bond}_t = \alpha_2 + \beta_{21}\,\mathrm{ln\_CD}_{t-1} + \gamma_{21}\mathrm{ln\_gov\_bond}_{t-1} + \delta_2\mathrm{ln\_bond}_t + e_t. \tag{13}$$

Because β_{21} and all constants are insignificant, I constrained these coefficients to 0 and re-estimated the coefficients. The results are presented in Table 9.

<Table 9 is here>

6. Test of Reverse Mortgage Sustainability

In this section, I will design a simulation model including all factors which affect the

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[19] Unit root test is not tabulated.

[20] $\mathrm{ln\_CD_t = ln\,(CD_t\,/CD_{t-1})}$ and $\mathrm{ln\_gov\_bond_t = ln\,(gov\_bond_t\,/gov\_bond_{t-1})}$

reverse mortgage loan balances, mortgage insurance premium balances (MIP), and house prices and run the simulation by house sizes and regions. In terms of mortality assumptions, I use the mortality table obtained in section 4 based on the Lee-Carter model. I use the mobility rate assumption, $0.251q_x+0.002(1-q_x)$, which is not significantly different from the HUD's assumption for the entire age group of 65 - 105 but reflects health-related and non-health-related mobility rates at each age group of 65-69, 70-74, and so forth. Predicted CD (91 days) rates and 3-year government bond rates are used as interest rates and risk-free rates to accumulate loan balances and discount net profits of the program to the present value.

The test will simulate net profits of different payment options, ages, and genders with different house price indices, that is, the nationwide house price index and house price indices of the subcategories explained in section 5. To test the sustainability of the program with a more realistic assumption, I will calculate weighted averages of net profits of the subcategories. The weights to calculate the average are the current outstanding guarantees of the program in each region. An individual house price is assumed to be 300 million KRW regardless of the size and region. Because the eligible house price is 900 million KRW or less in the Korean reverse mortgage program, I exclude large-sized homes in southern Seoul in the weighted average test because the price of a large house there is rarely less than 900 million KRW.

The test assumes that 1,000 policies are activated on January 1, 2015, for each gender, age, payment option, annuity type, and house price index. Policies are also assumed to be terminated at the middle of the year (the end of the 6th month of each year, or June 30). Single annuity types (male or female) are terminated when the borrowers die or move out, while joint annuity types are terminated when both spouses die or move out. Therefore, joint annuity types continue longer than single annuity types on average and will be more exposed to longevity risk.

Policies are terminated based on the survivor table (l_x, l_y, l_{xy}) calculated by mortality forecasts.[21] MIP_T and $Loss_T$ are determined when policies are terminated and immediately discounted by the risk-free rate. When all policies are terminated, MIP_0 and $Loss_0$ are summed and total profit or loss is calculated for each case (age, payment option, annuity type, and house price index). The number of iterations is 5,000.

7. Simulation Results

Table 10 shows the net profits of one thousand policies by ages, genders, payment options, and annuity types using the nationwide house price index. All cases show positive medians of net profits without an exception. However, policies of males at age 60 and 65 and policies of females and joint at age 60, 65, and 70 have negative means although the medians are higher than at older ages. This means that policies of longer duration are exposed to more crossover risks and very rare extreme losses may make the means negative. Most cases have positive net profits at the 5th percentile, which means that the probability of loss is less than 5% in the simulation using the nationwide house price index.[22]

<Table 10 is here>

In the tenure option, the medians of net profits are decreasing and net profits are less dispersed as age is increasing. The simulation results of the term payment options also show that the distributions of net profits are dispersed less widely for older ages. In any payment option at any age, similarities are observed between male, female, and joint annuity types. Because male mortality rates are higher than female mortality rates, the durations of female policies are longer than the durations of male policies and the durations of joint annuity types are the longest. For

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[21] The survivor table is not tabulated in this paper.

[22] Chen, Cox, and Wang (2010) show that HECM is sustainable in the US with 95% confidence.

example, in the tenure payment option at age 60, the distribution of net profits for females is dispersed more widely than the distribution for males, and the net profits of the joint annuity type are dispersed more widely than of single annuity types. This means that policies for younger ages are more exposed to crossover risks. Because mortality rates are not stochastic in the simulation, the result does not indicate longevity risks. This implies that policies of longer duration are more sensitive to the volatilities of interest rates and house price appreciation rates.

Table 11 shows the net profits of 1000 policies using the weighted average of the net profits of subcategories by age, gender, payment option, and annuity type. In contrast to the results shown in Table 10, all cases show negative medians of net profits without an exception. Even 95th percentiles are negative in most payment options and ages for female and joint policies. The results of the simulation using weighted average indicates that the simulation using nationwide house price index provides misleading results.

<Table 11 is here>

The net profits of subcategories are very different by house sizes and regions.[23] In Seoul and Gyong-gi province, most of the cases show negative medians of net profits. On the other hand, all cases in the six major cities show positive medians of net profits. These results reflect the recent different house price trends in these regions.

8. Conclusion

Because individual houses are collateralized for the reverse mortgage program, the data of individual house prices are more appropriate than house price indices. To reflect the individual house prices in the simulation, I estimated house price indices by house sizes and regions. Although the simulation results using the nationwide house price index show that the net profits

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[23] The simulation results of subcategories are not tabulated. They may be provided on request.

of the Korean reverse mortgage program are positive and the program is sustainable at the 95% confidence level, the simulation results using house price indices by sizes and regions conclude that the Korean reverse mortgage program is not sustainable. The result is due to the different house price appreciation rates by different sizes in different regions. It also implies that the nationwide house price index is not a good indicator of house prices to test the sustainability of the Korean reverse mortgage program because it does not reflect the price fluctuations of individual houses and most reverse mortgages are concentrated in Seoul and its satellite cities, where house price appreciation rates are much lower than in other regions.

This paper also concludes that policies of longer durations are more exposed to interest rate risk and house price risk. Therefore, longevity risk may inflate the effect of high interest rates and low house prices. Longevity risk may not be a big problem in the reverse mortgage program as long as interest rates are low and house price appreciation rates are high because more mortgage insurance premiums are collected and loan balances may be fully covered by house values. However, longevity risks inflate the negative effects of interest rate risks and house price risks on the sustainability of the reverse mortgage program when the economic environment moves toward high interest rates and low house prices.

