Reverse Mortgages and Retirement Income: Empirical Analyses of Different Age Cohorts in Taiwan

Abstract

This study examines the enhancement effect of reverse mortgages on the retirement income of different generations of homeowners in Taiwan. We employ Taiwan's population mortality rate and housing returns data to calculate the principal limit factor referred to by Szymanoski (1994). We then use the estimated housing prices to calculate the income levels that the elderly homeowners can obtain by taking advantage of these reverse mortgages, and thereby measure the extent of their impact on the retired homeowners' income replacement rate. Our results show that the average enhancement effect of the reverse mortgages on the retirement income replacement ratio of the elderly homeowners living alone is between 70% and 100%. As for the impacts among different age cohorts within the population, due to the fluctuations in housing prices in Taiwan, the enhancement effect of reverse mortgages on the retirement income of the younger generation of retired homeowners is greater than that in the case of the older generation of retired homeowners.

Keywords: Reverse Mortgage, Retirement Income, Principal Limit Factor, Replacement Rate

1. Introduction

As people live longer lives, the longevity risk that the elderly face in their retirement is correspondingly increasing, and the adequacy of their retirement income to meet their needs is also being severely tested. In particular, the aging of the global population and the increasingly low birth rate that have become more and more serious in recent years have also resulted in the financial crisis facing social insurance becoming more deeply entrenched, and this has heightened concerns over the economic security of the elderly in their retirement. Since the 1980s, the United States, the U.K. and other countries have developed a set of housing mortgage mechanisms to support the elderly in their old age. Elderly retired homeowners can take advantage of such residential mortgage financing extended to them by financial institutions to raise the level of their retirement income. This kind of housing mortgage mechanism is referred to as a "Reverse Mortgage". Differing from housing mortgages in general in which case borrowers must repay their debt in installments, reverse mortgages enable elderly homeowners to use their residential houses as collateral in exchange for the entire loan amount or a regular monthly amount. By not having to repay a fixed amount of principal or interest on a regular basis, these elderly homeowners can freely choose the method they will use to repay the loan in accordance with their personal financial situation, in order to guarantee their economic security during retirement. For those elderly that lack a retirement income, but who are "house rich and cash poor," the reverse mortgage can indeed lighten their economic burden after they retire. Because the ratio of home ownership among Taiwan's population exceeds 80%, reverse mortgages are a very suitable means of ameliorating the financial problems that the elderly population face after they retire. It is for this reason that, starting in 2013, Taiwan is launching reverse mortgage products, by first incorporating this product within its social welfare policies and initially targeting mostly single elderly over the age of 65 with no heir. The government will guarantee the payment of the annuity for a period of up to 30 years, in order to safeguard the standard of living of the elderly population after retirement. The results of empirical research conducted in other countries indicate that reverse mortgages have a significant enhancement effect on the retirement income of elderly homeowners.

Reverse mortgages have already been developed over a period of more than 20 years in the U.S., and Venti and Wise (1991) were the first to propose that a reverse mortgage could effectively improve the financial condition of single elderly after retirement. Mayer and Simons (1994) used the results of a 1990 survey to estimate that the United States had more than 6 million elderly homeowners who could increase their monthly income by at least 20% by means of a reverse mortgage, and that these mortgages could help more than 1.4 million borrowers to maintain their income level above the poverty line. Merrill, Finkek, and Kutty (1994) also pointed out that it was American homeowners over the age of 70 who reaped the greatest benefits through a reverse mortgage. However, although there were expectations that a significant number of elderly homeowners could benefit from reverse mortgages, prior to the year 2000 the size of the market in the United States was still limited. Caplin (2002) pointed out that there were many reasons why the reverse mortgage market in the U.S. had stagnated, of which the two most important factors likely to hinder the development of the market were the bequest motive and the elderly person moving out of the housing against which the loan had been made. Moreover, as Davidoff, Brown and Diamond (2005) and Brown (2007) pointed out, because reverse mortgages and annuity products are both highly complex financial instruments, the market development of these products has been far less than would in theory have been expected.

By using the Home Equity Conversion Mortgage (HECM) pricing structure as the basis, Ong (2008) calculated the increase in the retirement income levels of elderly homeowners in Australia resulting from reverse mortgages, and his findings indicated that the reverse mortgages on average resulted in a 71.3% increase in the net income of the borrowers. In particular, it was the group of single women over the age of 80 with high housing equity for whom the effect was all the more significant. In addition, Ong (2008) also pointed out that nearly 95% of poor homeowners were able by converting their home equity into retirement income to escape a life of poverty. Mitchell and Piggott (2004) explored the feasibility of developing reverse mortgages in Japan, and their findings indicated that, regardless of whether the loan was taken out as one lump sum or in the form of a stream of payments over the life of the borrower, reverse mortgages could always effectively raise the elderly homeowners' retirement income. If one considers elderly female homeowners as an example, the amount of the loan taken out in each year could reach up to 60% of the income replacement level.

Benedict (2009) pointed out that reverse mortgages could solve the problems of insufficient funds to pay for medical expenses and for care that the elderly faced after retirement. Davidoff (2009) explained that annuitized insurance products, long-term care insurance and reverse mortgage were the principal instruments that the elderly used to resolve their income needs after they retired. Neil and Neil (2009) argued that older people were generally inclined to continue living in the houses that they owned and hence reverse mortgages could indeed improve their income situation after retirement. Costa-Font, Gil, and Mascarilla (2010) in a study on 729 Spanish people aged 55 and above found that current income, the level of education, gender and age each affected the demand for reverse mortgages. Shan (2011) found that, due to the sharp rise in house prices from 2003 to 2007, the reverse mortgage market as a whole grew by almost 30 percent.

