# The Liability Regime of Insurance Pools and Its Impact on Pricing

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**Abstract**

By pricing the payoffs from an insurance pool in a contingent-claim approach the paper formally describes fairly priced premiums and equity contributions from the perspective of a pool's policyholder as well as the pool insurers' equity holders. The contingent-claim approach particularly distinguishes between two liability regimes which regulate the pool's indemnification when one or more pool insurers cannot meet their full obligations due to insolvencies. The distinction of both liability regimes and the corresponding effects on premiums and equity are more closely examined in a numerical example from which policy implications about the regimes' price and safety differences are drawn by considering asset correlations and the pool's risk sharing balance. Subsequent results and discussions argue for a preferred liability regime from the perspective of the policyholder if frictional costs and potential risk-shifting problems are taken into account.

**Keywords:** Default Put Option Value, Fair Premium, Insurance Pools, Pricing
**JEL classification:** G22; G28; G33

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# Introduction

Insurance markets which combine the demand and supply of risk coverage are a large part of the financial system and of immense importance for an efficient real economy. Typically, this market allows individuals to shift the uncertainty of their risks against the payment of a fixed insurance premium to the burden of a sound insurance company which generates profits due to the idea of risk diversification. Yet, there are examples where the market fails to establish a supply of risk coverage. Various research on insurance theory is particularly concerned about the failure of catastrophe insurance: Gollier (1997) names the possibility of asymmetric risk evaluation of events with low occurrence probability but severe impact impeding an equilibrium price accepted by insurer and consumer. Kousky and Cooke (2012) point out the enormous capital costs induced by fat-tailed loss distributions of catastrophe events leading to prohibitively high premiums. Similarly, Harrington and Niehaus (2003) exemplify numerically that high layers of coverage cause tax loadings being pretty high compared to the actually expected claim. Jaffee and Russell (1997) see the reasons for the absence of catastrophe insurance in institutional circumstances like accounting requirements or taxation that complicate the accumulation of sufficient risk reserves for insurance companies.

An example of catastrophe risk for which the conventional market commonly fails to provide insurance coverage but for which governments are diametrically interested in an established insurance solution is given by the risk of third party liability emerging from nuclear power. Typically, individual insurance companies refuse to underwrite this risk on their behalf. However, most countries have settled particular liability regulations for operator of nuclear power stations which comprehends the need for coverage solutions. In Germany, for instance, the operators of nuclear power stations are obliged to prove the existence of a compulsory cover for third party liability risk amounting to 2.5 billion EUR (cf. §9 AtDeckV[1]). Another example of special liability regulation for operators of nuclear power stations is given by the Price-Anderson Act which regulates the liability of operators of nuclear power plants in the United States and which requires a compulsory cover from the operators amounting to 12.6 billion USD (cf. Nuclear Energy Institute (2012)). In both cases nuclear power plant operators can meet their legal obligations by purchasing insurance coverage. However, as the ordinary insurance market had failed to provide coverage due to the high exposure emerging from nuclear risk, national insurance pools have been implemented in order to solve the divergence between supply and demand of insurance coverage implied by law.

Insurance pools are in general known as mutual organization of several insurance companies having been founded for the purpose of insuring a special type of risk which is frequently but not limited to nuclear risk.[2] In his definition Farny (2011) points out the legal and economic entity of the participating insurance company where their collaboration is indeed limited to the coverage of a special type of risk. With respect to the relation between pool and policyholder Farny (2011) distinguishes between reinsurance and co-insurance pools. While in the former case a policy is underwritten only by one primary insurer who is then ceding the risk to the pool, the latter case means that the risk is directly underwritten by all pool insurers. Both types of pools resemble simple reinsurance and co-insurance arrangement respectively, however, pools differ as the insurers' contribution is based on uniform terms and conditions for a longer time horizon. On this note Farny (2011) states that insurance pools actually constitute a restraint of competition which will become negligible if the insurance market fails to provide common coverage for some type of risks as it is the case for nuclear risk. The European Commission, for instance, meets this fact by assertively defining the legitimacy of insurance pools as a replacement of coverage on common

[1] *Atomrechtliche Deckungsvorsorge-Verordnung* which came into force on 25 January 1977 (BGBl. I S. 220) and was adjusted on 23 November 2007 (BGBl. I S. 2631).

[2] An overview about the landscape of insurance pools in Europe and their scope of coverage is provided by European Commission (2013).

insurance markets in one of its Block Exemption Regulations.[3]

The value of insurance pools is that resources of an insurance market can be concentrated in terms of capital, knowledge and experience allowing for underwriting heavy risks. Insurance pools make additionally sure that risks of the same type are collectively kept in one portfolio which can thereafter reach a critical size for benefiting from diversification effects. Those effects are highlighted by Kraut (2013) who argues, in line with capital allocation theory, for a fair sharing mechanism ensuring the conclusion of a pool agreement between several insurers. Optimal sharing mechanisms under varying risk assessment functions of the pool insurers are addressed by Fragnelli and Marina (2003) and Ambrosino et al. (2006). Our study takes diversification effects from pooling claims for granted as well as a fair sharing of premiums and claims as defined by zero net present values of the participating pool insurers.

In fact, we focus on an element which has, to our knowledge, not been addressed so far by the literature in the context of insurance pools, namely the different liability regimes under which those pools can operate on the market and which explicitly regulate how to share pool claims that cannot be indemnified by bankrupt pool insurers among the remaining solvent pool insurers. In one regime, hereafter called *regime of joint liability*, pool insurers commit themselves to give a guarantee to the policyholder ensuring that the share of one (or more) bankrupt co-insurers is indemnified jointly by the solvent insurers. In the second regime it is contrarily agreed that the policyholder cannot request a substitutional indemnification from solvent insurers as their liability is merely limited to their own share in the pool claims (hereafter called *regime of several liability*). As an example for a pool operating under a regime of joint liability we allude to the German Nuclear Reactor Insurance Association whilst the regime of several liability is for instance applied by the Austrian Insurance Pool for the Coverage of Terror Risks.

Apart from insurance pools the distinction between both liability regimes resembles the holding and integrated conglomerate structure, respectively, that is analyzed by Gatzert and Schmeiser (2011) in view of diversification benefits and insolvency risks of a parent company and its corresponding subsidiary. A confrontation of joint liability and several liability is also subject to the study of Cummins et al. (2002) that introduces a measure for market efficiency where the insurance market is said to be fully efficient if the single firms act, following our notation, in a regime of joint liability for indemnifying insured losses from a catastrophe event.

The remarkable importance to distinguish between both liability regimes become apparent by pointing to the duration of loss adjustment in long-tail business: If the claims settlement takes a long time with what loss reserves are carried at the balance sheets for a sustained period of time a pool insurer will be exposed within a regime of joint liability to a long-lasting counterparty risk.[4] Next to this the different liability regime recognizable affect the policyholder's counterparty risk as well since the payoffs in a case of insolvency obviously vary between both regimes. Apparently, this deviating risk exposure for both stakeholder groups, insurers and policyholder, within the two liability regimes should become noticeable in a risk-adequate pricing of the pool premium. In this context we refer to TIRSA (2010) containing the TIRSA Joint and Several Liability Endorsement which is suggested to be used by the member insurers of the rate service organization TIRSA if the policyholder demands for joint liability in co-insurance agreements. TIRSA (2010) commissions its member insurers to charge an extra fee (1 USD per 1000 USD sum insured) whenever joint liability is desired by the policyholder instead of several liability.

This work aims at examining the justification for such a premium difference by embedding the two different liability regimes into a model on which a fair pool premium can be priced. For modeling we use the contingent claims approach, firstly introduced by Doherty and Garven (1986), that allows to consider default risk in the context of option pricing theory explicitly. Subsequently, the model is used to conduct

[3] Commission Regulation (EU) No 267/2010 of 24 March 2010 on the application of Article 101(3) of the Treaty on the Functioning of the European Union to certain categories of agreements, decisions and concerted practices in the insurance sector.

[4] This is mainly relevant for but not limited to casualty business where the loss adjustment can take several decades. The settlement of complex claims from property business, however, may go on for years as well which is for instance illustrated by the loss indemnification of the World Trade Center as consequence of the terror attacks in 2001.

a numerical example which outlines price differences between both liability regimes. It will be illustrated that price differences are indeed confirmed by our model, yet a fixed price difference as it is suggested by TIRSA (2010) seems to be inappropriate by addressing asset correlation and the balance of risk sharing as factors that influences those differences and provoke for marginal cases a congruence of both regimes. Whilst this pertains to the risk-adequate fair premium, costs for taxation are incorporated as additional loading on the premium. Since the regime of joint liability will require less equity than the regime of several liability for reaching a safety level, that is for instance regulatory demanded for the pool business, the regime of joint liability arise as opportunity to reduce the cost loading for the policyholder.

Moreover, our numerical example considers a discussion on risk-shifting problems which can potentially occur through the interrelations between the pool's stakeholders. These problems that are materially researched by Jensen and Meckling (1976) and which find application in the work of, e.g. MacMinn (1993) and Green (1984), normally cause within insurance contracts adverse effects on the policyholder's position. The numerical example figures out that joint liability potentially diminishes those adverse effects – compared to several liability – since adverse effects are partially allocated to the burden of pool insurers that abandon risk shifting. The regime of joint liability, however, also intensifies the insurers' incentive to conduct a simultaneous risk shifting in order to immunize themselves against adverse effects.

The remaining study is organized as follows: The following section models pools with the two aforementioned liability regimes where a complete market on which insurance risks can be hedged is assumed. This allows to perform fair pricing by relying on option pricing theory as conducted by e.g., Doherty and Garven (1986), Cummins (1988), Sherris (2006) and Schmeiser and Wagner (2013). Subsequently, we carve out the distinguishing feature of both liability regimes and enrich the model by introducing corporate income tax as frictional costs. Based on the derived model we research a numerical example where different model parameters are varied under different perspectives. Besides, the discussion on the numerical example results is enriched by policy implications as well as by alluding to risk-shifting problems in both regimes.