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Table 1: Monthly Payments of Reverse Mortgage Program by Ages and Payment Options

Age	Tenure	Term		
		15 years	20 years	25 years
60	685,240	937,540	792,800	709,440
65	822,950	1,065,170	897,670	
70	999,390	1,202,610		
75	1,236,500			
80	1,565,380			

Note: The amounts are calculated at www.hf.go.kr assuming that house price is 300 million KRW, as of January 1st, 2015.

Table 2: Fitted Values of a_x and b_x

Age	a_x			b_x		
	Total	Male	Female	Total	Male	Female
0	-4.38762	-4.34815	-4.43435	0.08712	0.08810	0.08240
1	-5.55728	-5.51395	-5.60868	0.09431	0.09630	0.08837
5	-5.88836	-5.78195	-6.02494	0.09863	0.09955	0.09460
10	-6.12781	-6.00573	-6.28180	0.08775	0.08849	0.08428
15	-5.45549	-5.21844	-5.80623	0.07285	0.07179	0.07436
20	-5.18489	-4.96165	-5.50708	0.06877	0.06803	0.06968
25	-5.03676	-4.81078	-5.37209	0.06011	0.05701	0.06407
30	-4.83561	-4.59400	-5.19837	0.05593	0.05456	0.05789
35	-4.49770	-4.21408	-4.94184	0.05014	0.04879	0.05295
40	-4.06766	-3.73688	-4.60052	0.04827	0.04758	0.05210
45	-3.65583	-3.30131	-4.22651	0.04575	0.04535	0.04996
50	-3.28763	-2.92251	-3.85876	0.04535	0.04490	0.04903
55	-2.92617	-2.54808	-3.48700	0.04496	0.04546	0.04674
60	-2.53558	-2.15745	-3.04363	0.04358	0.04430	0.04354
65	-2.12024	-1.75437	-2.54356	0.03912	0.04021	0.03752
70	-1.69802	-1.36913	-2.01439	0.03137	0.03206	0.02957
75	-1.26293	-0.97765	-1.47946	0.02600	0.02753	0.02296

Table 3: Fitted and Forecasted Values of k_t

Year	Total	Male	Female	Year	Total	Male	Female
1970	12.2471	11.0955	13.8692	2012	-20.9656	-19.4645	-25.4272
1971	11.9544	10.9433	13.4489	2013	-21.7564	-20.1921	-26.3628
1972	11.6650	10.7935	13.0344	2014	-22.5472	-20.9197	-27.2984
1973	11.3777	10.6469	12.6252	2015	-23.3379	-21.6473	-28.234
1974	11.0933	10.5028	12.2209	2016	-24.1287	-22.3749	-29.1697
1975	10.8116	10.3620	11.8226	2017	-24.9195	-23.1025	-30.1053
1976	10.5327	10.2232	11.4287	2018	-25.7103	-23.8302	-31.0409
1977	10.2562	10.0876	11.0399	2019	-26.501	-24.5578	-31.9765
1978	9.9820	9.9547	10.6556	2020	-27.2918	-25.2854	-32.9122
1979	9.7099	9.8241	10.2758	2021	-28.0826	-26.013	-33.8478
1980	9.2181	9.2313	9.8529	2022	-28.8734	-26.7406	-34.7834
1981	8.7303	8.6420	9.4413	2023	-29.6642	-27.4682	-35.7191
1982	8.2460	8.0559	9.0402	2024	-30.4549	-28.1959	-36.6547
1983	7.7655	7.4729	8.6499	2025	-31.2457	-28.9235	-37.5903
1984	7.0606	6.7412	7.9215	2026	-32.0365	-29.6511	-38.5259
1985	6.3610	6.0125	7.2102	2027	-32.8273	-30.3787	-39.4616
1986	5.4352	5.1290	6.2686	2028	-33.618	-31.1063	-40.3972
1987	4.5149	4.2498	5.3335	2029	-34.4088	-31.834	-41.3328
1988	3.6651	3.3581	4.4862	2030	-35.1996	-32.5616	-42.2684
1989	2.8183	2.4682	3.6457	2031	-35.9904	-33.2892	-43.2041
1990	1.9477	1.5201	2.7975	2032	-36.7812	-34.0168	-44.1397
1991	1.0854	0.5901	1.9570	2033	-37.5719	-34.7444	-45.0753
1992	0.3270	0.0234	1.0264	2034	-38.3627	-35.472	-46.0109
1993	-0.4259	-0.5373	0.1021	2035	-39.1535	-36.1997	-46.9466
1994	-1.0500	-1.1955	-0.5552	2036	-39.9443	-36.9273	-47.8822
1995	-1.6713	-1.8527	-1.2065	2037	-40.7351	-37.6549	-48.8178
1996	-2.4846	-2.6068	-2.1372	2038	-41.5258	-38.3825	-49.7535
1997	-3.2983	-3.3583	-3.0700	2039	-42.3166	-39.1101	-50.6891
1998	-4.0920	-4.1138	-3.9270	2040	-43.1074	-39.8378	-51.6247
1999	-4.8795	-4.8639	-4.7792	2041	-43.8982	-40.5654	-52.5603
2000	-5.9924	-5.8730	-6.0689	2042	-44.6889	-41.293	-53.496
2001	-7.1114	-6.8836	-7.3663	2043	-45.4797	-42.0206	-54.4316
2002	-8.2985	-8.0626	-8.6447	2044	-46.2705	-42.7482	-55.3672
2003	-9.3902	-9.0489	-9.9823	2045	-47.0613	-43.4758	-56.3028
2004	-11.0406	-10.5040	-12.0736	2046	-47.8521	-44.2035	-57.2385
2005	-12.6986	-11.9630	-14.1772	2047	-48.6428	-44.9311	-58.1741
2006	-14.2426	-13.2415	-16.3058	2048	-49.4336	-45.6587	-59.1097
2007	-15.4512	-14.4707	-17.7463	2049	-50.2244	-46.3863	-60.0454
2008	-16.8849	-15.6165	-19.9794	2050	-51.0152	-47.1139	-60.981
2009	-18.1651	-16.9656	-21.4653	2051	-51.8059	-47.8416	-61.9166
2010	-18.7149	-17.4376	-22.4386	2052	-52.5967	-48.5692	-62.8522
2011	-20.1748	-18.7368	-24.4915	2053	-53.3875	-49.2968	-63.7879