In this study, we use the operating mechanism of the U.S. Home Equity Conversion Mortgage (HECM) as a basis for calculating the ratio of the amount that the borrower can borrow to the amount of the housing mortgage, or the so-called principal limit factor (PLF). In addition, we also employ the reverse mortgage insurance pricing model proposed by Szymanoski (1994), while at the same time using Taiwan's population mortality rate, housing returns, Taiwan's ten-year government bond rate, and other parameters to calculate the proportion that Taiwan's elderly homeowners can borrow on one occasion. We then use data from the Survey of Family Income and Expenditure Taiwan to further analyze the enhancement effect that reverse mortgage products will have on the retirement income replacement rate for different age cohorts of households.

The results of the simulation conducted in this study indicate that, based on maintaining the financial soundness of reverse mortgage insurance, and by assuming a lending rate of 4.72%, if one takes a 65-year-old male borrower as an example, the principal limit factor sought is 0.542, and the reverse mortgage can lead to a 55.60% increase in the borrower's income replacement ratio after retirement. The study goes further to estimate that, in the case of single or couple-only (without children) households for three different age cohorts, the enhancement effect from taking advantage of reverse mortgages on the income replacement rate averages around 70% to 130%. In regard to the different age cohorts, we find that, due to the fluctuations in housing prices, the enhancement effect of reverse mortgages on the retirement income of the younger generation of retired homeowners is greater than that in the case of the older generation of retired homeowners.

The remainder of this paper is organized as follows. In the next section, we describe the reverse mortgage. In the third section, we present the methodology. The results of the simulations and the empirical analyses are discussed in the fourth and fifth section, and in the final section we conclude.

2. Reverse Mortgages

Since the 1970s, the United Kingdom, the United States, and many other countries have been developing a housing mortgage mechanism, known as the "Reverse Mortgage" or "Housing Endowment," which enables elderly homeowners to consume some of their home equity but still maintain their ownership of and residence in the home. In a typical reverse mortgage arrangement (see Chen et al.[1]), the lender advances a lump sum or periodic payments to elderly homeowners. The loan accrues with interest and is settled using the sale proceeds of the property when the borrowers die, or sell or vacate their homes to live elsewhere. There are various styles of loan services provided by financial institutions, including:

(a) Lump-sum payments: the borrower receives a fixed amount of the entire available principal at the closing of the loan;

(b) Tenure payments: equal monthly payments are made as long as the borrower lives;

(c) Term payments: equal monthly payments are made for a fixed period of months selected by the borrower;

(d) Line of credit: installments are paid to the borrower at times and in amounts of the borrower's choosing until the line of credit is exhausted.

On the other hand, despite substantial government subsidies and protection, many lenders have been unable to generate enough profit to justify maintaining the costs or risks associated with this specialized product and have exited the market. In addition to low origination fees and the uncertainties arising due to regulatory and legal problems, the lenders are also exposed to the following risks: (a) interest rate risk (see Boehm and Ehrhardt[2]); (b) longevity risk (see Chen et al.[3]); (c) housing price risk (see Mitchell and Piggott[4]); and (d) borrower maintenance risk (see Ong[5]). The overall risk measure can be referred to as the crossover

[1] Chen, Cox, and Wang (2010).
[2] Boehm and Ehrhardt (1994).
[3] Chen et al. (2010).
[4] Mitchell and Piggott (2004).
[5] Ong (2008).

risk (see Chinloy and Megbolugbe[6] and Wang et al.[7]) and arises because the outstanding loan balance may not be repaid in full when the loan is terminated because the loan balance may be larger than the property's value and the lender will recover only up to the sale price of the property. The crossover risk is usually insured via mortgage insurance premiums, and the fair premium is determined by the present value of the non-recourse provision (see Chen et al.[8])

3. Methodology

Because the lending institutions that engage in the business of extending reverse mortgages are subject to volatility in interest rates and in the value of the house used as collateral as well as the longevity risk associated with the borrower, the lender will require a certain percentage of the risk premium to cover possible loan losses that might arise. In view of the reverse mortgages extending for periods that may exceed twenty or thirty years, there is a high degree of uncertainty inherent in the fluctuations in interest rates and in the value of the house. If the lender expects the future value of the house that is mortgaged to be lower than the outstanding balance on the loan, it will choose to give up this loan business. To enable elderly homeowners to increase their retirement living income by means of reverse mortgages, the U.S. Federal Housing Administration has developed loan insurance mechanisms to eliminate the critical risk faced by lending institutions and thereby increase their willingness to issue such mortgages to ensure that the borrower can in a timely manner receive the loan installments. Szymanoski (1994) examines the risks faced by the issuers of reverse mortgages, and refers to a HECM pricing model that is used to calculate the principal limit factor. The model takes into consideration the largest loan-to-value ratio underlying the longevity risk of the borrower, the risk inherent in the fluctuations in the prices of the mortgaged houses and the interest rate risk, in order to eliminate the collateral risk.

In this paper, we refer to the HECM pricing model proposed by Szymanoski (1994) as our basic framework in order to calculate the proportion of the value of the house that elderly homeowners in Taiwan can borrow in the form of a lump sum payment by means of a reverse mortgage with loan insurance. Under the condition that a balanced budget be maintained for the loan insurance fund, we solve the principal limit factor corresponding to an age-specific category of borrowers for a certain interest rate level. In what follows we introduce the pricing model framework and the related parameter settings.