# Model Framework

## Stochastic Payoffs to Equity and Policyholders

Our model comprises a point-to-point consideration of stylized balance sheets, that is, we look at the assets' and liabilities' values at time $t = 0$ and $t = 1$, respectively, where the dynamics for the incremental behavior of both balance sheet items are going to be defined as stochastic differential equations (cf., e.g., Cummins (1988)). We agree upon that a company is supposed to be bankrupt if at time $t = 1$ the company's assets are exceeded by the company's liabilities. Furthermore, we assume for the sake of simplicity that the pool insurers do not underwrite business in addition to the pool activities. Hence, the structure of the balance sheet is affected by the assets being invested as well as by the liabilities emerging from the pool business where the liabilities' size will depend on the pool's liability regime.

The insurance pool is supposedly composed of $n$ insurers which are indexed by $i = 1, \ldots, n$. Each insurer $i$ comes with equity holders contributing an amount of $E_0^{\star,i}$ at time $t = 0$. Likewise, at time $t = 0$ insurer $i$ receives a premium amount of $P_0^{\star,i}$ for its participation in the pool. Hence, insurer $i$ has assets available at time $t = 0$ equaling to

$$A_0^{\star,i} = E_0^{\star,i} + P_0^{\star,i}$$

which are invested by insurer $i$ itself. The superscripted symbol "$\star$" always serves as place holder for $J$ (joint liability) and $S$ (several liability), respectively, indicating the two possible liability regimes.

At time $t = 1$ the policyholder has a stochastic claims amount $C_1$. Those insured claims are based on occurred losses where the size is independent of the pool's liability regime.[5] However, if the claim size exceeds the pool's available funds the policyholder's true indemnification is reduced to the available funds which in turn depend on the liability regime at hand. Hence, the policyholder's position at time $t = 1$ reads

$$P_1^\star = C_1 - D_1^\star$$

where $D_1^\star$ denotes the part of the insured claims not covered by the pool due to insufficient funds (so called default put option; cf. Doherty and Garven (1986) and Butsic (1994)).

In a regime of several liability those insolvency related costs for the policyholder have the specification

$$D_1^S = \sum_{i=1}^{n} \left[\alpha_i C_1 - A_1^{S,i}\right]^+$$

where $\alpha_i$ is the percentage share of insurer $i$ in the policyholder's pool claims $C_1$ and $A_1^{S,i}$ denotes the stochastic value of the assets at time $t = 1$. For joint liability the analogous specification reads

$$D_1^J = \left[C_1 - \sum_{i=1}^{n} A_1^{J,i}\right]^+ .$$

At time $t = 1$ insurer $i$ has to face liabilities which are composed of its share on the pool claims $C_1$ as well as of its possible guarantee for other pool insurers that cannot indemnify their claims shares thoroughly where the latter depends on the liability regime at hand. The liability of insurer $i$ at time $t = 1$ is thus defined as

$$L_1^{\star,i} = \alpha_i C_1 + G_1^{\star,i}. \tag{1}$$

Since the regime of several liability is just characterized by an exclusion of a mutual assumption of failed liabilities, we consequently have $G_1^{S,i} = 0$. The precise definition of $G_1^{J,i}$ is going to be outlined later.

The equity holders of insurer $i$ receive at time $t = 1$ the assets in excess of liabilities by simultaneously benefiting of limited liability which together results in the payoff

$$E_1^{\star,i} = [A_1^{\star,i} - L_1^{\star,i}]^+.$$

## Valuation in a Contingent Claims Setting

In line with Cummins (1988) we value the different stakes using risk-neutral pricing, i.e. any expectations being used for the purposes of pricing are computed under the risk-neutral measure $\mathbb{Q}$. For fairness in bilateral contracts (without transactions costs) between the policyholder and one insurer (organized as a stock company) it is sufficient to ensure that solely one party, so either the policyholder or the equity holders, takes a fair position at time $t = 0$ which we perceive as net present value of zero for premium or equity payments. The fairness for the other party is then directly implied.

In terms of a pool agreement with the participation of multiple insurers the position of the policyholder will be fair at time $t = 0$, i.e. a zero net present value is provided, if the payed premium $P_0^\star$ coincides with the present value of $P_1^\star$ that is given by

$$\Pi_P^\star = \mathbb{E}^{\mathbb{Q}}\left[e^{-r} P_1^\star\right] = \mathbb{E}^{\mathbb{Q}}\left[e^{-r} C_1\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-r} D_1^\star\right] = \Pi_C - \Pi_D^\star. \tag{2}$$

[5] We consider $C_1$ as the aggregated loss of the pool portfolio. Actually, there will be $k$ losses each with a loss amount of $C_1^j, j = 1, \ldots, k$ so that $C_1 = \sum_{j=1}^k C_1^j$. For researching diversification effects from pooling claims it would be necessary to deal with $C_1^j$. Since those effects are not the focus of our examination we take them for granted and as incorporated in the later distribution specification of $C_1$.

The fairness condition $\Pi_P^\star = P_0^\star$ for the policyholder yet does not result automatically in a fair situation for the equity holders of the pool insurers. In order to ensure fairness the premium share $P_0^{\star,i}$ of insurer $i$ in the overall pool premium must equal to the present value of its effective pool contribution which comprises the liabilities less the amount of liabilities exceeding the value of assets. We write this present value as

$$\begin{aligned}
\Pi_P^{\star,i} &= \mathbb{E}^{\mathbb{Q}}\left[e^{-r}P_1^{\star,i}\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{-r}L_1^{\star,i}\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\left[L_1^{\star,i} - A_1^{\star,i}\right]^+\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\alpha_i C_1\right] + \mathbb{E}^{\mathbb{Q}}\left[e^{-r}G_1^{\star,i}\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\left[L_1^{\star,i} - A_1^{\star,i}\right]^+\right] \\
&= \alpha_i \Pi_C + \Pi_G^{\star,i} - \Pi_D^{\star,i}.
\end{aligned}$$

The condition $P_0^{\star,i} = \Pi_P^{\star,i}$ will equivalently hold if the equity holders' initial contribution $E_0^{\star,i}$ conforms to the present value of the payoff to the equity holders at time $t = 1$ which is

$$\Pi_E^{\star,i} = \mathbb{E}^{\mathbb{Q}}\left[e^{-r}E_1^{\star,i}\right] = \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\left[A_1^{\star,i} - L_1^{\star,i}\right]^+\right]. \tag{3}$$

A fair position for all equity holders implies a fair position for the policyholder as the present values of the insurer's pool contribution $P_1^{\star,i}$ sum up to the present value of the overall pool payment $P_1^\star$.

In summary, a fair position for all equity holders implies a fair position for the policyholder whilst the reversion is not true in general. Consequently, if it is ensured that the pool premium $P_0^\star$ is fairly priced from the perspective of the policyholder it will be essential for the fairness of the equity holders that the premium is appropriately allocated among the pool insurers.

Typically in practice, the premium share which we define here as

$$\beta_i := \frac{P_0^{\star,i}}{P_0^\star},$$

is identical with the share $\alpha_i$ in the pool claims.[6] If we assume that fairness is at hand for all equity holders, which yields for every insurer $i$

$$\beta_i = \frac{\Pi_P^{\star,i}}{\Pi_P^\star} = \frac{\alpha_i \Pi_C + \Pi_G^{\star,i} - \Pi_D^{\star,i}}{\Pi_P^\star},$$

the practically relevant constraint $(\alpha_1, \dots, \alpha_n) = (\beta_1, \dots, \beta_n)$ can only be fulfilled if and only if

$$\alpha_i = \frac{\Pi_D^{\star,i} - \Pi_G^{\star,i}}{\Pi_D^\star} \tag{4}$$

holds true for every insurer $i$. This requirement means that under fairness in terms of our fair pricing setup the condition $\alpha_i = \beta_i$ will only appear if the percentage allocation of premium and pool claims matches the proportion between the value of the pool's default put option and the values of the insurers' individual default put option reduced by the value of the provided guarantee for failing co-insurers. If the equality of Equation (4) is not fulfilled a fair position for all stakeholders can still occur but not under the equality constraint $\alpha_i = \beta_i$ for all $i$.

[6] See also European Commission (2013, p. 76).

A potential deviation between $\alpha_i$ and $\beta_i$ under a fairly priced pool contract for which the premium is fairly allocated by $\beta_i$ turns out to be

$$\alpha_i = \beta_i + \frac{\Pi_D}{\Pi_C}(\kappa_i - \beta_i). \tag{5}$$

where

$$\kappa_i := \frac{\Pi_D^{\star,i} - \Pi_G^{\star,i}}{\Pi_D^{\star}}.$$

The second term on the right hand side of Equation (5) determines the necessary difference between risk and premium share so that the pool contract is fair for all stakeholders. If an insurer's premium participation is smaller than its share in the pool's default put option its share in the pool claims must be implicitly higher than its premium share. Yet a small value of $\Pi_D$ compared to $\Pi_C$ will materially deflate this deviation.

Under the constraint $\alpha_i = \beta_i$ for all insurer $i$ the condition $\kappa_i = \beta_i$, for all $i$, is a necessary but not sufficient condition for a fairly priced pool contract in terms of zero net present values. Consequently, if there is an insurer $i$ for which $\kappa_i \neq \alpha_i = \beta_i$ is observed it will be proved at once that the pool contract cannot be fairly priced for all stakeholders. This will for instance be the case if the pool is composed of two insurers where merely one insurer can fail on its liabilities from the pool business.

## The Default Mechanism in a Regime of Joint Liability

The distinctive feature of joint liability as boundary to several liability is the insurers' commitment to guarantee the other insurers' failed indemnification in the case of insolvencies. Technically, the guarantee is a contingent liability which is regulated within the scope of IFRS by IAS 37. In accordance of this standard it is not contemplated that the liability is recognized in the balance sheet, however, it should be integrated as annotation into the disclosure. This is accomplished, for instance, by Munich Re which writes on page 123 of its Annual Report 2013 under the paragraph "Contingent liabilities, other financial commitments":

"*...As a member of the German Reinsurance Pharmapool and the German Nuclear Insurance Pool, we are committed - to the extent of our proportional share - to assuming the payment obligations of another pool member if the latter is not able to meet these obligations...*"

The last statement points out that the guarantee is allocated amongst the solvent pool insurers in line with their proportional share in the pool.