Table 4: Forecasted Log of Mortality Rates for Old Ages

	Total				Male				Female			
	60	65	70	75	60	65	70	75	60	65	70	75
2011	-3.415	-2.909	-2.331	-1.788	-2.988	-2.508	-1.970	-1.494	-4.110	-3.462	-2.739	-2.042
2012	-3.449	-2.940	-2.356	-1.808	-3.020	-2.537	-1.993	-1.514	-4.151	-3.497	-2.766	-2.063
2013	-3.484	-2.971	-2.380	-1.829	-3.052	-2.566	-2.016	-1.534	-4.191	-3.533	-2.794	-2.085
2014	-3.518	-3.002	-2.405	-1.849	-3.084	-2.595	-2.040	-1.554	-4.232	-3.568	-2.822	-2.106
2015	-3.553	-3.033	-2.430	-1.870	-3.116	-2.625	-2.063	-1.574	-4.273	-3.603	-2.849	-2.128
2016	-3.587	-3.064	-2.455	-1.890	-3.149	-2.654	-2.086	-1.594	-4.314	-3.638	-2.877	-2.149
2017	-3.622	-3.095	-2.480	-1.911	-3.181	-2.683	-2.110	-1.614	-4.354	-3.673	-2.905	-2.171
2018	-3.656	-3.126	-2.504	-1.931	-3.213	-2.713	-2.133	-1.634	-4.395	-3.708	-2.932	-2.192
2019	-3.691	-3.157	-2.529	-1.952	-3.245	-2.742	-2.156	-1.654	-4.436	-3.743	-2.960	-2.214
2020	-3.725	-3.188	-2.554	-1.973	-3.278	-2.771	-2.180	-1.674	-4.477	-3.778	-2.988	-2.235
2021	-3.760	-3.219	-2.579	-1.993	-3.310	-2.800	-2.203	-1.694	-4.517	-3.813	-3.015	-2.257
2022	-3.794	-3.250	-2.604	-2.014	-3.342	-2.830	-2.226	-1.714	-4.558	-3.848	-3.043	-2.278
2023	-3.828	-3.281	-2.628	-2.034	-3.374	-2.859	-2.250	-1.734	-4.599	-3.884	-3.071	-2.300
2024	-3.863	-3.312	-2.653	-2.055	-3.407	-2.888	-2.273	-1.754	-4.639	-3.919	-3.098	-2.321
2025	-3.897	-3.343	-2.678	-2.075	-3.439	-2.917	-2.296	-1.774	-4.680	-3.954	-3.126	-2.343
2026	-3.932	-3.374	-2.703	-2.096	-3.471	-2.947	-2.320	-1.794	-4.721	-3.989	-3.154	-2.364
2027	-3.966	-3.404	-2.728	-2.116	-3.503	-2.976	-2.343	-1.814	-4.762	-4.024	-3.181	-2.386
2028	-4.001	-3.435	-2.752	-2.137	-3.536	-3.005	-2.366	-1.834	-4.802	-4.059	-3.209	-2.407
2029	-4.035	-3.466	-2.777	-2.158	-3.568	-3.034	-2.390	-1.854	-4.843	-4.094	-3.237	-2.429
2030	-4.070	-3.497	-2.802	-2.178	-3.600	-3.064	-2.413	-1.874	-4.884	-4.129	-3.264	-2.450
2031	-4.104	-3.528	-2.827	-2.199	-3.632	-3.093	-2.436	-1.894	-4.925	-4.164	-3.292	-2.472
2032	-4.139	-3.559	-2.852	-2.219	-3.664	-3.122	-2.460	-1.914	-4.965	-4.199	-3.320	-2.493
2033	-4.173	-3.590	-2.877	-2.240	-3.697	-3.151	-2.483	-1.934	-5.006	-4.235	-3.347	-2.515
2034	-4.208	-3.621	-2.901	-2.260	-3.729	-3.181	-2.506	-1.954	-5.047	-4.270	-3.375	-2.536
2035	-4.242	-3.652	-2.926	-2.281	-3.761	-3.210	-2.530	-1.974	-5.088	-4.305	-3.403	-2.558
2036	-4.276	-3.683	-2.951	-2.302	-3.793	-3.239	-2.553	-1.994	-5.128	-4.340	-3.430	-2.579
2037	-4.311	-3.714	-2.976	-2.322	-3.826	-3.268	-2.576	-2.014	-5.169	-4.375	-3.458	-2.601
2038	-4.345	-3.745	-3.001	-2.343	-3.858	-3.298	-2.600	-2.034	-5.210	-4.410	-3.486	-2.622
2039	-4.380	-3.776	-3.025	-2.363	-3.890	-3.327	-2.623	-2.054	-5.250	-4.445	-3.513	-2.644
2040	-4.414	-3.807	-3.050	-2.384	-3.922	-3.356	-2.646	-2.074	-5.291	-4.480	-3.541	-2.665
2041	-4.449	-3.838	-3.075	-2.404	-3.955	-3.385	-2.670	-2.094	-5.332	-4.515	-3.569	-2.686
2042	-4.483	-3.868	-3.100	-2.425	-3.987	-3.415	-2.693	-2.115	-5.373	-4.550	-3.596	-2.708
2043	-4.518	-3.899	-3.125	-2.445	-4.019	-3.444	-2.716	-2.135	-5.413	-4.586	-3.624	-2.729
2044	-4.552	-3.930	-3.149	-2.466	-4.051	-3.473	-2.740	-2.155	-5.454	-4.621	-3.652	-2.751
2045	-4.587	-3.961	-3.174	-2.487	-4.084	-3.502	-2.763	-2.175	-5.495	-4.656	-3.679	-2.772
2046	-4.621	-3.992	-3.199	-2.507	-4.116	-3.532	-2.786	-2.195	-5.536	-4.691	-3.707	-2.794
2047	-4.656	-4.023	-3.224	-2.528	-4.148	-3.561	-2.810	-2.215	-5.576	-4.726	-3.735	-2.815
2048	-4.690	-4.054	-3.249	-2.548	-4.180	-3.590	-2.833	-2.235	-5.617	-4.761	-3.762	-2.837
2049	-4.724	-4.085	-3.273	-2.569	-4.212	-3.619	-2.856	-2.255	-5.658	-4.796	-3.790	-2.858
2050	-4.759	-4.116	-3.298	-2.589	-4.245	-3.649	-2.880	-2.275	-5.699	-4.831	-3.818	-2.880

Table 5: Unit-root test statistics

Variables	ADF		PP	
	t statistic	p-value	t statistic	p-value
$\ln\_HPI_t$	-7.731	<0.001	-81.881	<0.001
$\ln\_IR_t$	-12.133	<0.001	-230.628	<0.001

Note: 1. ADF is augmented Dickey-Fuller test and PP is the Phillips-Perron test.
2. ln_HPI is the log of nationwide house price index appreciation rates.
3. ln_IR is the log of 3 year government bond rate (AA-) changes.