3.1 Pricing Model

Based on the assumption of risk neutrality, with our primary objective being to maintain a balanced budget on the Loan Insurance Fund, we refer to Szymanoski (1994)'s HECM pricing model in order to solve for the highest loan-to-value ratio that the lending institution can apply, or the so-called principal limit factor (PLF). This indicates, based on this proportion

[6]Chinloy and Megbolugbe (1994).
[7]Wang, Valdez, and Piggot (2007).
[8]Chen et al. (2010).

of loans, that the sum of the net present values of the expected future loan losses will be less than or equal to the expected sum of the net present values of the future loan insurance premiums, so that the Loan Insurance Fund will in the future remain solvent, as shown in Equation (1) below:

$$\sum_{t=0}^{\infty} E[L(t)](1+i)^{-t} \leq \sum_{t=0}^{\infty} E[P(t)](1+i)^{-t} \tag{1}$$

where $E[.]$ is the expected value, $L(t)$ is the actual loss in period t, i is the discount rate, and $P(t)$ is the premiums charged in period t. In what follows, we define what is meant by the expected loan insurance premiums and expected loan losses in Equation (1).

The borrower should bear the cost of the premiums in each period including the initial loan premium and each annual premium. In general, the initial loan premium is 2% of the price of the house in the initial period, while the annual premium is 0.5% of the loan outstanding in the then current period. The loan outstanding in period 0 is the maximum claim amount multiplied by the principal limit factor. However, beginning from period 1, the loan outstanding in each period consists of the loan outstanding in the previous period plus interest and the loan insurance premium in the current period. When calculating the expected loan insurance premium in period t, it is necessary to use the premium received in period t ($P(t)$) as well as the probability that loan contracts will still not have been terminated in period t. In this study, we assume that within the period of the loan, elderly homeowners will not move out of their houses that have been mortgaged, and also that they will not take the initiative to terminate the contract. In other words, only when the borrower dies will the loan contract finally be terminated. Therefore, in period t, the probability of the loan contract not having been terminated will be the same as the probability of the borrower being alive in that period. For this reason, on the right-hand side of Equation (1), the expected loan insurance premium in period t ($P(t)$) can be expressed as follows:

$$E[P(t)] =_t p_x \cdot P(t) \tag{2}$$

where $_tp_x$ is the probability that the borrower, who applies for a loan at age x, will still be alive t years later.

After obtaining the expected loan insurance premiums in each period, we will now explain the left-hand side of Equation (1), and the process for solving the expected loan loss. Based on our assumptions, when the borrower dies, the reverse mortgage contract will be terminated, and so this is the point in time when the reverse mortgage loan loss is realized. At the time when the borrower dies, the accumulated loan outstanding is higher than the price of the housing mortgage at that time. So the contract termination rate is the probability ($_td_x$) of the borrower who applied for the loan at age x dying within the period between t and $(t+1)$, which can be expressed as $_td_x =_t p_x -_{t+1} p_x$. Another condition for the loan loss to occur is that the price of the housing mortgage at the time the contract is terminated must be less than the accumulated loan outstanding during the period of the loan. The probability that the price of the housing mortgage is less than the loan outstanding can be deduced from the geometric Brownian motion. The geometric Brownian motion model assumes that the single-period housing return is normally distributed with expected value μ, and variance σ^2, where μ and σ are both constants. For this reason, by taking the single-period expected housing return μ, and variance σ^2 and the time factor and multiplying each of them together (linearly), we can find that the housing return t periods later is normally distributed with expected value μt, and variance $\sigma^2 t$. Therefore, the probability $(A(t))$ that the loan outstanding will be greater than the price of the housing mortgage can be expressed as: $A(t) = [1/\sigma\sqrt{t}][1/\sqrt{2\pi}] \int_{-\infty}^{\ln[b(t)]} e^{-0.5[(y-\mu t)/\sigma\sqrt{t}]^2} dy$, where $b(t) = BAL(t)/H_0$. $BAL(t)$ is the outstanding balance at time t and H_0 is the initial house value. By multiplying the contract termination rate by the probability that the loans outstanding will be greater than the price of the housing mortgage, we can obtain the probability of a loan loss occurring $({}_t d_x \cdot A(t))$.

Next, we can further solve for the extent of the loan loss, that is, we can proceed to estimate the amount of the expected loan loss. When the loan contract is terminated, that the price of the house that is mortgaged is lower than the loan outstanding is an indicator of the extent of the loan loss. The calculation is based on the current loan outstanding less the expected price of the mortgaged house, that is, the magnitude of the loan loss. However, it is expected that the value of the mortgaged house is calculated based on the conditional expected value that this asset's value is less than that of the loan outstanding.Under the assumption of a lognormal distribution, unconditional expected values are generally expressed as $E[H(t)] = H_0 e^{\mu t + 0.5\sigma^2 t}$, where $E[H(t)]$ is the expected mortgaged housing price in period t, based on there being no conditional limitations. In order to solve for the conditional expected value of the mortgaged housing price, Szymanoski (1994) uses the factor β to indicate how the generally unconditional expected mortgaged housing price can be converted to the conditional expected value of the housing price[9]. If $E[H(t)]$ is multiplied by the factor β, we can obtain the situation, when a loan loss occurs, where the expected mortgaged housing price is lower than the loan outstanding in period t. To sum up, the product of the probability of a loan loss occurring and the magnitude of the loan loss is as shown on the left-hand side of Equation (1). The expected loan loss in period t is as shown in Equation (3):

$$E[L(t)] =_t d_x \cdot A(t) \cdot \{BAL(t) - \beta E[H(t)]\} \tag{3}$$

From the above, we can then calculate, based on a certain loan interest rate, the loan-to-value ratio that can be used to determine how much a borrower aged x can borrow. In addition, based on this level of borrowing, the Loan Insurance Fund budget can be balanced, in order to ensure its financial soundness.