For a formalistic specification we assume that the guarantee $G_1^{J,i}$ of insurer $i$ at time $t = 1$ is subordinated compared to the liability from the pool claims $\alpha_i C_1$, thus the payoff from the guarantee is limited to the amount $[A_1^{J,i} - \alpha_i C_1]^+$. In general, we assume that the liability amount from the guarantee cannot exceed $[A_1^{J,i} - \alpha_i C_1]^+$.[7] This agreement allows to deduce that the default put value of insurer $i$ does not include parts from the guarantee, i.e.

$$D_1^{J,i} = [L_1^{J,i} - A_1^{J,i}]^+ = [\alpha_i C_1 - A_1^{J,i}]^+$$

which yields

$$\sum_{i=1}^{n} G_1^{J,i} = \sum_{i=1}^{n} D_1^{J,i} - D_1^J = \sum_{i=1}^{n} \left[\alpha_i C_1 - A_1^{J,i}\right]^+ - \left[C_1 - \sum_{i=1}^{n} A_1^{J,i}\right]^+.$$

[7] Beyond our model world, if it comes to an insolvency proceeding, this rather technical assumption would mean, that an insurance company cannot be hold liable for any guarantee exceeding its funds. If, within the scope of joint liability, at least one pool insurer is solvent the policyholder must resort to this insurer.

A formalistic specification of $G_1^{J,i}$ must consider that the guarantee of insurer $i$ can gradually increase if a first allocation of an insurer's shortfall is incomplete due to the limited funds of other pool insurers and so forth. We meet this fact by defining the following recursion

$$\begin{aligned} G_1^{J,i}(0) &= 0 \\ G_1^{J,i}(k+1) &= G_1^{J,i}(k) + \min\left\{ \frac{\alpha_i}{\sum_{j \in M(k)} \alpha_j} \sum_{j=1}^{n} \left( \left[\alpha_j C_1 - A_1^{J,j}\right]^+ - G_1^{J,j}(k) \right), \left[A_1^{J,i} - \alpha_i C_1 - G_1^{J,i}(k)\right]^+ \right\} \end{aligned}$$

where $M(k)$ is defined as the set of solvent insurers after $k$ allocation steps, i.e.

$$M(k) := \left\{ i : A_1^{J,i} > \alpha_i C_1 + G_1^{J,i}(k) \right\}.$$

In order to avoid any non-defined results we additionally agree upon

$$M(k) = \emptyset \Rightarrow G_1^{J,i}(k+1) = G_1^{J,i}(k)$$

for all insurers $i$. Hence, once the funds of all insurers have been exhausted there cannot be a further reallocation of unpaid pool claims which means a non-zero value for $D_1^J$. In general, the defined recursion must terminate at latest for index $n-1$, hence for any insurer $i$ the payoff from the guarantee in a regime of joint liability eventually reads

$$G_1^{J,i} = G_1^{J,i}(n-1).$$

## Introducing Corporate Income Tax

The above setting neglects any frictional costs. As firstly introduced by Doherty and Garven (1986), however, it is feasible to extend the setting by taking corporate income taxation into account. We assume that the insurance companies composing the pool are taxed due to their income emerging from their balance sheets at time $t = 1$. Hereby, it is particularly assumed, that the pool itself is not subject to taxation.[8] The income of insurer $i$ is the change of equity between $t = 0$ and $t = 1$ whereat for taxation only a positive income becomes relevant. Hence, the base for taxation of insurer $i$ reads

$$B_1^{\star,i} = \left[\widetilde{E}_1^{\star,i} - \widetilde{E}_0^{\star,i}\right]^+ = \left[\widetilde{A}_1^{\star,i} - \widetilde{A}_0^{\star,i} + \widetilde{P}_0^{\star,i} - \widetilde{L}_1^{\star,i}\right]^+$$

where tilde on the variables indicates the consideration of the presumed taxation (all values before tax outflows at time $t = 1$) in the model. If $\tau$ denotes the tax rate insurer $i$ will have a tax burden payable to the state amounting to

$$T_1^{\star,i} = \tau B_1^{\star,i}.$$

Consequently, the equity holders of insurer $i$ resume a position after taxation at time $t = 1$ that is

$$\mathscr{E}_1^{\star,i} = \left[\widetilde{A}_1^{\star,i} - \widetilde{L}_1^{\star,i}\right]^+ - T_1^{\star,i}.$$

In the context of risk-neutral pricing, equity holders will only contribute to the insurance company at time $t = 0$ if this contribution $\widetilde{E}_0^{\star,i}$ equals the present value:

$$\Pi_{\mathscr{E}}^{\star,i} = \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\mathscr{E}_1^{\star,i}\right] = \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\left[\widetilde{A}_1^{\star,i} - \widetilde{L}_1^{\star,i}\right]^+\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-r}T_1^{\star,i}\right] = \Pi_{\widetilde{E}}^{\star,i} - \Pi_T^{\star,i}.$$

[8] In practice, the state of the pool's taxation is certainly depending on the pool's organizational form. The German Nuclear Reactor Insurance Association, for instance, is organized as a GbR (*Gesellschaft des bürgerlichen Rechts*) and thus not obliged to pay income tax itself. Income from the business of a GbR is taxed within the scopes of the individual taxation of the insurers establishing the GbR (cf. Lang et al. (2010, p. 18f.)) which is thus in line with our model proceeding.

As it is concluded by Gatzert and Schmeiser (2008, p. 52) the fairness condition $\widetilde{E}_0^{\star,i} = \Pi_{\mathscr{E}}^{\star,i}$ on the equity holders' initial investment into insurance company $i$ results in a premium for the policyholder that reads

$$\begin{aligned}\Pi_{\widetilde{P}}^{\star,i} &= \mathbb{E}^{\mathbb{Q}}\left[e^{-r}C_1\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-r}\left[C_1 - \sum_{i=1}^{n}\widetilde{A}_1^{\star,i}\right]^{+}\right] + \mathbb{E}^{\mathbb{Q}}\left[e^{-r}T_1^{\star,i}\right].\\ &= \Pi_C - \Pi_{\widetilde{D}}^{\star} + \Pi_T^{\star}.\end{aligned}$$

Since the present value of the policyholder's payoff is merely $\Pi_C - \Pi_{\widetilde{D}}^{\star}$, the policyholder confronts a non-zero net present value. More precisely, it is the policyholder who entirely carries the tax burden as surcharge to its premium.[9]

Gatzert and Schmeiser (2008) examine the capital structure of a single insurer after taxation has been introduced. In order to make a comparison between the states of non-taxation and taxation, respectively, it is assumed that an insurer's safety level is ought to remain constant for both states. For our purposes, we analogously require the condition

$$\Pi_{\widetilde{D}}^{\star} = \Pi_D^{\star} \tag{6}$$

so that the policyholder experiences the same safety level for the pool indemnification independent of the tax situation. We ensure the condition of Equation (6) by requiring the even stronger constraint that also the value of the insurer's individual default put option as well as their guarantees remain unaffected from taxation, i.e.

$$\Pi_{\widetilde{D}}^{\star,i} - \Pi_{\widetilde{G}}^{\star,i} = \Pi_D^{\star,i} - \Pi_G^{\star,i}$$

should hold for all insurer $i$. Therefore it is in turn sufficient to call for

$$\begin{aligned}\widetilde{A}_0^{\star,i} &= A_0^{\star,i}\\ \Leftrightarrow \widetilde{E}_0^{\star,i} + \widetilde{P}_0^{\star,i} &= E_0^{\star,i} + P_0^{\star,i}\\ \Leftrightarrow \left(E_0^{\star,i} - \Pi_T^{\star,i}\right) + \left(P_0^{\star,i} + \Pi_T^{\star,i}\right) &= E_0^{\star,i} + P_0^{\star,i}\end{aligned}$$

as initial condition for the insurers' assets. Effectively, the introduction of taxation, whilst the insurers' safety levels are kept fixed, merely leads to a reallocation of the source of initial funds from the equity holders contribution which is reduced by $\Pi_T^{\star,i}$ to an increased premium of the policyholder. The pool liability of insurer $i$ as well as its indemnification at time $t = 1$ are not affected by taxation under the assumptions that are used in this section. This incidentally presages the order of calculations: At first one would calculate premium and equity in a tax-free world whose total value is then in turn used in a second step. This second step introduces income taxation in order to derive the present values of every insurers' tax burden. Thirdly, equity and premium from the first step are adjusted by the resulting tax loading from step two.

# Numerical Example

## Distributional Assumptions

For performing a numerical example it is necessary to agree upon the specification of the distribution on the underlying processes. We have to specify on the one hand the processes driving the outcome of the

[9] The implied assumption that insurance companies can pass the entire tax burden to the policyholder is reasoned by Doherty and Garven (1986) and also applied by e.g. Harrington and Niehaus (2003). In order to ensure insurance demand by policyholders in spite of a negative net present value it is necessary to assume that policyholders, in contrast to equity holders, show some degree of risk aversion and cannot replicate future payoffs.

insurers' asset portfolios, on the other hand the process determining the size of the pool claims $C_1$ at time $t = 1$. Using the setup of Merton (1976), the latter's underlying is going to be modeled as a jump-diffusion process evolving between $t = 0$ and $t = 1$ where the process' value at time $t = 1$ determines $C_1$. By admitting jumps to the process of the pool claims we comply with the pool business that is ordinarily subject to major claim events with low frequency but high severity.