Table 6: Lag selection criteria

Lag order	AIC	HQIC	SBIC
1	-9.84785	-9.82003	-9.77815*
2	-9.86429	-9.81793	-9.74812
3	-9.87684	-9.81194	-9.71421
4	-9.95656	-9.87312	-9.74747
5	-9.98470	-9.88272*	-9.72915
6	-9.96747	-9.84695	-9.66545
7	-9.99140*	-9.85233	-9.64291
8	-9.98679	-9.82918	-9.59184

Note: AIC is Akaike’s information criterion, HQIC is Hannan and Quinn information criterion, and SBIC is Schwarz’s Bayesian information criterion.

Table 7-1: Coefficients of VAR(5) model

Variable	Lag	Coefficient						std. err.
		1	2	3	4	5	constant	
ln_HPI$_t$	ln_HPI	0.7946***	0.0121	-0.3343***	0.1819***	0.1591***	0.0007	0.0073
		(0.0540)	(0.0688)	(0.0658)	(0.0683)	(0.0546)	(0.0005)	
	ln_IR	-0.0046	0.0029	-0.0282***	0.0010	-0.0176**		
		(0.0074)	(0.0077)	(0.0078)	(0.0078)	(0.0074)		
ln_IR$_t$	ln_HPI	0.2252	0.6219	0.2679	-0.2352	0.3344	-0.0094***	0.0541
		(0.3992)	(0.5079)	(0.4858)	(0.5043)	(0.4032)	(0.0034)	
	ln_IR	0.2864***	-0.1593***	0.0875	-0.0562	0.0583		
		(0.0550)	(0.0572)	(0.0577)	(0.0579)	(0.0547)		

Note: 1. ln_HPI is the log of nationwide house price index appreciation rate.
2. ln_IR is the log of the 3 year government bond rate (AA-) change
3. Standard errors are applied to one step ahead forecasting.

Table 7-2: Coefficients of VAR(5) model

Variable	Lag	Coefficient						std. err.
		1	2	3	4	5	constant	
ln_HPI$_t$	ln_HPI	0.8003***		-0.3305***	0.1818***	0.1585***	0.0007	0.0073
		(0.0403)		(0.0571)	(0.0672)	(0.0520)	(0.0005)	
	ln_IR			-0.0267***		-0.0171***		
				(0.0072)		(0.0071)		
ln_IR$_t$	ln_HPI		0.9336***				-0.0084***	0.0540
			(0.2615)				(0.0032)	
	ln_IR	0.2760***	-0.1196**					
		(0.0545)	(0.0540)					

Note: 1. ln_HPI is the log of nationwide house price index appreciation rate.
2. ln_IR is the log of the 3 year government bond rate (AA-) change
3. Insignificant coefficients in Table 7-1 are constrained to zero.
4. Standard errors are applied to one step ahead forecasting.

Table 8: Models for house price appreciation rates by sizes and regions

Subcategory	Unit root	White Noise	Coefficients ρ_1	ρ_2	ρ_3	α	std. err.
Northern Seoul Small	No	No	0.7898*** (0.0248)				0.0060
Northern Seoul Medium	No	No	0.9177*** (0.0447)				0.0018
Northern Seoul Large	No	No	0.4749*** (0.0937)		0.3960*** (0.1102)	0.0033* (0.0018)	0.0020
Southern Seoul Small	Yes	No	-0.2567*** (0.0721)				0.0033
Southern Seoul Medium	Yes	Yes					0.0030
Southern Seoul Large	No	No	0.7057*** (0.0656)			0.0051*** (0.0016)	0.0032
6 Major Cities Small	Yes	No	-0.2607** (0.1293)				0.0020
6 Major Cities Medium	Yes	No	-0.3835*** (0.1461)				0.0015
6 Major Cities Large	No	No	0.4193*** (0.1139)	0.2486** (0.1140)			0.0018
Gyong-Gi Small	Yes	Yes					0.0019
Gyong-Gi Medium	Yes	Yes					0.0018
Gyong-Gi Large	No	No	0.6492*** (0.1002)			0.0063*** (0.0009)	0.0026

Note: 1. Observations are the difference between the log of nationwide house price appreciation rate and the logs of house price appreciation rates by sizes and regions.
2. If unit root is no, the model is AR(1), AR(2), or AR(3)
3. If unit root is yes and white noise is no, the model is ARIMA(1,1,0).
4. If unit root is yes and white noise is yes, the differences of observations are white noise.
5. Insignificant coefficients are constrained to zero and significant coefficients are re-estiimated.

Table 9: Coefficients of VARX model for CD (91 days) rates and 3 year government bond rate

	Coefficients			std. err.
	β	γ	δ	
ln_CD_t	0.4119*** (0.0498)	0.1351*** (0.0521)	0.4448*** (0.0408)	0.0377
$ln\_gov\_bond_t$		0.1529*** (0.0442)	0.6499*** (0.0406)	0.0376

Note: 1. Insignificant coefficients are constrained to zero and other coefficients are re-estimated.
2. Constants are insignificant.