[9] $\beta = [1/A(t)][1/\sqrt{2\pi}] \int_{-\infty}^{U(t)} e^{-0.5(y-\sigma\sqrt{t})^2} dy$, where $U(t) = \{\ln[b(t)] - \mu t\}/\sigma\sqrt{t}$. For the derivation of the related formula, please refer to Ross (1996) and Szymanoski (1990).

3.2 Parameter setting

From the HECM pricing model proposed by Szymanoski (1994) we can learn that the housing returns rate, contract termination rate (that is, the mortality rate), lending rate, and other parameters affect the equilibrium relationship between the profit and loss on the Loan Insurance Fund. These parameters also represent the sources of critical risk that the lenders are facing. In this study, we use the geometric Brownian motion to simulate the changes in the prices of mortgaged houses. For the borrower's mortality rate we refer to the Taiwan annuity life table, and for the risk-free interest rate we use the ten-year interest rate on government bonds. The main sources of information and the explanation of the parameter setting are as follows:

(1) Housing prices simulation

This study uses a geometric Brownian motion to simulate the prices of mortgaged houses, with one of the main features of the geometric Brownian motion being the assumption that future changes in asset prices are independent of past price movements. In addition, it is assumed that the asset prices are not negative, and that they are log-normally distributed. Therefore, it is assumed that the movements in the asset prices of mortgaged houses satisfy the following random equation: $dH/H = \mu dt + \sigma dz$, $dz = \varepsilon\sqrt{dt}$, where dH is the instantaneous change in housing prices; μ is the expected housing returns within a unit period of time (dt); σ is the standard deviation of the housing returns within a unit period of time (dt); z is a random variable that follows a standard Brownian motion, and dz is its instantaneous change where $\varepsilon \sim N(0, 1)$.

Because Taiwan does not have an officially prepared housing price index database, we use real estate price indices for Taiwan from the first quarter of 1971 to the fourth quarter of 2008 as our sample data[10], in order to calculate the mean rate of return and standard deviation for Taiwan's real estate prices and thereby simulate the extent of the movements in housing prices[11]. We observed the trend in the real estate price index in Taiwan from 1971 to 2008. The data show that housing prices in Taiwan started to rise substantially in 1987, but that they began to decline in 1993, only resuming their upward trend again in 2004. In this study, we use Taiwan's housing price index to calculate a mean housing return (μ) of 0.004611 and a standard deviation (σ) of 0.003665.

(2) Contract termination rate

According to the HECM loan contract provisions, when either the borrower dies, more

[10] Data from 1971 Q1 to 1999 Q4 are based on a real estate pricing survey commissioned by the Construction and Planning Agency, Ministry of the Interior in 1999 and conducted by National Chengchi University, while data from 1999 Q2 to 2008 Q4 are based on Cathay Real Estate Price Indices announced every quarter by the Cathay Real Estate Development Company and National Chengchi University's Taiwan Real Estate Research Center. Because the base periods on which the two sets of data are calculated are different, we first adjust the data so that the first quarter of 1999 is the base period for the calculation (= 100), and after that perform the calculation.

[11] In his fixed pricing model, Szymanoski (1994) in determining the expected value (μ) and standard deviation (σ) of housing returns in both cases adopts U.S. housing returns assumptions, where it is assumed that the expected rate of return is 0.04 and the standard deviation is 0.1.

than 12 months passes after he has moved out of the mortgaged residential property, or the house is auctioned off to repay debts, the contract is deemed to have terminated. In this study we assume that it is only when the borrower dies that the contractual relationship between both lender and borrower is considered to have been terminated. We do not consider the cases where elderly homeowners directly auction off their houses, or relocate to a nursing home for medical reasons. In other words, we assume that the probability that the borrower will die is the same as the probability that the loan contract will be terminated, and that the probability of the borrower dying is obtained from the annuity life table[12]. It is also assumed that people live up to a maximum age of 110 years old.

(3) Lending rates

Boehm and Ehrhardt (1994)'s empirical results show that reverse mortgage products carry higher interest rate risk than housing mortgages and interest-bearing bonds in general. In terms of their interest rate flexibility, reverse mortgage loans have relatively high interest rate sensitivity compared to other types of fixed income instruments, and their prices are vulnerable to movements in interest rates and can be volatile. Therefore, interest rate fluctuations will affect the soundness of the Loan Insurance Fund. The pricing model adopted in this study uses the ten-year interest rate on government bonds plus a lender's risk margin to calculate the principal limit factor. The data are sourced from the Taiwan Economic Journal Data Bank.

4. The Enhancement Effect of Reverse Mortgages on the Retirement Income of Homeowners in Taiwan

By means of the solution process referred to in the previous section, we can calculate the principal limit factor corresponding to a borrower of a specific age for a certain loan interest rate. By multiplying the principal limit factor by the maximum claim amount, and subtracting the loan insurance premium, closing costs, and other loan costs, we obtain the borrower's lump sum payment (LS). In addition, we further annuitize this lump sum payment to determine the tenure payment (TP) which will be paid over the remainder of the borrower's lifetime. From this we can calculate the income replacement level for the elderly homeowner that can be enhanced by the reverse mortgage.

In this study, we take a retired 65-year-old single male homeowner as an example and assume that the mean (μ) of the return on housing prices in Taiwan prices is 0.004611 and that the standard deviation (σ) is 0.003665, that the price of the mortgaged house in the initial period is NT$8 million ($H_0 = 8,000,000$), and that the lending rate is 4.72%[13]. The contract termination rate is as described in the annuity life tables and the monthly salary before retirement is assumed to be $42,018[14]. From these figures we can calculate the amount

[12] In this study, the borrower's mortality rate is based on the 1997 Taiwan Annuity Life Table announced by Taiwan's Ministry of Finance.