In order to introduce the jump component, we define by $J(t)$ a compound Poisson process

$$J(t) = \sum_{j=1}^{N(t)} Y_j$$

where $Y_j$ are i.i.d random variables and $N(t)$ is a Poisson process with intensity $\lambda$. The jump-diffusion process under the real-world measure $\mathbb{P}$ is the solution for the stochastic differential equation

$$\mathrm{d}C(t)/C(t-) = \mu_C \,\mathrm{d}t + \sigma_C \,\mathrm{d}W_C^{\mathbb{P}}(t) + \mathrm{d}J(t) \tag{7}$$

that is taken from Merton (1976) with initial condition $C(0) = C_0$ as the face value of the pool claims and $W_C^{\mathbb{P}}(t)$ as a Brownian motion under $\mathbb{P}$. At the arrival time $t_j$ of the $j$th jump the claims' value $C(t_j-)$ instantaneously before the jump is compounded by $Y_j$. Hence, $Y_j$ controls for the magnitude of the jump effect on the claims size.

By setting $E^{\mathbb{Q}}[Y_j - 1] = m$ the solution to Equation (7) under the risk-neutral measure $\mathbb{Q}$ is derived by Merton (1976) and reads

$$C(t) = C_0 \exp\Big(\left(r - \lambda m - \sigma_L^2/2\right) t + \sigma_C W_C^{\mathbb{Q}}(t)\Big) \prod_{j=1}^{N(t)} Y_j$$

where $W_C^{\mathbb{Q}}, N$ and $Y$ are supposed to be independent. Cause of the applied drift $r - \lambda m$, we can deduce (in line with, e.g. Gatzert and Schmeiser (2008)) that $e^{-r}C(t)$ satisfies the martingale property. The unique representation of $C(t)$ under the risk-neutral measure $\mathbb{Q}$ is then based on the assumption of Merton (1976) that the jump risk is diversifiable and thus does not require an additional return compensation. Otherwise, as jumps prevent from arranging a replicating portfolio with riskless payoff, the market would be incomplete and the uniqueness of the risk-neutral measure not ensured.[10]

For the distribution of $Y_j$ we assume $\log(Y_j) \sim N(a, b^2)$ so that the expected jump size turns out to be

$$\mathbb{E}^{\mathbb{Q}}[Y_j] = \exp\left(a + b^2/2\right).$$

By assuming a lognormal distribution for $Y_j$ the jump size will take values greater than zero. As aforementioned the purpose of incorporating jumps is to adequately represent claims from catastrophe risks, yet the values of $Y_j$ can be less than 1 which would reduce liabilities. Those realizations are interpreted as recognizable benefits from winding up loss reserves that have been settled too high in a first loss assessment.

The processes resulting in the distribution of $A_1^{\star,i}, i = 1, \ldots, n$ are modeled as ordinary Geometric Brownian motions which stay unaffected from jumps. The stochastic differential equation for the process which drives $A_1^{\star,i}$ is given by

$$\mathrm{d}A^{\star,i}(t) = \mu_{A,i} \,\mathrm{d}t + \sigma_{A,i} \,\mathrm{d}W_{A,i}^{\mathbb{P}}(t)$$

[10] Cummins (1988) considers the assumption of diversifiable jumps for insurance claims as less problematic than for stock prices. If this assumption, however, were not entirely applicable the present value calculations could be biased.

with initial condition $A^{\star,i}(0) = A_0^{\star,i}$. As special case of Equation (7) the solution to the last stochastic differential equation under the risk-neutral measure reads

$$A^{\star,i}(t) = A^{\star,i}(0) \exp\left(\left(r - \sigma_{A,i}^2/2\right) t + \sigma_{A,i} W_{A,i}^{\mathbb{Q}}(t)\right). \tag{8}$$

The distinction between the different liability regimes, marked by $\star$, is solely necessary due to the different initial values $A^{\star,i}(0)$. The return process itself is not different which means in particular that the insurers' investment strategies are not regime-depending. The applied indexes for the volatility parameters, however, as well as for the Brownian motions highlight that the investments of the different insurance companies are managed individually and can possibly vary with respect to the assumed risk.

Finally, we allow in our numerical examination for correlations between $n+1$ occurring Brownian motions $W_{A,1}^{\mathbb{Q}}, \ldots, W_{A,n}^{\mathbb{Q}}, W_C^{\mathbb{Q}}$ where the notation is as follows:

$$\begin{aligned} Cor\left[W_{A,i}^{\mathbb{Q}}, W_{A,j}^{\mathbb{Q}}\right] &= \rho_A^{ij} \\ Cor\left[W_{A,i}^{\mathbb{Q}}, W_L^{\mathbb{Q}}\right] &= \rho_L^{i}. \end{aligned}$$

## Pool Composition and Numerical Procedure

In order to get illustrative results we reduce complexity by setting $n = 2$. In this way, the risk share $\alpha_1$ of insurer 1 already defines the risk share of insurer 2 by $\alpha_2 = 1 - \alpha_1$. We conform to a practically relevant situation by additionally requiring for all cases $\alpha_i = \beta_i$. The marginal cases $\alpha_i = 0$, for $i = 1$ or 2, coincide for both liability regimes and mean that the policyholder's risk is conventionally carried by one insurer.[11]

A meaningful comparison between both liability regimes is ensured by relying on the value of the default put option as a reference. If $\Pi_D^S = \Pi_D^J$ holds the risk-neutral policyholder will face an identical safety level for two contracts - one having been concluded in a regime of several liability and one in a regime of joint liability. As the face value $\Pi_C$ of the pool claims does not depend on the regime at hand a coincident safety level results in the same fair premium for both liability regimes (see Equation (2)). For reaching the equilibrium of a competitive market it then remains to find for both insurers the initial equity contribution such that Equation (3) is fulfilled for $i = 1$ and 2. As the interdependencies within the regime of joint liability rule out closed-form solutions we simulate the pool claims and payoffs from the insurers' investments in 1,000,000 simulation runs and take the averaged realizations as basis for the approximated fair value of $E_0^{\star,i}$ by numerically solving

$$\Pi_P^{\star,i} - P_0^{\star,i} = 0$$

for $i = 1$ and 2 simultaneously. These fair combinations equivalently satisfy Equation (3) and can subsequently be used for calculating the tax loading under an universally assumed tax rate of $\tau = 35$ percent. In an other numerical procedure we fix equity $E_0^{\star,i}$ and derive the fair pool premium $P_0^{\star}$ by a fixed point iteration which emphasizes different prices for both regimes due to varying safety levels.

## Reference Case – Concordant Asset Volatility and No Correlation

For the assets' volatilities we select in a first step identical parameters for both insurers. More precisely, we determine $\sigma_{A,1} = \sigma_{A,2} = 20$ percent which is ought to signalize that both insurers execute investment strategies with the same degree of risk. For the diffusion part of the process $C(t)$ we set a moderate volatility of $\sigma_C = 10$ percent whilst the jump part is going to be characterized by $\lambda = 0.1$, $a = 0.4$ and

[11] If the condition $\alpha_i = \beta_i$ were not effective an occurring combination of $\alpha_i = 0$ and $\beta_i > 0$ would correspond in a regime of joint liability to a pure guarantee of insurer $i$ where $\beta_i$ defines the compensation for this value performance.

Table 1: The parameter setting for the reference case

| $\sigma_{A,1}$ | $\sigma_{A,2}$ | $\sigma_C$ | $\lambda$ | $a$ | $b$ | $C_0$ | $r$ | $\rho_L^2$ | $\rho_L^2$ | $\rho_A^{12}$ | $\Pi_D^\star$ | $\Pi_P^\star$ | $\tau$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20% | 20% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 0.5 | 99.5 | 35% |

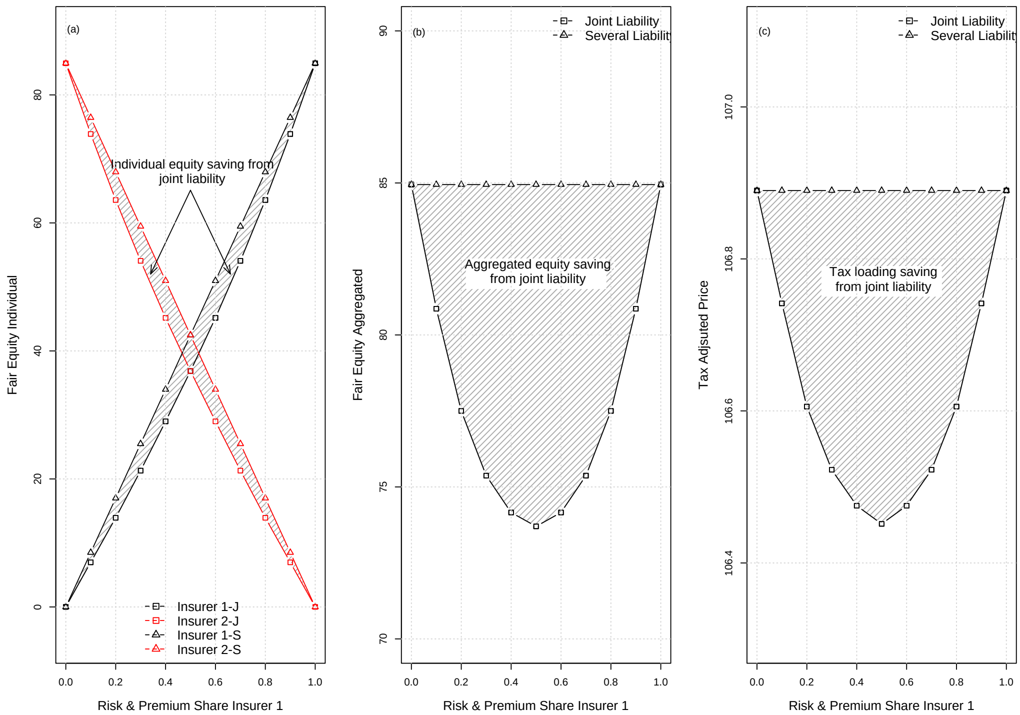


Figure 1: (a) shows the fair equity on an individual base for both regimes, (b) shows it on an aggregated base and (c) shows the tax adjusted price. Figures are based on the reference case.