Table 10: Net Profits (Nationwide House Price Index, One thousand policies)

Option			Tenure					Term (20 years)		Term (15 years)		
Age			60	65	70	75	80	60	65	60	65	70
MALE	Mean		-5,000	-376	2	10	9	-6,074	-431	-6,691	-473	1
	Median		21	17	14	12	10	22	18	23	19	16
	Minimum		-20,144,435	-1,812,202	-40,495	-1,500	-553	-24,364,210	-2,066,513	-26,562,666	-2,260,691	-47,002
	Maximum		283	157	75	33	19	322	173	348	177	67
	Percen-tile	5%	11	10	9	7	6	12	11	13	11	10
		10%	15	13	11	10	8	16	14	17	15	13
		90%	27	22	17	13	11	30	23	31	25	19
		95%	33	25	19	14	11	36	27	38	28	21
FEMALE	Mean		-27,477	-3,240	-179	7	10	-33,452	-3,706	-37,078	-4,080	-210
	Median		27	22	19	15	13	28	24	29	25	21
	Minimum		-93,832,895	-14,228,967	-895,669	-18,179	-1,668	-113,488,056	-16,225,304	-123,728,182	-17,749,609	-1,038,808
	Maximum		451	300	151	52	27	514	331	554	335	126
	Percen-tile	5%	11	9	5	2	-2	13	11	13	12	10
		10%	17	16	13	11	8	19	17	20	18	15
		90%	36	29	23	18	14	39	31	41	33	26
		95%	44	35	26	20	15	48	37	51	40	29
JOINT	Mean		-31,704	-3,594	-187	7	11	-38,587	-4,109	-42,739	-4,523	-219
	Median		30	26	21	17	14	32	27	33	28	23
	Minimum		-110,442,659	-15,883,429	-933,247	-19,088	-2,084	-133,577,070	-18,111,902	-145,629,848	-19,813,455	-1,082,409
	Maximum		551	373	186	64	33	629	411	679	416	157
	Percen-tile	5%	11	9	4	0	-4	14	12	14	13	10
		10%	19	17	15	12	8	21	19	21	19	17
		90%	42	34	27	21	16	45	36	47	38	30
		95%	51	40	31	23	17	56	43	59	46	34

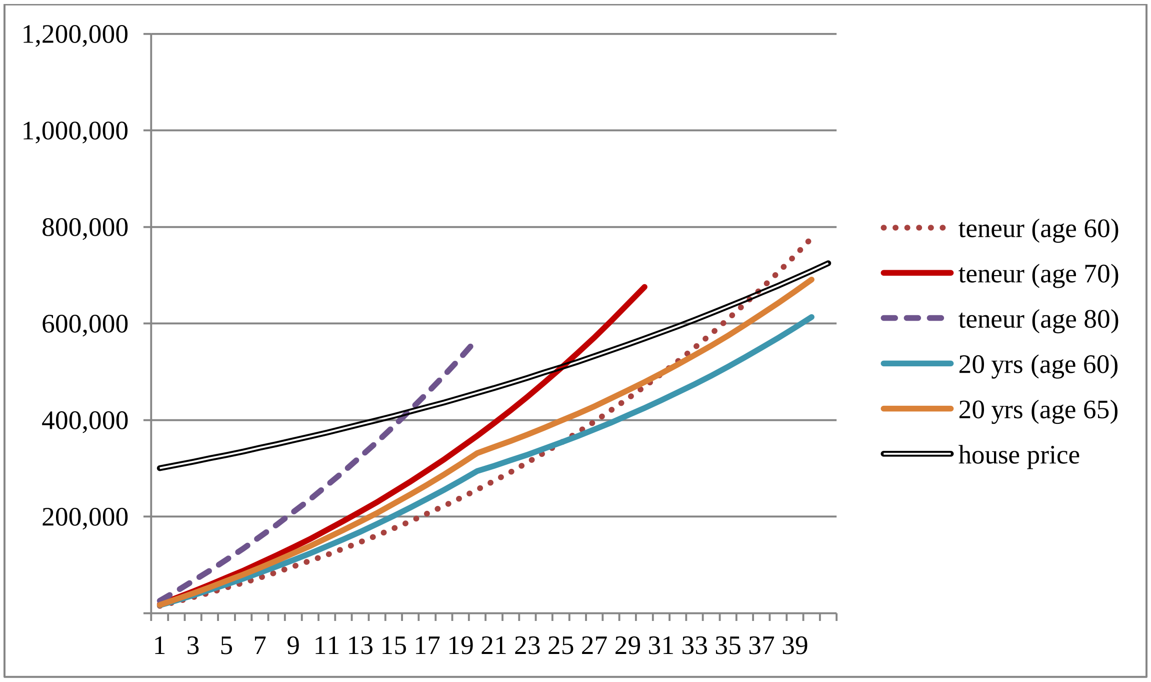
Note: 1. KRW amounts are scaled by one billion.
2. House price is assumed to be 300 million KRW regardless of sizes and regions.

Table 11: Net Profits (Weighted Average, One thousand policies)

Option			Tenure					Term (20 years)		Term (15 years)		
Age			60	65	70	75	80	60	65	60	65	70
MALE	Mean		-4,954	-413	-33	-16	-8	-6,025	-469	-6,639	-512	-38
	Median		-34	-27	-20	-13	-7	-31	-28	-29	-27	-23
	Minimum		-19,721,426	-1,775,614	-38,160	-1,493	-558	-23,939,431	-2,029,463	-26,137,382	-2,223,316	-44,511
	Maximum		16	12	11	10	9	18	13	19	15	12
	Percen -tile	5%	-93	-73	-55	-39	-25	-98	-78	-96	-80	-63
		10%	-73	-60	-46	-32	-20	-72	-62	-70	-63	-52
		90%	-7	-5	-2	1	3	-5	-4	-3	-3	-2
		95%	-1	0	3	4	5	1	1	3	3	3
FEMALE	Mean		-26,958	-3,251	-232	-40	-25	-32,910	-3,711	-36,525	-4,083	-261
	Median		-50	-46	-39	-31	-22	-40	-40	-36	-37	-35
	Minimum		-91,360,019	-13,998,057	-875,309	-16,918	-1,665	-111,013,129	-15,993,573	-121,252,710	-17,517,320	-1,017,911
	Maximum		14	12	11	10	10	19	15	21	16	13
	Percen -tile	5%	-140	-118	-95	-74	-55	-138	-115	-137	-115	-95
		10%	-102	-93	-79	-62	-47	-91	-87	-87	-85	-77
		90%	-15	-13	-11	-7	-4	-8	-9	-5	-7	-7
		95%	-7	-6	-4	-1	1	-1	-2	1	-0	-0
JOINT	Mean		-31,116	-3,611	-253	-51	-33	-37,970	-4,118	-42,109	-4,528	-282
	Median		-60	-56	-49	-40	-30	-47	-49	-42	-44	-44
	Minimum		-107,623,481	-15,621,366	-910,757	-17,744	-2,082	-130,755,349	-17,848,754	-142,807,472	-19,549,568	-1,059,249
	Maximum		14	12	11	11	10	21	16	23	18	15
	Percen -tile	5%	-165	-144	-118	-94	-72	-162	-139	-160	-137	-116
		10%	-122	-113	-98	-79	-61	-106	-104	-101	-100	-93
		90%	-19	-18	-15	-11	-6	-10	-13	-7	-10	-10
		95%	-10	-9	-7	-4	-1	-3	-4	0	-2	-2