[13] Assuming that the lender's risk return is 2% and that the ten-year interest rate on government bonds is 2.72% (as of June 2008), the loan interest rate is assumed to be the sum of the two, i.e., 4.72%.

[14] According to the results of the survey on employees' salaries conducted by the Directorate-General of Budget, Employment and Statistics in September 2008, the average salary of employees in the industry

of the reverse mortgage that can be applied for.

Based on the above parameter conditions and according to the results of our simulation analysis, to maintain the financial soundness of the reverse mortgage insurance, the principal limit factor that we have calculated is 0.542, meaning that for a 65-year-old male borrower who applies for a loan when the lending rate is 4.72%, the amount that a lending institution can lend against a house is 54.2% multiplied by the price of that property. In order to reflect the characteristics of excessively costly reverse mortgage loans, we deduct the costs associated with the loan from the amount that can be lent, which include the loan insurance premium in the initial period (assumed to be 2% of the price of the house in the initial period), start-up costs (also assumed to be 2% of the price of the house in the initial period) and the closing costs (assumed to be NT$30,000), and then calculate the borrower's lump-sum amount. We then annuitize the lump-sum amount by using the life annuity payment method (as in Equation (4)) so that we can find out the amount that the elderly homeowner will obtain as a result of the reverse mortgage over the course of his life. The term r in Equation (4) is the risk-free interest rate, and ω is the highest age to which people are assumed to live. Finally, we use the income replacement rate to measure the extent to which the reverse mortgage enhances the retirement income level. The numerical simulation results are as shown in Table 1.

$$TS = LS / \sum_{t=0}^{\omega-x-1} {}_tp_x \frac{1}{(1+r)^t} \tag{4}$$

The results presented in Table 1 show that for a 65-year-old male borrower, after deducting related transaction costs and loan insurance expenses in the initial period, the lump-sum payment is NT$2,980,000, and the tenure payment to be made each year is NT$280,367. Finally, by dividing the tenure payment by the annual salary before retirement, we can calculate the ability of the reverse mortgage to enhance the borrower's income replacement rate by 55.60%. Thus the income enhancement effect can be expected to resolve the elderly homeowner's problem of insufficient funds for retirement caused by a lack of cash and other liquid assets to meet the cost of living expenses. The financial gap faced by the retiree can thus be filled and his economic security safeguarded in old age.

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and service sectors was NT$42,018 per person. We therefore use this salary level in our study to represent the borrower's monthly salary before retirement.

Table 1. Calculation of loan amount applicable to a 65-year-old borrower

Lending rate	4.72%
Principal limit factor	0.542
Initial house price	NT$8,000,000
Initial loan insurance premium	(160,000)
Origination fee	(160,000)
Closing costs	(30,000)
Lump sum	$3,986,000
Tenure payment	$322,726
Annual salary before retirement	$504,216
Income replacement rate	64.01%

Source of data: Results of this study's simulation. The 2006 exchange rate is US$1= NT$32.53.

In addition, the amount of housing price risk faced by the lending institutions depends on the expected value or mean and the standard deviation of the mortgaged housing price's rate of return. If the house cannot sufficiently maintain its value and its price is subject to large fluctuations, upon the maturity of the loan contract the value of the mortgaged assets will be directly affected. In this study, we therefore use different housing price rates of return and standard deviations to simulate and analyze the impact of these fluctuations on the principal limit factor and income replacement rate. The numerical simulation results are as shown in Table 2:

Table 2 Principal limit factors and income replacement rates corresponding to different housing price rates of return and standard deviations

	$\sigma = 0.003665$			
μ	0.01	0.02	0.03	0.04
Principal limit factor	0.583	0.682	0.706	0.747
Income replacement rate	69.27%	81.99%	85.07%	90.34%
	$\mu = 0.004611$			
σ	0.01	0.02	0.03	0.04
Principal limit factor	0.504	0.482	0.405	0.385
Income replacement rate	59.12%	56.30%	46.41%	43.84%

Source of data: Results of this study's simulation

From Table 2 above we can see that, by holding other parameters fixed and assuming that $\sigma = 0.003665$, the higher that the average housing price rate of return (μ) becomes, the principal limit factor obtained also becomes larger. From this we know that the higher that the average housing price rate of return becomes, the easier it will be for the expected future value of the mortgaged house to settle the accumulated amount of the loan outstanding upon the expiration of the contract, and so the loan-to-value ratio will be higher, which will correspond to a higher income replacement rate. This concurs with the conclusion reached by Shan (2011) that rising housing prices contribute to the development of the reverse mortgage market. When the other parameters are held fixed and it is assumed that $\mu = 0.004611$, when the standard deviation of the housing rate of return becomes larger, the principal limiting factor obtained will become lower and the income replacement rate will also decline. These results indicate that when housing prices are subject to a greater degree of fluctuation, as far as the lending institutions are concerned, the likelihood of claims will increase and their premiums receivable will also go up. Therefore, when the uncertainty associated with the rates of return on mortgaged property prices increases, the principal limiting factor will be reduced and the income replacement rate will consequently decline.

5. Empirical Analyses of Different Age Cohorts

Based on the premise that no bequest motive is considered in relation to the housing, in this study we turn our attention to three different cohorts of households, consisting of single persons or a husband and wife only born from 1921 to 1925 (the twenties), those born from 1931 to 1935 (the thirties), and those born from 1941 to 1945 (the forties). We use their income before retirement and income after retirement to calculate their income replacement rate, and then employ reverse mortgages estimate the enhancement effect on this income replacement rate.