$b = 0.1$. In terms, this means that every ten years a jump increasing the liabilities by about 50 percent must be expected.

As starting point of the numerical example a reference case is simulated with the parameter setting shown in Table 1 for different allocation schemes that are specified by $\alpha_1 = 0, 0.1, \ldots, 1$. A risk share $\alpha_1$ close to 0 or 1 means a more unbalanced segmentation of the pool business between both pool insurers whilst as $\alpha_1$ goes to 0.5 the participation of both insurers becomes more balanced. Besides, we initially assume no correlation, that is $\rho_L^1 = \rho_L^2 = \rho_A^{12} = 0$, and choose for the risk-free rate $r = 3$ percent.

By applying this setting for the different values of $\alpha_i$ in both liability regimes we overall obtain results for 20 different coverage alternatives.[12] The alternatives' comparability is ensured by throughout setting $\Pi_D^\star = 0.5$. Any frictional costs are excluded in the first instance so that each alternative has the same value for a risk-neutral policyholder. Implicitly, to make the contract fair, the equity holders' initial investment will change with respect to the allocation scheme as well as with respect to the regime at hand. We are going to point out this by writing $E_0^{\star,i} = E_0^{\star,i}(\alpha)$.

Plot (a) of Figure 1 demonstrates the fair individual equity $E_0^{\star,i}(\alpha)$ of insurer 1 and 2, respectively,

[12] In each of both regimes insurer 1 can have available eleven different shares $\alpha_1 = 0, 0.1, \ldots, 1$ where the marginal cases $\alpha_1 = 0$ and $\alpha_1 = 1$ yield the same results for both liability regimes.

under the parameter setting of Table 1. A higher share $\alpha_i$ implies a larger value for the fair equity of insurer $i$. As both insurers possess the same investment strategy, measured by volatility, the identity

$$E_0^{\star,1}(\alpha_1, \alpha_2) = E_0^{\star,2}(\alpha_2, \alpha_1)$$

can be deduced which means that the the individual equity is symmetric in both liability regimes. As distinctive feature for both regimes we observe that for an equal share the fair equity in a regime of several liability is greater than the one of joint liability which means

$$E_0^{S,i}(\alpha_1, \alpha_2) \geq E_0^{J,i}(\alpha_1, \alpha_2)$$

for $i = 1$ and 2. The interpretation of this spread is twofold: On the one hand, an identical safety level, that is $\Pi_D^J = \Pi_D^S$ as assumed, requires in the regime of joint liability less equity. On the other hand, it signalizes that the guarantee of the joint liability lowers the value of the payoff to the equity holders who in turn invest less equity for reaching a zero net present value.

Plot (b) of Figure 1 illustrates the aggregated value of $E_0^{\star,1}$ and $E_0^{\star,2}$, which we denote by $E_0^{\star,A} := E_0^{\star,1} + E_0^{\star,2}$. For the regime of several liability we explicitly observe that

$$E_0^{S,A}(\alpha) = c$$

holds true for any allocation scheme $\alpha$. Assuming a competitive market, this means, that irrespective of a balanced or unbalanced sharing among the pool insurers the required aggregated capital to run the pool in a regime of several liability is always the same. In other words, the equity ensuring the safety level under the allocation $\alpha = (1.0, 0.0)$, that is one insurer bears the risk alone, is the same as the necessary aggregated equity for the allocation $\alpha = (0.5, 0.5)$ when risk sharing is most balanced.[13]

On the contrary, we see that in a regime of joint liability the safety level of the contract can be maintained for a reduced aggregated equity if the the risk sharing between both insurers becomes more balanced within the pool. Hence, in relation to several liability the pool insurers save equity in a regime of joint liability as $\alpha$ goes to $(0.5, 0.5)$. Apparently, at this point of allocation the guarantee serving as equity substitute within the regime of joint liability has its highest effectiveness which in turn means a lower payoff for equity holders and thus a minimized value for $E_0^{J,A}$.

## Introducing Market Frictions

In a complete market without any frictional costs those regime-depending savings in aggregated equity are immaterial since the initial individual equity $E_0^{\star,i}$ is determined for both insurers in a fair manner,

[13] Under the presumed distributions and specifications that $\alpha = \beta$ as well as $\sigma_{A,1} = \sigma_{A,2}$ this observation can be verified by the following argumentation: Let $\widehat{A}_0^{S,1} = \widehat{E}_0^{S,1} + \widehat{P}_0^{S,1}$ be the initial assets of insurer 1 if the allocation reads $(1.0, 0.0)$. As insurer 1 bears the risk solely it earns the complete premium at time $t = 0$, thus $\widehat{P}_0^{S,1} = P_0^S$. In this case it holds that $\widehat{\Pi}_D^{S,1} = \Pi_D^S$. Now, suppose a new allocation $\alpha$ for which the initial assets are then given by $A_0^{S,i} = E_0^{S,i} + P_0^{S,i} = E_0^{S,i} + \beta_i P_0^S$ for insurer $i$. As the constraint $\alpha_i = \beta_i$ is provided we deduce from Equation (4) that

$$\Pi_D^{S,i} = \alpha_i \Pi_D^S = \alpha_i \widehat{\Pi}_D^{S,1}$$

must hold. This equality in conjunction with the assumption that $\sigma_{A,1} = \sigma_{A,2}$ yields the implied identity

$$A_0^{S,i} = \alpha_i \widehat{A}_0^{S,1} = \alpha_i \widehat{E}_0^{S,1} + \alpha_i P_0^S$$

which in turn leads to the equality $E_0^{S,i} = \alpha_i \widehat{E}_0^{S,1}$. As this holds for both insurers and since the allocation shares sum up to unity we can conclude that the aggregated equity must always equal $\widehat{E}_0^{S,1}$. The argumentation can be applied to the general case of $n$ pool insurers, too. As the value of $\widehat{E}_0^{S,1}$ is the same for any $n$ we can moreover deduce that the level of the aggregated capital is independent of $n$, i.e. an increase in $n$ does not reduce the overall capital for reaching the predetermined safety level.

that is, all stakeholders face a net present value of zero on their initial contribution irrespective of the applied allocation scheme and liability regime, respectively.

However, if any accumulation of equity generates costs a minimization of capital expenditure might be of interest for the stakeholder of the insurance contract. We want to illustrate this by introducing a tax rate of $\tau = 35$ percent applied to the incomes of both pool insurers. As previously presumed insurance companies can transfer their tax burden to the account of the policyholder for whom a certain degree of risk aversion can be assumed. Such an implicit risk aversion justifies the policyholder's willingness to pay a price greater than the fairly and risk-neutrally priced pool premium $P_0^\star$.[14]

Plot (c) of Figure 1 shows the tax adjusted price of the insurance contract that has been calculated in line with the parameters of Table 1. We see that the price in a regime of several liability is invariant with respect to the allocation scheme $\alpha$.[15] As opposed to that, in the regime of joint liability a convex but non-constant shape of this tax-adjusted price is observable. Coinciding with the aggregated equity the minimal tax loading is reached at the most balanced risk sharing $\alpha = (0.5, 0.5)$.

Altogether a regime of joint liability provides some tax advantage due to a lower accumulation of equity. Consequently, in our examined setting the policyholder would prefer from 20 different contracts all established under the same safety level the one from a pool operating under joint liability with a risk allocation of $(\alpha_1, \alpha_2) = (0.5, 0.5)$. The advantageous feature of joint liability for the policyholder to reduce friction costs which occur in our example by means of corporate income taxation as an increasing function of equity is yet marginalized if the risk is more unevenly shared between both insurers.

## Introducing Correlated Asset Portfolios

So far the reference case has excluded any correlation between the processes $A^{\star,1}$ and $A^{\star,2}$, i.e. $\rho_A^{12} = 0$. We revoke this assumption by varying the value of $\rho_A^{12}$ on the interval $[-1, 1]$ and observe the effects on equity and frictional costs in both liability regimes. Whilst in the last the safety level has been kept constant under a changing allocation scheme $\alpha$ we now fix the risk sharing at the most balanced risk sharing $(\alpha_1, \alpha_2) = (0.5, 0.5)$ and instead augment our examination with the inclusion of other values for $\Pi_D^\star$.

The left hand plot of Figure 2 illustrates the combination of fair premium and fair aggregated equity $E_0^{\star,A}$ by the gray plane and the associated black line. The plane hereby contains all combinations for joint liability under different asset correlations $\rho_A^{12}$. The black line corresponds to the combinations in a regime of several liability. As the pool claims' face value is fixed at 100 a decrease of $\Pi_D^\star$ is by means of Equation (2) equivalent to a higher premium on the vertical axes and figuratively equivalent to a higher safety level.

For the regime of joint liability we conclude that a fixed equity but increasing asset correlation yield a shrinking safety level for the policyholder which is reflected by a lower fair premium. Inversely, for a fixed premium we deduce that the resulting aggregated equity is increasing in $\rho_A^{12}$ for the regime of joint liability, i.e. the stronger the asset portfolios of both insurers are correlated the more equity is required to keep the safety level constant. Obviously, two highly correlated asset portfolios amplify the

[14] As risk-neutral pricing implies the possibility to replicate payoffs a risk-neutral policyholder would actually not accept to pay premiums above the fair price derived from strict risk-neutral pricing. This would effectively rule out cases in which the policyholder pays a premium that includes a tax component as additional loading. However, given that the policyholder is possibly not operating as professional investment manager it is suitable to argue that replicating strategies are only manageable for insurance companies having access to all instruments of the capital market. The risk-neutral price is thus a normative value from the equity holders' perspective where the actual market price can deviate.

[15] The reasoning is analogous to the argumentation in footnote 13 why the aggregated equity is a constant. In fact, the constant tax loading appears due to the fact that the individual equity $E_0^{\star,i}$ is a proportion of $\alpha_i$ from the aggregated equity. This allows the conclusion that

$$\Pi_T^{S,i} = \alpha_i \widehat{\Pi}_T^{S,1}$$

if $\widehat{\Pi}_T^{S,1}$ denotes the (overall) tax loading in the marginal case $\alpha = (1.0, 0.0)$.