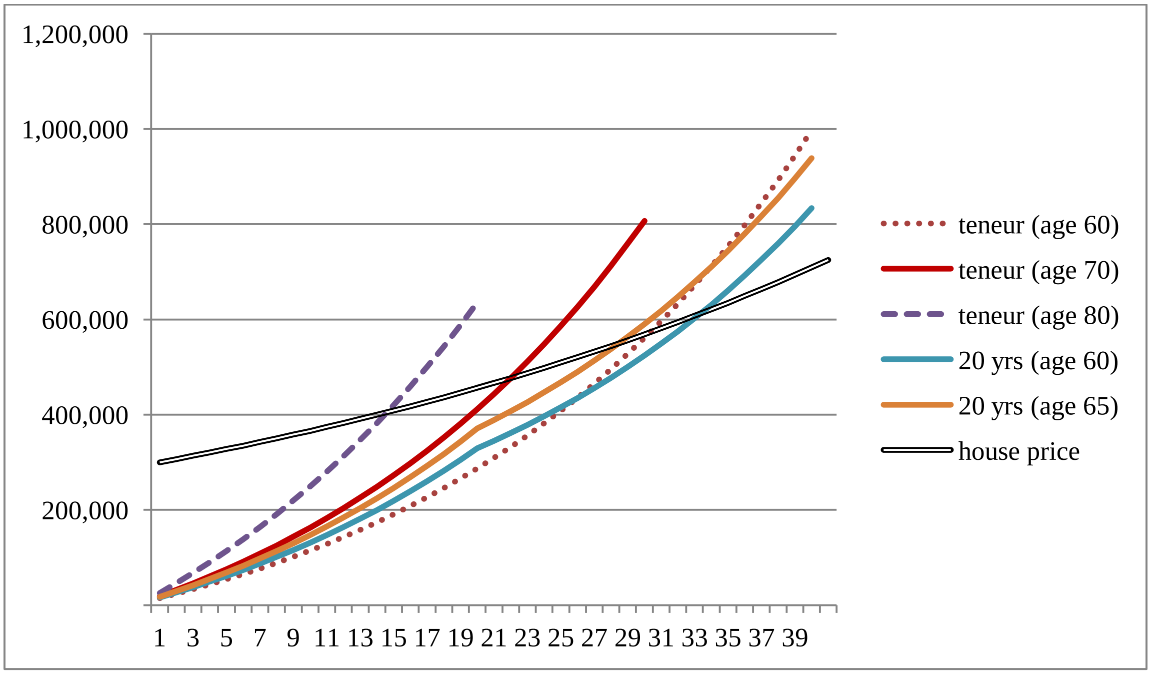
Note: 1. KRW amounts are scaled by one billion.
2. House price is assumed to be 300 million KRW regardless of sizes and regions.
3. Weighted average is the weighted average of 11 categories except southern Seoul (large) based on the amount of the outstanding guarantee by regions.

Figure 1-1: Crossover Risks in Reverse Mortgage (1)



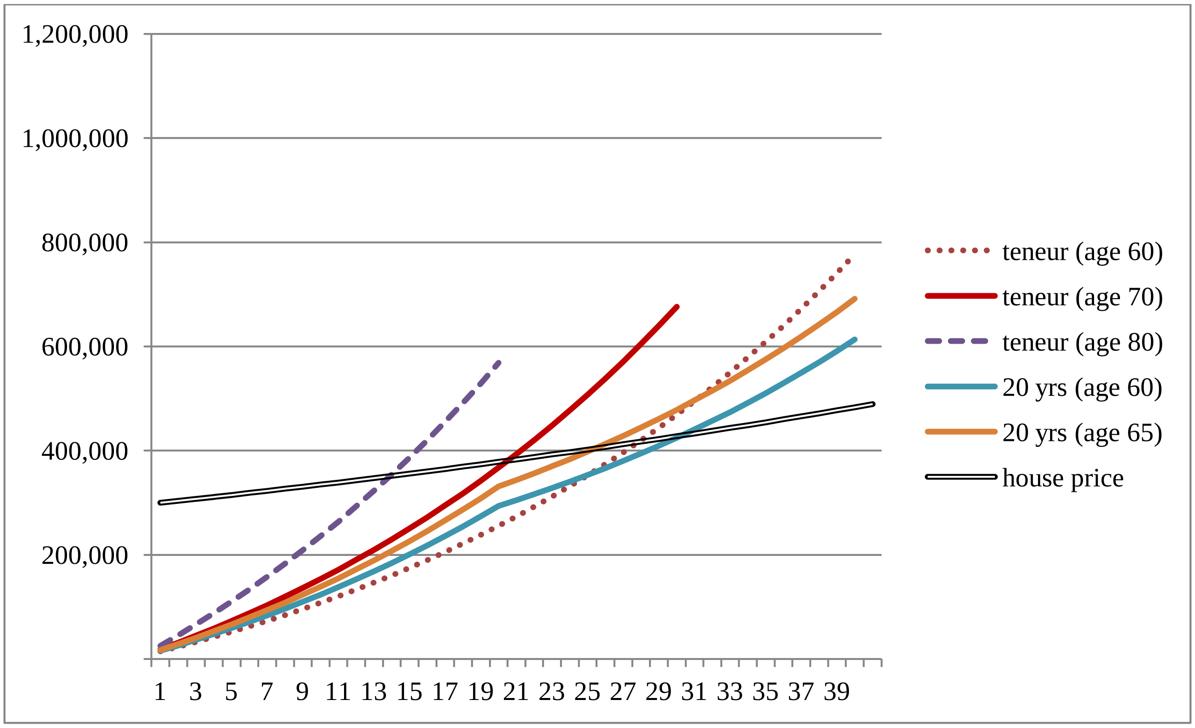
Note: 1. Interest rate is 3.23% and house price appreciation rate is 2.23%.
2. The amount is scaled by 1,000 KRW.

Figure 1-2: Crossover Risks in Reverse Mortgage (2)



Note: 1. Interest rate is 4.23% and house price appreciation rate is 2.23%.
2. The amount is scaled by 1,000 KRW.