This paper uses data spanning the period from 1985 to 2010 which allows us to estimate household income both before and after retirement. The empirical data are sourced from the Survey of Family Income and Expenditure (SFIE) compiled by the Directorate-General of Budget, Accounting and Statistics (DGBAS) of the Executive Yuan in Taiwan. The survey annually investigates individuals and families who have lived in Taiwan since 1970. Its units of measurement are the household and the individual, and the sample accounts for around 0.2% of all Taiwanese households, or approximately 14,000 households and 60,000 individuals in each year.

Because the data contained in the Survey of Family Income and Expenditure are based on random sampling, they are not truly panel data. Therefore, in order to be able to more accurately compare the changes in the income of the elderly before and after retirement, we refer to the approach adopted by Banks ,Blundell, and Tanner (1998) which is to construct a pseudo-panel dataset. To this end we first take the individual cross-sectional data for each year and from those data select households that are made up of either single individuals or husband and wife couples. We then, based on the household head's "year of birth," separate the data into three different age groups, that is, households whose heads were born as part of the same age group are deemed to belong to the same age group. From the data for 1985-2010 included in this study, for every five years we select a representative year, and a total of six such years are selected, namely, 1985, 1990, 1995, 2000, 2005 and 2010. From each of those years, we then extract those households whose heads were born between 1921 and 1925 (the twenties), between 1931 and 1935 (the thirties), and between 1941 and 1945 (the forties), and by averaging the relevant variables for these three specific age groups track the changes in the average values of these variables for these age groups over the years. We then further focus attention on and compare the average income of household heads between the ages of 60 and 64 with that for household heads between the ages of 65 and 69 by calculating the income replacement rates for different cohorts of households. The sampling years and the sample sizes are as shown in Table 3.

Table 3. Sampling years and sample sizes

Year / Year of Birth	1985	1990	1995	2000	2005	2010
1921-1925	60-64 (719)	65-69 (575)	70-74	75-79	80-84	85-89
1931-1935	50-54	55-59	60-64 (647)	65-69 (472)	70-74	75-79
1941-1945	40-44	45-49	50-54	55-59	60-64 (424)	65-69 (565)

Note: The figures not in parentheses in the table refer to the age range of the cohort in the current year. The figures in parentheses denote the size of the sample for the respective cohort.

From Table 3 we can see that the first age group is made up of households whose heads were born between 1921 and 1925. In 1985, they were aged between 60 and 64, and there are 719 of them in our sample. It is these who in our study are deemed to be close to the age of retirement. In 1990, they were between 65 and 69 years old, and there are 575 of them in the sample. They are then considered to be part of an age group that has already retired. The second age group is made up of households whose heads were born between 1931 and 1935, and in 1995 they were aged between 60 and 64. The number of them in our sample is 647, as they are regarded in this study as the age group approaching the age of retirement. In 2000, they were between the ages of 65 and 69, and there are a total of 472 of them in our sample. In this study they are viewed as belonging to the group that has already retired. The third age group is made up of households whose heads were born between 1941 and 1945, and in 2005 they were aged between 60 and 64. There are a total of 424 in our sample, and they are seen as the age group approaching the age of retirement. In 2010, they were between the ages of 65 and 69, and the number of them in the sample is 565. They are regarded as part of the age group that has already retired. By comparing the income of those in the same age group both before and after they retire, we can more accurately calculate the income replacement rate after those belonging to that age group retire.

We now explain how we estimate the values of the houses, as well as how we calculate the yearly tenure payment that the homeowners can obtain by means of a reverse mortgage. In this study, we make use of the DGBAS's Survey of Family Income and Expenditure data for Taiwan, and by means of the "Imputed Net Rent from Owner-occupied Dwellings" item included in the survey data estimate the value to a household of owning a home. Past studies[15] have pointed out that the estimated value of a residential property can be obtained by multiplying the monthly rent by a rent multiplier[16]. Gallin (2004) suggested that rentals and real estate prices should maintain a long-run equilibrium relationship. Hui and Zheng

[15]Grabler et al. (1956), Shelton (1968) and Muth (1969), Hendershott and Shilling (1982), and Meese and Wallace (1994).

[16]Rent as a proportion of the housing price is referred to as the rent multiplier.

(2012) found that, relative to other real estate markets, housing prices and rents in Hong Kong's owner-occupied residential property market were the most highly correlated. Lin (1993) estimated the average monthly rent multiplier in Taiwan to be somewhere between 200 and 350. However, the monthly rent multiplier in Taiwan appears to be on the high side when compared with monthly rent multipliers in general in the U.S.[17]. Lin (1993) and Tseng, Chang, and Hua (2005) believed that the main reason for the seemingly high monthly rent multiplier in Taiwan was that in Taiwan homeowners generally have higher expectations regarding the future capital gains on the property, and so they are willing to provide residential services at relatively low rents, thereby resulting in a high monthly rent multiplier. These authors also indicated that house prices were from the households' point of view able to meet their consumption and investment demand, and that the rents could only satisfy the demand for residential consumption.

In this study, in considering the bank's start-up costs and closing fees, we assume the monthly rent multiplier to be 150,200 and 250 based on three different scenarios, and by regarding the amounts for the item "Imputed Net Rent from Owner-occupied Dwellings" for owner-occupier households included in the Survey of Family Income and Expenditure as the imputed monthly rents of those households, we then estimate the value of the dwelling as being equal to the property's imputed rent multiplied by the monthly rent multiplier. We then refer to the principal limit factor derived from our simulation in the previous section, and by assuming the homeowner's lump-sum payment to be 50%, 60% and 70% of the value of the house we once again refer to the Taiwan annuity life table and use Equation (4) to determine the annual tenure payment that the elderly homeowner can obtain from the reverse mortgage[18].