Table 2: Parameters for examining effects of changing asset correlation

| $\sigma_{A,1}$ | $\sigma_{A,2}$ | $\sigma_C$ | $\lambda$ | $a$ | $b$ | $C_0$ | $r$ | $\rho_L^1$ | $\rho_L^2$ | $\tau$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 20% | 20% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 35% |

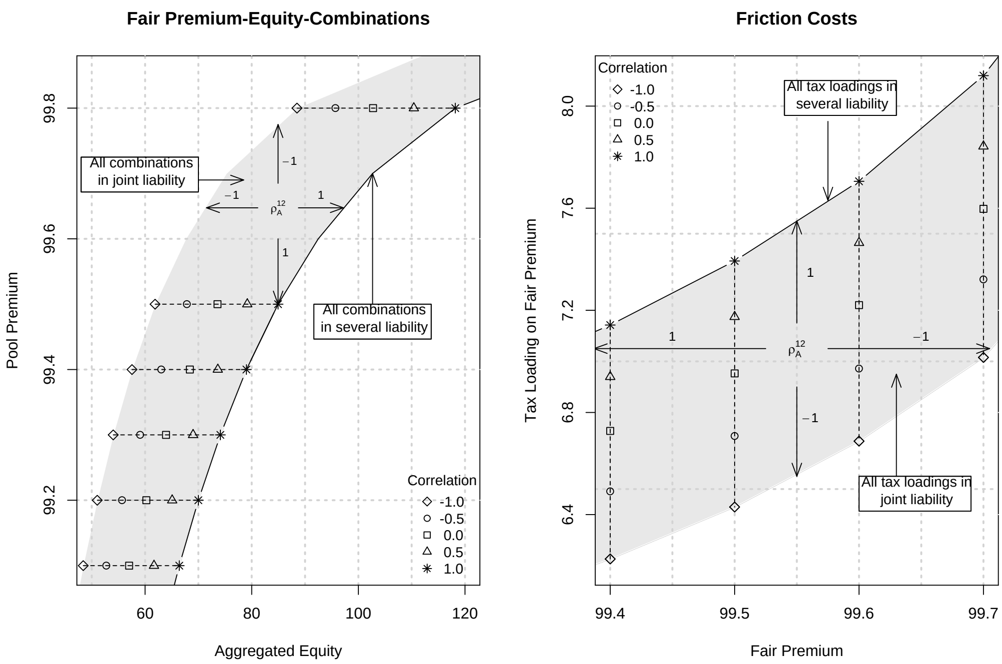


Figure 2: The gray plane contains all combinations for joint liability, the black line contains the combinations for several liability

probability that both insurers default at the same time. In this case the regime of joint liability will loose its distinctive feature since it becomes unlikely that the default of one insurer is offset by the solvency of the other insurer. This explains why a decreasing correlation comes along with an increasing premium in a regime of joint liability though the equity of both insurers remains unchanged. Lower correlation implies a lower probability that both insurers default at the same time so that the safety level, and hence the premium as well, will rise.

As opposed to that, for several liability all combinations of fair premium and aggregated equity can be found on the black line delimiting the gray plane to the right implying that the fair combination of equity and premium is not depending on the correlation of the insurers' asset portfolios. This becomes nearby by taking into account that $\rho_A^{12}$ does not occur in any calculations for the values from several liability. Intuitively, in a regime of several liability an increasing asset portfolio just means that shortfalls with low severity for the policyholder will less likely occur whereat the probability for shortfalls with higher severity increases – in average, however, the policyholder's insolvency costs remain constant.[16] Since the black line delimits the gray plane to the right the regime of several liability may be considered as the limit case of join liability if $\rho_A^{12}$ goes towards 1.

[16]Take the easing example in which $D_1^{S,1}$ and $D_1^{S,2}$ are binomially distributed with success probability $p$. The expected value of $D_1^{S,1} + D_1^{S,2} = D_1^S$ equals $2p$ irrespective of the correlation of $D_1^{S,1}$ and $D_1^{S,2}$. What changes is the sample space for $D_1^S$ which contains for the case of no correlation the events 0, 1 and 2 with corresponding probabilities $(1-p)^2, 2p(1-p)$ and $p^2$, for the case of perfect correlation it just contains 0 and 2 with the corresponding probabilities $(1-p)$ and $p$.

Table 3: Parameters for examining effects of changing allocation schemes

| $\sigma_{A,1}$ | $\sigma_{A,2}$ | $\sigma_C$ | $\lambda$ | $a$ | $b$ | $C_0$ | $r$ | $\rho_L^1$ | $\rho_L^2$ | $\tau$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 20% | 20% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 35% |

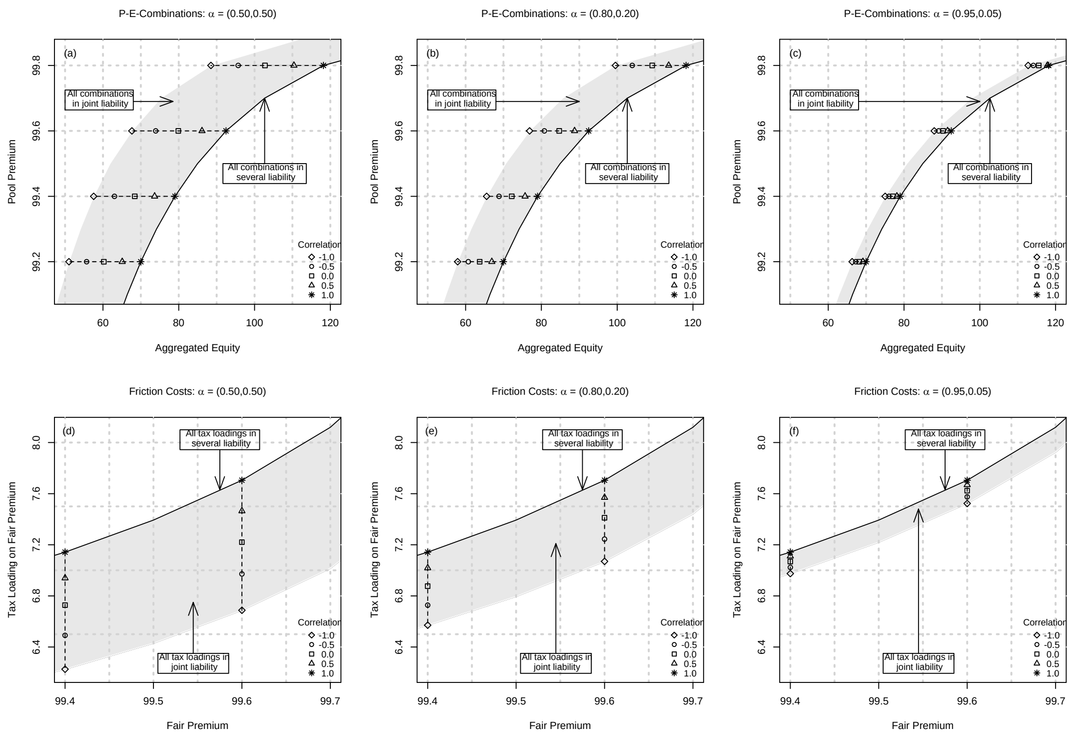


Figure 3: The plots (a) - (c) correspond to the left hand plot of Figure 2 under varying allocation schemes which are $\alpha = (0.5, 0.5), (0.8, 0.2)$ and $(0.95, 0.05)$. For these allocation schemes the plots (d) - (e) corresponds to the right hand plot of Figure 2.

The effects from asset correlation on friction costs are shown in the right hand plot of Figure 2 which combines the fair tax-free premium on the horizontal axes with the corresponding tax loading on the vertical axes. Equivalent to the premium-equity-combinations we see that the combination of premium and tax-loading depends on the asset correlation in a regime of joint liability. For a given tax-free premium of, e.g. 99.5 the tax loading ranges between 6.43 and 7.39 where the tax loading is increasing in $\rho_A^{12}$. Consequently, a predefined safety level will require a higher tax loading from the policyholder if the correlation of the insurers' assets is large. On the contrary the only dimension with an impact on the tax loading in a regime of several liability is that of the safety level whereas the asset correlation itself is non-relevant for the size of these friction costs.

## Changing Allocation Schemes

In Figure 3 we research the effects on the premium-equity-combinations and friction costs of Figure 2 if the allocation schemes, i.e. the balance of the risk sharing, changes. Figure 3(a) - 3(c) shows that the gray plane highlighting the potential range of deviation between both liability regimes will narrow if the risk sharing within the pool becomes more unbalanced between both insurers. For a fixed amount of aggregated equity and more unbalanced risk sharing the fair premium for a regime of joint liability decreases which means that the safety level of the contract declines. With the default risk of the policyholder in mind a more balanced risk sharing between commensurate insurance companies would be preferable in a regime of joint liability as the potential insolvency costs of one insurer that have to be resumed by the other insurer would be in the right proportion. By having an unbalanced risk sharing, on the contrary, the default of the larger company generates insolvency costs which presumably overstrain the smaller insurer's capacities so that part of these costs redounds upon the policyholder. Correspondingly, this justifies a difference in the fair premium between a balanced and an unbalanced insurance pool within a regime of joint liability.

This distinction is not recognizable for the regime of several liability where the safety level will not change for varying risk sharing if the aggregated equity is kept fixed. Without any element that ensures reciprocal compensation among insurers for bankruptcy a risk-neutral pricing framework does not value any effects from multiple risk sharing.

Overall, the regime of several liability can be viewed as limit case for joint liability as $\alpha$ goes towards $(1.0, 0.0)$. This converging property of both regimes is also reflected in the friction costs where the advantage of lower friction costs in a regime of joint liability by virtue of lower equity amounts entirely vanishes as $\alpha$ goes towards $(1.0, 0.0)$ (cf. Figure 3(d) - 3(f)).