Figure 1-3: Crossover Risks in Reverse Mortgage (3)



Note: 1. Interest rate is 3.23% and house price appreciation rate is 1.23%.
2. The amount is scaled by 1,000 KRW.

Figure 2: Comparison of Log of Fitted Mortality with Log of Real Mortality in Selected Ages

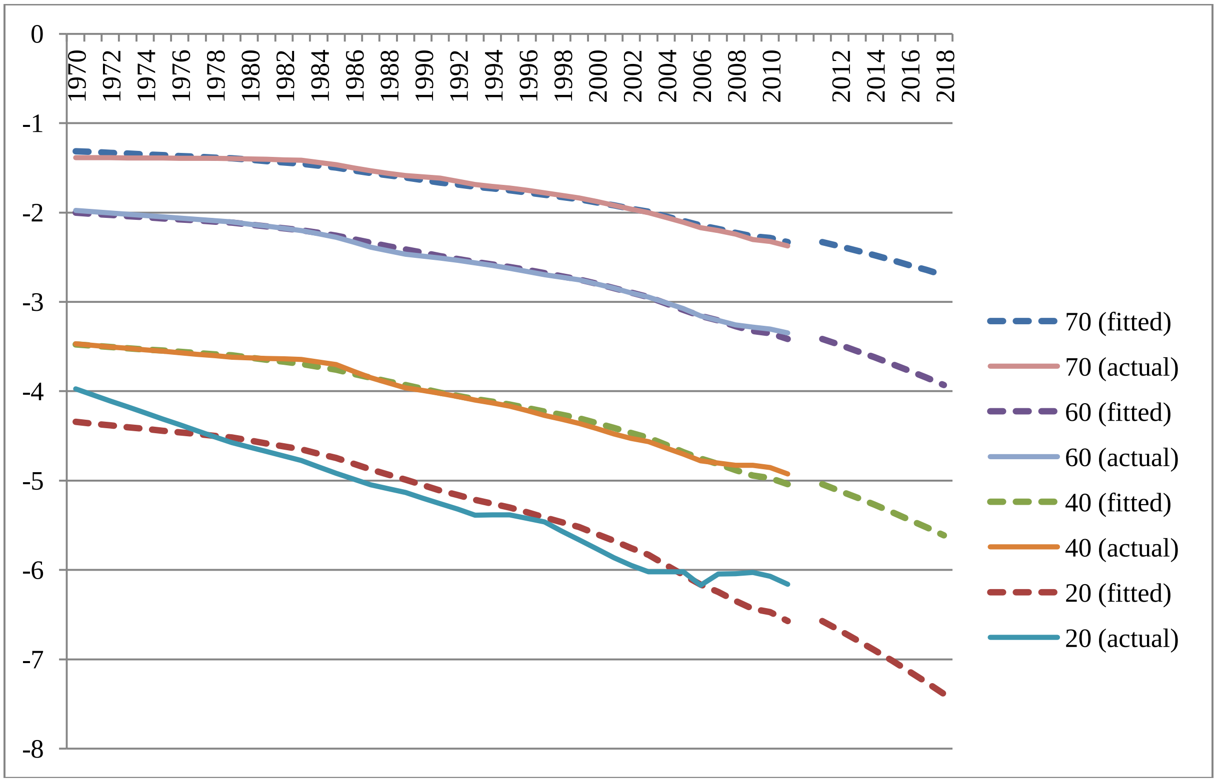


Figure 3: Mortality Rate Improvement by Genders and Selected Ages

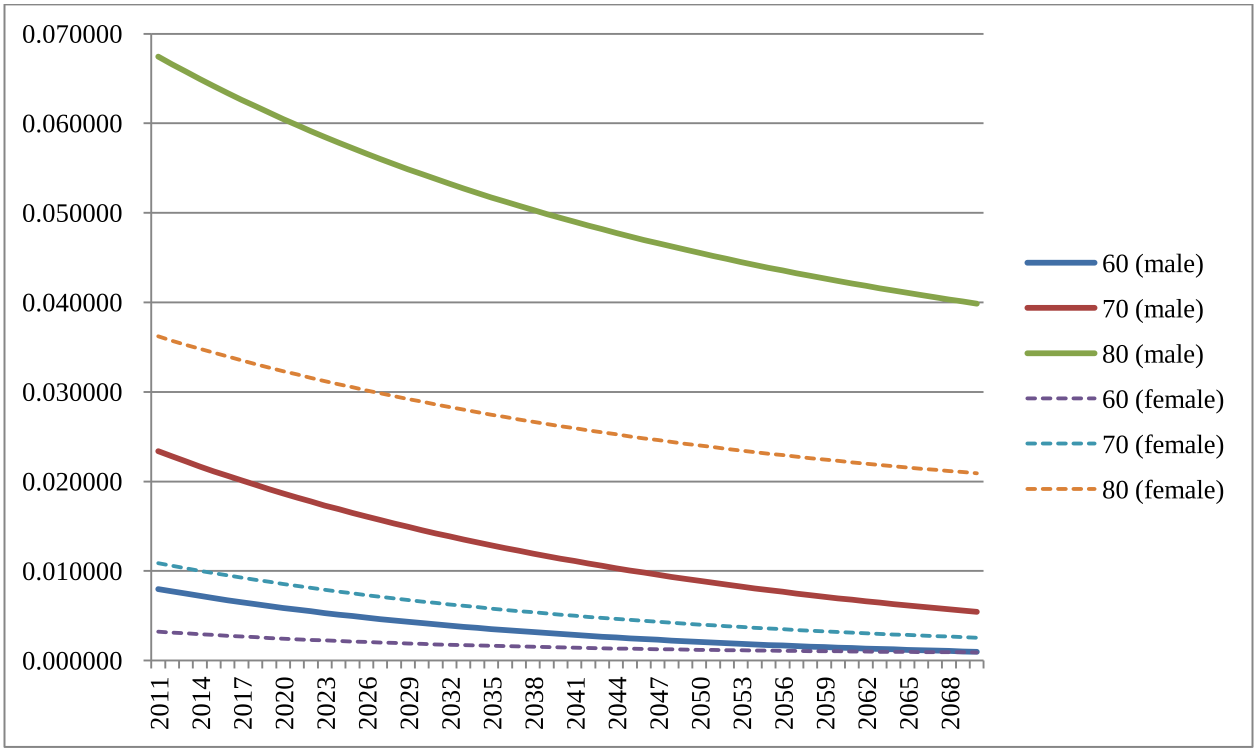
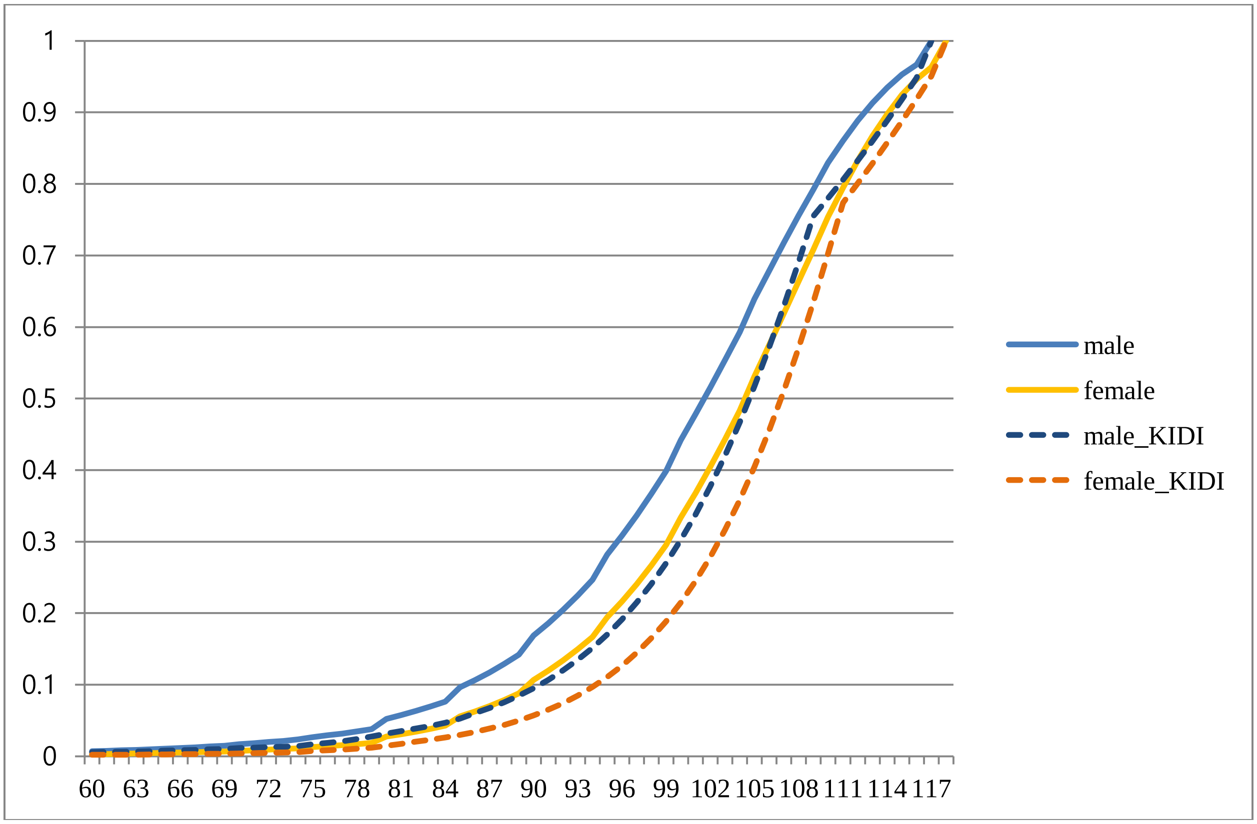