In this study, we use the income both before and after retirement for three different cohorts of households made up of either single individuals or husband-and-wife-only couples, and respectively calculate the income replacement rate after retirement for each cohort. Our results are as shown in Table 4. From this table we can see that, for households consisting of single individuals or husband-and-wife-only couples for cohorts born in the 1920s, the pre-retirement income is about NT$320,000, whereas the income after retirement is about NT$340,000, and the income replacement rate after retirement is 103.46%. For those households living in owner-occupied housing the average pre-retirement income is about NT$345,000, and after retirement their average income is also close to NT$345,000, with their post-retirement income replacement rate being 100.12%. For those households not living in owner-occupied housing, their average income before retirement is about NT$280,000, and after retirement about NT$320,000, indicating that their income replacement rate after retirement is 114.59%.

[17] According to the experience of the U.S. residential property market, for detached houses, the house price is 100 times the monthly rent, which has been called the 1%-rule. For apartment-style housing, the monthly rent multiplier is about 70.

[18] It is assumed that r is the lending rate ($r = 4.72\%$, which includes the risk-free rate of 2%); ω is the maximum lifespan (assumed to be 110), and ${}_tp_x$ is the probability that a person aged x will live another t years.

For households made up of either single individuals or husband-and-wife-only couples where the household head was born in the 1930s, the pre-retirement income increases significantly to NT$730,000, but their income is about NT$450,000 after retirement, reflecting an income replacement rate after retirement of 61.52%. As to those households living in owner-occupied housing the average pre-retirement income is about NT$750,000, and close to NT$430,000 after retirement, indicating an income replacement rate after retirement of 57.35%. For those households not living in owner-occupied housing, their average income prior to retirement is about NT$590,000, and after retirement about NT$570,000, giving them an income replacement rate after retirement of 97.07%.

For households made up of either single individuals or husband-and-wife-only couples where the household head was born in the 1940s, the pre-retirement income is about NT$640,000, but their income after retirement is around NT$490,000, so that their income replacement rate after retirement is 61.52%. As to those households living in owner-occupied housing the average pre-retirement income is about NT$650,000, and close to NT$510,000 after retirement, reflecting an income replacement rate after retirement of 78.69%. For those households not living in owner-occupied housing, their average income prior to retirement is about NT$550,000, and after retirement about NT$410,000, giving them an income replacement rate after retirement of 74.03%.

Table 4 Calculation of income replacement rate for different cohorts (for all, owner-occupier and non-owner-occupier households)

Unit: NT dollars

	Pre-retirement income	Post-retirement income	Income replacement rate
Born 1921-1925			
Full sample	329,496	340,990	103.49%
Owner-occupier	345,270	345,678	100.12%
Non-owner-occupier	281,554	322,636	114.59%
Born 1931-1935			
Full sample	737,103	453,497	61.52%
Owner-occupier	757,547	434,440	57.35%
Non-owner-occupier	587,965	570,723	97.07%
Born 1941-1945			
Full sample	637,104	494,734	77.65%
Owner-occupier	646,424	508,720	78.69%
Non-owner-occupier	552,039	408,697	74.03%

Note: The 2006 exchange rate is US$1=NT$32.53.

From the above explanation we can see that for people born in the 1920s, the income replacement rate is 114.59%, while it is 97.07% and 74.03% for those born in the 1930s and 1940s, respectively. These results indicate that the income after retirement for those households not living in owner-occupied housing is lower than that before retirement, and that the income replacement rate is exhibiting a gradual downward trend. However, for those living in owner-occupied housing, the income replacement rates for the different age cohorts reveal relatively large fluctuations, as evidenced by an income replacement rate of 100.12% for those people born in the 1920s, which then fell sharply to 57.35% for those born in the 1930s, before rising to 74.03% for those born in the 1940s. A possible reason for this is that people born in the 1930s were faced by a period of time before they retired in which the Taiwan stock market rose sharply. It is possible that during this time they mortgaged their house in exchange for additional cash from the bank which they invested in the stock market, resulting in a substantial increase in investment income. This caused their income before retirement to be relatively high. However, after they retired they were confronted with the Asian financial crisis, and their investment income was dramatically reduced, causing their total income after retirement to be relatively low, and resulting in a relatively low income replacement rate after retirement.

Finally, for those retired homeowners with their own homes, by using the annual tenure payments obtained from the "Reverse Mortgages" we estimate the extent to which the income replacement rate is enhanced after retirement, and the results of our calculations are as shown in Table 5.

Table 5 Comparisons of converting owner-occupied residences into reverse mortgages (1) Estimated house price (2) Monthly loanable amount (3) Percentage of income replacement rate enhanced

	Born 1921-1925–Retired 1990			Born 1931-1935–Retired 2000			Born 1941-1945–Retired 2010		
Imputed Rent Multiplier	150	200	250	150	200	250	150	200	250
Estimated House Price (NT$)	3,647,282	4,863,043	6,078,803	8,675,341	11,600,000	14,500,000	10,600,000	14,100,000	17,600,000
Loan-to-Value Ratio	Monthly Loanable Amount (NT$)								
50%	12,028	16,037	20,046	27,925	37,234	46,542	33,766	45,021	56,276
60%	14,433	19,244	24,056	33,511	44,681	55,851	40,519	54,025	67,531
70%	16,839	22,452	28,065	39,096	52,127	65,159	47,272	63,029	78,786
	Enhancement Effect on Income Replacement Rate (%)								
50%	41.80%	55.74%	69.67%	44.24%	58.98%	73.73%	62.68%	83.57%	104.47%
60%	50.16%	66.88%	83.61%	53.08%	70.78%	88.47%	75.22%	100.29%	125.36%
70%	58.52%	78.03%	97.54%	61.93%	82.57%	103.22%	87.75%	117.00%	146.26%

Note: (1) In estimating house prices, imputed rents of 150 times, 200 times and 250 times for owner-occupied homes and other buildings are assumed. (2) Loan-to-Value Ratio: the amounts that may be borrowed against the house are assumed to be 50%, 60% and 70amount: it is assumed that the lending rate is 4.72% (which includes a risk-free rate of 2%), and that the estimation is based on a 65-year-old male and a 65-year-old female as per the Taiwan annuity life table. (4) The income replacement rate is based on a comparison between the annual tenure payment that can be obtained as a result of the reverse mortgage and the income of a particular cohort before retirement. (5) The 2006 exchange rate is US$1=NT$32.53.