## Policy Implications

TIRSA (2010) commissions its members to charge an extra rate (1 USD per 1000 USD sum insured) if the liability regime of co-insurance switches from several liability to joint liability. Our simulations applied to insurance pools as a special form of co-insurance confirm that price differences in a competitive market are indeed justified as the policyholder's insolvency costs that are measured by the pool DPO $\Pi_D^\star$ are lower in a regime of joint liability than in a regime of several liability. Given that the insurers' individual equity is fixed at 50 and assumed that there is no correlation between assets Table 4 exemplifies that a regime of joint liability justifies an extra charge of 0.1. If tax loading is added the extra charge will yet increase to 0.13.

However, if safety requirements come to the fore, for instance by regulators who limit the feasible insolvency costs for the policyholder, this will result in different capital requirements for the different liability regimes. Whilst $\Pi_D^J = 0.22$ needs an individual equity of 50 per insurer, the same DPO value for a regime of several liability requires some equity per insurer between 55 and 60 (see Table 4). If friction costs play a role the regime of several liability becomes the more expensive alternative for the policyholder.

Price differences between both regimes are driven by the correlation of the insurer's asset portfolios. If correlation increases, justified price differences between both regimes will have to shrink. Table 4 shows that a positive correlation of $\rho_A^{12} = 0.5$ will halve the difference between $\Pi_P^J$ and $\Pi_P^S$ if both insurers' equity initially amounts to 50. Analogously, the spread of the tax adjusted premiums $\Pi_{\tilde{P}}^J$ and $\Pi_{\tilde{P}}^S$ will close if the correlation becomes large. Hence, policyholder should be aware in practice that any price spreads between both regimes as it is outlined by TIRSA (2010) may become obsolete if the involved insurers possess high positive correlation in their investments.

Moreover, it can be pointed out by the simulation results that any argued price differences between both regimes should be regarded with suspicion if the risk is unequally shared among the insurers. For instance, if a small and a large insurer bear the risk of the pool in proportion to their size the mechanism of joint liability diminishes. Consequently, a fixed price difference between both liability regimes would be unjustified if the pool's composition and allocation scheme can gradually vary from more to less balanced risk sharing.

Table 4: Further simulations based on the parameters of Table 1 under an assumed allocation scheme of $\alpha = (0.5, 0.5)$

| $E_0^{\star,i}(i=1,2)$ | | 30.00 | 35.00 | 40.00 | 45.00 | 50.00 | 55.00 | 60.00 | 65.00 | 70.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\rho_A^{12} = 0.0$ | | | | |
| $\Pi_D^\star$ | J | 0.81 | 0.56 | 0.40 | 0.29 | **0.22** | 0.17 | 0.13 | 0.10 | 0.09 |
| | S | 1.12 | 0.80 | 0.58 | 0.43 | 0.32 | **0.25** | **0.19** | 0.15 | 0.12 |
| $\Pi_P^\star$ | J | 99.19 | 99.44 | 99.60 | 99.71 | **99.78** | 99.83 | 99.87 | 99.89 | 99.91 |
| | S | 98.88 | 99.20 | 99.42 | 99.57 | **99.68** | 99.75 | 99.81 | 99.85 | 99.88 |
| $\Pi_T^\star$ | J | 6.35 | 6.80 | 7.22 | 7.63 | 8.03 | 8.43 | 8.83 | 9.21 | 9.61 |
| | S | 6.29 | 6.75 | 7.18 | 7.60 | 8.01 | 8.41 | 8.81 | 9.20 | 9.60 |
| $\Pi_{\widetilde{P}}^\star$ | J | 105.54 | 106.23 | 106.82 | 107.34 | **107.82** | 108.27 | 108.70 | 109.11 | 109.52 |
| | S | 105.17 | 105.95 | 106.60 | 107.17 | **107.69** | **108.16** | **108.62** | 109.05 | 109.48 |
| | | | | | | $\rho_A^{12} = 0.5$ | | | | |
| $\Pi_D^\star$ | J | 0.95 | 0.68 | 0.49 | 0.36 | 0.27 | 0.20 | 0.16 | 0.13 | 0.10 |
| | S | 1.12 | 0.80 | 0.58 | 0.43 | 0.32 | 0.25 | 0.19 | 0.15 | 0.12 |
| $\Pi_P^\star$ | J | 99.05 | 99.32 | 99.51 | 99.64 | **99.73** | 99.80 | 99.84 | 99.87 | 99.90 |
| | S | 98.88 | 99.20 | 99.42 | 99.57 | **99.68** | 99.75 | 99.81 | 99.85 | 99.88 |
| $\Pi_T^\star$ | J | 6.33 | 6.78 | 7.21 | 7.62 | 8.03 | 8.42 | 8.82 | 9.21 | 9.60 |
| | S | 6.29 | 6.75 | 7.18 | 7.60 | 8.01 | 8.41 | 8.81 | 9.20 | 9.60 |
| $\Pi_{\widetilde{P}}^\star$ | J | 105.38 | 106.10 | 106.72 | 107.26 | **107.76** | 108.22 | 108.66 | 109.08 | 109.50 |
| | S | 105.17 | 105.95 | 106.60 | 107.17 | **107.69** | 108.16 | 108.62 | 109.05 | 109.48 |

## Risk Shifting under Information Asymmetry

The present-value-calculations in the previous sections assumed a known asset volatility for both insurers. In bilateral contracts between a single insurer (organized as stock company) and a single policyholder it is essential for the purposes of fair pricing that the policyholder is informed about the insurer's intended investment risk. If the policyholder is interpreted as bondholder this will become reproducible by Jensen and Meckling (1976) who points out that investments being riskier than assumed at the date of pricing are beneficial for the equity holders at the expense of bondholders by reason of the equity holders' limited liability. In an insurance pool without own funds every participating insurer can decide its investment strategy itself, yet the decision on investment risk does not only affect the policyholder but potentially the co-insurers as well. The following numerical example is ought to illustrate that those effects are regime depending.

We therefore assume that equity holders and the policyholder base their investment decisions as well as pricing at time $t = 0$ on the values of Table 1 under a balanced risk and premium sharing, i.e. $\alpha = \beta = (0.5, 0.5)$, where correlation $\rho_A^{12}$ can vary between -1 and 1. In particular, all stakeholders suppose an uniform asset volatility of 20 percent for both insurers. However, it is supposed that insurer 2 will have the possibility to shift its assets into more risky investments which would result in an effective asset volatility of $\sigma_{A,2} = 35$ percent.

A deviation between effective asset volatility and the asset volatility assumed for pricing will be merely possible if the market features some degree of information asymmetry between equity holders of the different insurers and the policyholder. In our numerical example this might be conceivable if both, insurer 1 as well as the policyholder, refer to historical parameter estimates for the sake of pricing but are

Table 5: Ex-ante parameters are used for deriving $E_0^{\star,i}$, ex-post parameters apply after risk shifting

| | $\sigma_{A,1}$ | $\sigma_{A,2}$ | $\sigma_C$ | $\lambda$ | $a$ | $b$ | $C_0$ | $r$ | $\rho_L^2$ | $\rho_L^2$ | $\rho_A^{12}$ | $P^\star$ | $\alpha_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ex-ante | 20% | 20% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |
| ex-post (single) | 20% | 35% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |
| ex-post (double) | 35% | 35% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |

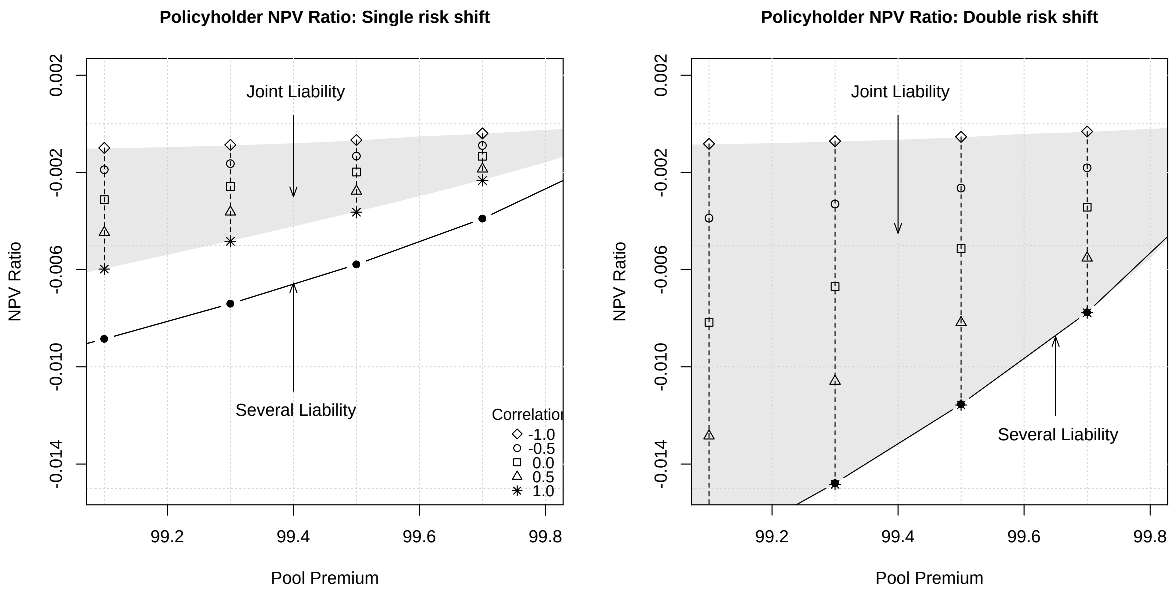


Figure 4: Illustration of $\nu_P^\star$ owing to risk shifting. The left hand plot shows the situation after an one-sided risk shift of insurer 2. The right hand plot corresponds to a simultaneous risk shift of both insurers.

not informed about the prospective intention and possibility of insurer 2 to utilize more risky investments. For different premium levels the adopted parameters of Table 1 result in triples $(E_0^{\star,1}, E_0^{\star,2}, P_0^\star)$ which depend on the assumed liability regime and asset correlation. If insurer 2 choose the riskier investments, however, the present value triple $(\Pi_E^{\star,1}, \Pi_E^{\star,2}, \Pi_P^\star)$ will in general not be equal to $(E_0^{\star,1}, E_0^{\star,2}, P_0^\star)$ indicating non-zero net present values. This presumed setting will thus generate a market disequilibrium which appears as discrepancy between the policyholder's and equity holders' positions at time $t = 0$ and the corresponding payoffs at time $t = 1$.[17]

We measure these discrepancies for the different stakeholders by the net present value ratios (NPV ratios) $\nu_E^{\star,i}$ and $\nu_P^\star$ which we define for different liability regimes as

$$\nu_E^{\star,i} := \frac{\Pi_E^{\star,i} - E_0^{\star,i}}{E_0^{\star,i}}$$

[17] Like in footnote 14 we argue that the policyholder is not or only partially able to replicate payoffs. Otherwise the policyholder can handle the problem of risk shifting by skipping insurance and replicating payoffs itself. Moreover, insurance coverage might be compulsory like the third party liability insurance for nuclear risk so that, though the opportunity of risk shifting exists, the policyholder cannot get around dealing with insurance pools.