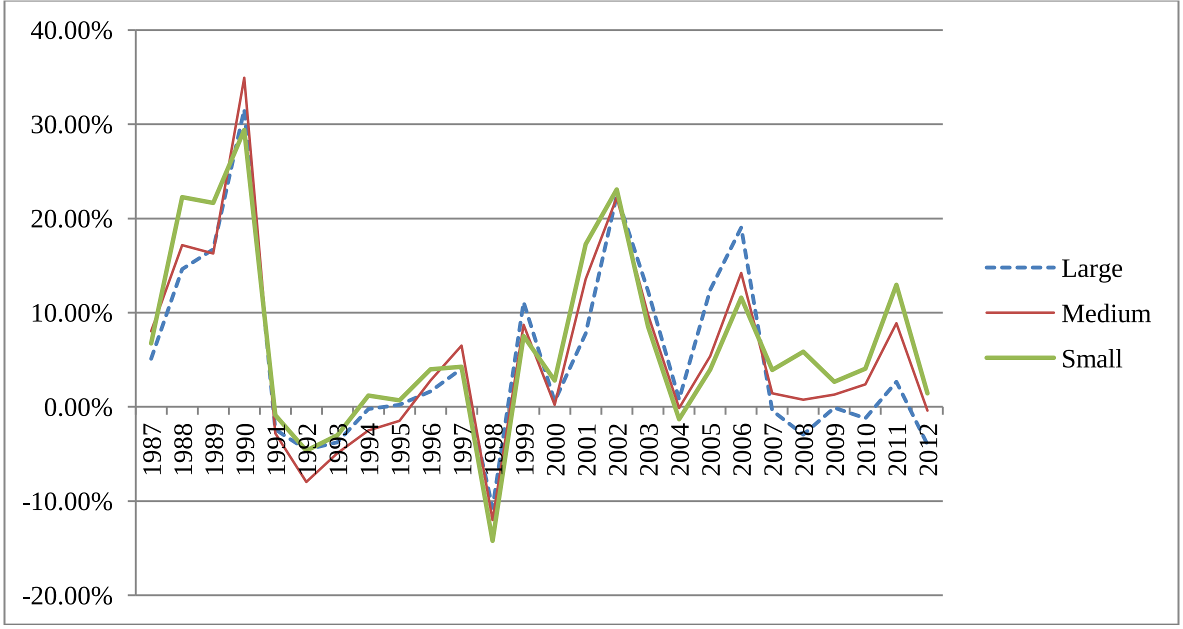
Figure 4: Korean Population Mortality Rates and Reference Annuity Mortality Rates



Note: 1. These mortality rates reflect mortality improvement so that these are applicable to male and female of age 60 as of 2014.

2. q_{117}=1 for male and q_{118}=1 for female are assumed.

Figure 5: Apartment Price Index Yield Rate by Size



Note: small < 62.8㎡ ≤ medium < 95.9㎡ ≤ large

Figure 6: Apartment Price Index Yield Rate by Size and Region