Table 5 lists estimates of the average housing price level for retired homeowners who convert their owner-occupied residences using a "reverse mortgage," the monthly amount that the homeowners can obtain as well as the magnitude of the increase in the income replacement rate. The results show that for those homeowners born between 1921 and 1925 who convert their homes by means of a reverse mortgage, based on the assumption of a rent multiplier of 150 and that between 50% and 70% of the estimated value of the house can be borrowed, the average amount of cash that can be obtained each month ranges from about NT$12,000 to NT$16,000. In terms of the income replacement rate after retirement, this can be increased by between 41% and 58%. By assuming that the rent multiplier is 200, the average amount of cash that can be obtained each month ranges from about NT$16,000 to NT$22,000. As for the income replacement rate after retirement, this can be increased by between 55% and 78%. If the rent multiplier is assumed to be 250, the average amount of cash that can be obtained each month ranges from about NT$20,000 to NT$28,000, and the income replacement rate after retirement can be enhanced by between 69% and 97%.

For those people born between 1931 and 1935, by assuming a rent multiplier of 150 and that between 50% and 70% of the estimated value of the house can be borrowed, the average amount of cash that can be obtained each month ranges from about NT$28,000 to NT$39,000. As for the income replacement rate after retirement, this can be increased by between 44% and 61%. By assuming that the rent multiplier is 200, the average amount of cash that can be obtained each month ranges from about NT$37,000 to NT$52,000. As for the income replacement rate after retirement, this can be increased by between 59% and 83%. If the rent multiplier is assumed to be 250, the average amount of cash that can be obtained each month ranges from about NT$47,000 to NT$65,000, and the income replacement rate after retirement can be enhanced by between 74% and 103%.

As for those born between 1941 and 1945, by assuming a rent multiplier of 150 and that between 50% and 70% of the estimated value of the house can be borrowed, the average amount of cash that can be obtained each month ranges from about NT$34,000 to NT$47,000. In regard to the income replacement rate after retirement, this can be increased by between 63% and 87%. By assuming that the rent multiplier is 200, the average amount of cash that can be obtained each month ranges from about NT$45,000 to NT$63,000. As for the income replacement rate after retirement, this can be increased by between 84% and 117%. If the rent multiplier is assumed to be 250, the average amount of cash that can be obtained each month ranges from about NT$56,000 to NT$78,000, and the income replacement rate after retirement can be enhanced by between 104% and 146%.

Based on the above data, the average amount by which the income replacement rate is enhanced as a result of the reverse mortgage ranges from 70% to 100%, showing that it is indeed effective in helping owner-occupied residential homeowners improve their financial situation after they retire. Furthermore, when different age cohorts of homeowners are considered, due to Taiwan's housing prices c ontinuing to rise over time, we find that the enhancement effect of reverse mortgages on the retirement income of the younger generation of retired homeowners is greater than that in the case of the older g eneration of retired homeowners.

6. Conclusions

In this study, we use the HECM pricing model established by Szymanoski (1994) as our principal framework, and calculate the principal limit factor corresponding to elderly homeowners in Taiwan, following this by calculating the extent to which the income replacement level of the retired homeowners can be enhanced by a reverse mortgage. Our empirical results for Taiwan show that the enhancement effect on the income of these elderly living in owner-occupied housing that is brought about by the reverse mortgages averages between some 70% and 100%, which is consistent with the findings of Mitchell and Piggott (2004) and Ong (2008), indicating that reverse mortgages can effectively enhance the financial situation of the elderly after they retire. When taking different age cohorts into consideration, we also find that, because Taiwan's housing prices have continued to rise year after year, the enhancement effect of reverse mortgages on the retirement income of the younger generation of retired homeowners is greater than that in the case of the older generation of retired homeowners.

Past studies have also found that, for reverse mortgages to be able to be successfully implemented, it is extremely important to establish a sound loan insurance mechanism. It is only because of the loan insurance safeguard that both lenders and borrowers need not concern themselves with the various risks associated with reverse mortgages, thereby increasing both the supply and demand for them. At present, reverse mortgages in Taiwan are being handled with social welfare policy in mind, and we therefore recommend that the Taiwan government make this financial instrument more widely available. In the future, high-quality financial institutions can be allowed to issue reverse mortgages to meet the needs of more elderly people, while the Government at the same time provides reverse mortgage loan insurance. In this way, not only can it be ensured that the borrowers will continue to receive the payments to be made to them because of the loans, but it will also be possible for losses from loans provided by the lending institution to be effectively controlled. In addition, if the amount of the reverse mortgage loan can enjoy an exemption from tax, the extent to which the income of the elderly homeowners can be enhanced by the reverse mortgages will not only be more significant, but at the same time will also have the effect of giving the elderly homeowners a greater incentive to apply for such loans. For this reason, we recommend that the Government provide favorable tax treatment to encourage elderly homeowners to convert their home equity into highly liquid currency, and thereby stabilize their income after they retire to an adequate degree.

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