Table 6: Ex-ante parameters are used for deriving $E_0^{\star,i}$, ex-post parameters apply after risk shifting

| | $\sigma_{A,1}$ | $\sigma_{A,2}$ | $\sigma_C$ | $\lambda$ | $a$ | $b$ | $C_0$ | $r$ | $\rho_L^2$ | $\rho_L^2$ | $\rho_A^{12}$ | $P^\star$ | $\alpha_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ex-ante | 20% | 20% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |
| ex-post (single) | 20% | 35% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |
| ex-post (double) | 35% | 35% | 10% | 0.1 | 0.4 | 0.1 | 100 | 3% | 0.0 | 0.0 | 0.0 | 99.5 | 50% |

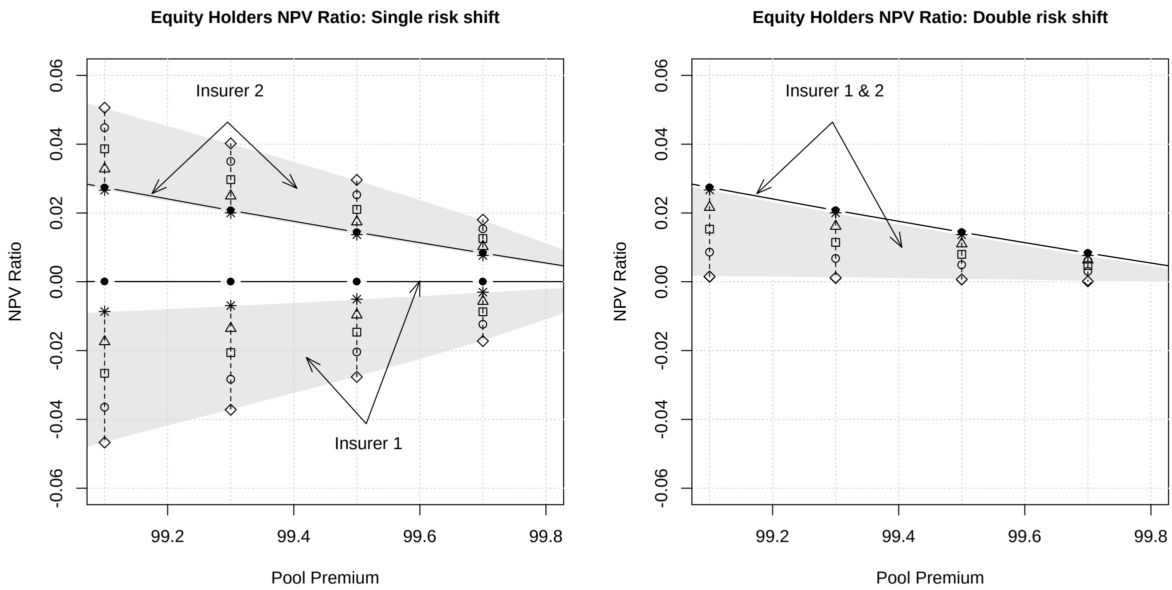


Figure 5: Illustration of $\nu_{E_1}^\star$ and $\nu_{E_2}^\star$ owing to risk shifting. The left hand plot shows the situation after an one-sided risk shift of insurer 2. The right hand plot corresponds to a simultaneous risk shift of both insurers.

for both insurers $i = 1, 2$ and as

$$\nu_P^\star := \frac{\Pi_P^\star - P_0^\star}{P_0^\star}$$

for the policyholder. Given this definition a negative value of $\nu_P^\star$ means that the policyholder's paid premium is above the present value of the payoff from the contract – an unfair situation for the policyholder by virtue of risk-neutral pricing. Analogously, the equity holders' investments at time $t = 0$ as contribution to insurer $i$ possess a risk-inadequate return if $\nu_E^{\star,i} < 0$.

Figure 4 depicts on the left hand side the behavior of $\nu_P^\star$ as function of $P_0^\star$ where it is distinguished between both liability regimes and different asset correlations. We observe that for all considered cases the ratio $\nu_P^\star$ is throughout below zero signalizing that the policyholder always suffers a negative net present value from the risk shift of insurer 2. Like in the previous section we observe that correlation in a regime of joint liability influence the outcome. For a fixed value of $P_0^J$ the value of $\nu_P^J$ varies on a range which is delimited by perfect negative and positive correlation. As opposed to that for the regime of several liability the ratio $\nu_P^S$ is independent of the asset correlation. Moreover, we deduce from the plot that $0 > \nu_P^J > \nu_P^S$, i.e. the policyholder is always more adversely affected from risk shifting of insurer 2 in a regime of several liability than in a regime of joint liability.

Whilst the policyholder always suffers from risk shifting insurer 2 is the benefiting party. The left hand plot of Figure 5 shows consistently that $\nu_E^{\star,2} > 0$ . At the same time the plot points out that for the equity holders of insurer 1 there is only an effect from risk shifting in a regime of joint liability. For any pool premium $P_0^S$ in a regime of several liability the value of $\nu_E^{S,1}$ remains at 0. Hence, risk shifting of insurer 2 is not affecting the originally zero net present value of insurer 1 in the pool. This does not hold true for joint liability for which we have for all pool premiums and asset correlations $\nu_E^{J,1} < 0$. Apparently, the expenses of a positive net present value of insurer 2 is shared in a regime of joint liability between both the policyholder as well as the equity holders of insurer 1. Contrarily, in a regime of several liability the policyholder has to bear the risk shift of insurer 2 exclusively.

Furthermore the policyholder's position in a regime of several liability seems to be more vulnerable to risk shifting than in a regime of joint liability. Even for a prefect positive correlation the NPV ratio $\nu_P^J$ is at least 30 percent greater than $\nu_P^S$. The deflated negative NPV ratio in a regime of joint liability is due to the burden of insurer 1 to be liable for insurer 2 if its riskier investments result in a negative outcome. Thus the regime of joint liability constitutes at first a mechanism protecting the policyholder against too adverse effects from risk shifting of individual insurers.

However, if the the riskier investment is attainable for both insurers the interdependencies in a regime of joint liability will push both insurers into the riskier investments – in fact not only to exploit the asymmetric advantages from limited liability but also to immunize itself against a negative net present value from the possibility of risk shifting in the other insurer's asset portfolio. Given that the risky investment with a volatility of 35 percent were also attainable for insurer 1 we deduce from the right hand plot of Figure 5 that the NPV ratio turns out to be positive for insurer 1 and 2 if both access the risky investment. The right hand plot of Figure 4 in turn illustrates that this incentive for a simultaneous risk shifting intensifies the adverse effects on the policyholder's position.

## Summary

In this article, insurance pools as a special vehicle for risk coverage solutions have been modeled where the model distinguishes between both liability regimes that regulate the bankruptcy of one or more pool insurers. The regime of joint liability as one of both regimes requests financial solidarity among the insurers. Admittedly, this solidarity is an enhancing feature for the policyholder so that premium differences between both liability regimes are justified. Our model confirms those premium differences in a numerical example with two insurers. If both insurers have a fixed equity available the regime of joint liability justifies a higher premium due to implicitly lower insolvency costs for the policyholder. Inversely, if expected insolvency costs that are measured by the default put option are fixed, for instance due to regulatory requirements, the regime of joint liability takes for fairness by means of zero net present values lower equity by insurers. The resulting equity saving gets an economic pertinence if capital commitment becomes costly. By incorporating taxes as friction costs into our model the equity saving in a regime of joint liability proves to reduce costs for the policyholder.

Premium differences between both regimes are, however, not fixed but can vary with respect to some specific attributes. In this article, asset correlation and the balance of risk sharing have been outlined in the numerical example as factors that can potentially shrink premium differences. An increased correlation between the insurers' assets causes less effectiveness of the bankruptcy mechanism in a regime of joint liability and thus results in less justified premium differences between both regimes. Furthermore a pool that is composed of unbalanced large and small insurers likewise necessitates a reduced premium difference between both regimes. In this way, the article provides insights about factors determining price differences which may be useful if the liability regime becomes subject of negotiations or is utilized for arguing pretendedly justified price differences.

Finally, the numerical example considered the effects on the stakeholders' positions if informations asymmetries enable risk shifting in the insurers' asset portfolios. Apparently, the regime of joint liability protects the policyholder compared to the regime of several liability from too adverse effects if a single insurer decides to assume riskier assets after pricing has already been conducted under deviating presumptions. In this case, the policyholder shares the adverse effects from risk shifting with pool insurers that abandon risk shifting and thus suffer negative net present values since their guarantee to cover the insolvency risk of the risk shifting insurer is not adequately compensated any more. As opposed to that in a regime of several liability a single risk shift solely affects the policyholder whereas the remaining insurers stay unaffected. On the other hand, a regime of joint liability may force all insurers to conduct a risk shifting in order to hedge themselves against the adverse effects from single risk shifting. In this manner joint liability would intensify the adverse effects from risk shifting on the policyholder's position.

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