Escaping the Guarantee Trap

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Abstract

We develop an asset-liability model of a stylized German life insurance company to study the performance of participating endowment life insurance policies (characterized by a fixed minimum guarantee and a surplus participation) in different interest rate environments. Our study highlights the extraordinary financial risk that these policies pose on German and many European life insurance companies due to the deferred adjustment of their guarantee levels to different interest rate environments; a phenomenon that we refer to as the guarantee trap. To reduce interest rate dependency and to increase the financial stability of life insurers in the long-run, we investigate a more flexible guarantee mechanism with an adaptable shorter-term (temporary) guarantee. We perform a market consistent valuation of fixed and temporary guarantee products, identify key drivers of default risk and analyze the payout to both policyholders and shareholders. Furthermore, the model design allows us to study the interaction of existing policies with newly sold products with either fixed or temporary guarantees. Our results demonstrate that a temporary guarantee successfully reduces the financial risk associated with traditional fixed interest rate guarantees since the insurer's duration gap diminishes. A welfare analysis shows that both policyholders and shareholders benefit from temporary guarantees.

Keywords: Participating Endowment Life Insurance, Interest Rate Guarantee, Temporary Guarantee, Hull-White model, Product Design, Solvency II

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1 Introduction

In many European jurisdictions such as Germany, Switzerland and Italy, one of the most important and widespread pension products is the endowment life insurance policy, which includes a year-by-year guarantee (so-called cliquet-style guarantee) and a (yearly) profit participation (cf. Eling and Holder (2013a)).[1] While these products typically exhibit a long contract life of 30 years or more, the financial market does not supply enough investment opportunities with similar maturities.[2] As a result, life insurance companies selling these guarantees are not able to perfectly match the duration of their asset and liability side. In the literature, this effect is commonly referred to as duration gap.[3]

At the inception of an endowment life insurance contract, the (maximum) guaranteed rate is set by the regulator and depends on the presently achievable interest rates.[4] However, a contract's guarantee typically does not change until its maturity. Consequently, life insurers have to cope with the same guarantee level in different interest rate environments. We refer to this phenomenon as the *guarantee trap*.

Whereas for high interest rates policyholders with lower guarantee contracts still benefit from high returns via profit participation, the current product design does not offer a balancing mechanism for times of low interest rates. Instead, the insurer is forced to provide the (still high) guarantee, thus, putting both policyholders and shareholders at risk.

In general, life insurers can cope with the latter situation by adjusting their assets and/or liabilities. However, the upcoming regulatory framework Solvency II makes alternatives on the asset side, such as high yielding bonds or stocks, less palatable. Specifically, to be able to afford a more risky asset allocation, insurers would have to increase their equity capital base. However, it is rather unlikely to find investors willing to provide equity capital that is (at least in the short-run) mainly used to pay already existing guarantees. Moreover, raising equity capital does not seem to be a long-term solution to the guarantee trap phe-

[1] An endowment policy is a life insurance contract designed to pay a lump sum after a specific term (on its 'maturity'). In Germany, more than 50% of all premiums were to endowment life insurance and pension insurance. Cf. German Federal Financial Supervisory Authority (BaFin).

[2] The average maturity of all German government bonds in 2010 was 7.4 years. Source: Eurostat.

[3] See Bank for International Settlements (2011).

[4] In Germany, Austria and Switzerland the guarantee is based on 60% of the 10-year moving average of the 10 year sovereign bond, cf. Eling and Holder (2013a, p. 5).

nomenon, which is essentially caused by the nature of the product design. Therefore, to increase the financial stability of life insurers in the long-run, it will be crucial to change the fundamental product characteristics of (future) endowment life insurance policies.

Briys and de Varenne (1997) were among the first to analyze the impact of guaranteed interest rates on life insurers' default risk. The financial risk of traditional products has further been investigated by Grosen and Jorgensen (2000), who provide a comprehensive discussion on different contractual features of participating policies.

As Kling et al. (2007, p. 177) conclude, cliquet-style guarantees in connection with high minimum participation rates and a market value oriented accounting (as required by Solvency II) "*pose an unmanageable risk to insurers*". Hence, a more flexible guarantee mechanism that abstains from long-dated fixed guarantees in favor of adaptable shorter-term guarantees in combination with profit participation could provide a way out of the guarantee trap. On the one hand, policyholders would still benefit from high interest rates. On the other hand, life insurers would no longer suffer from interest rate changes since the guarantee would adjust accordingly and the duration gap diminishes. For example, with decreasing interest rates, a downward adjustment of the guarantee would benefit policyholders since a lower default risk reduces the volatility of their payout. This rationale is also supported by the results of Bohnert et al. (2014), who show that a higher guarantee does not necessarily imply a higher policyholder benefit.

Alternative guarantee designs have been analyzed by Eling and Holder (2013b) as well. The authors study different guarantee levels from the policyholder perspective. Among others, they analyze a temporary fixed guarantee that changes to capital preservation (0% guarantee) during the contract term. They are able to calibrate the analyzed products such that from the policyholders' perspective there is no substantial difference between different guarantee designs in terms of expected utility.

In addition, Reuß et al. (2013) study alternative product designs as well as their impact on life insurers' financial situation. The authors demonstrate how life insurers can reduce their financial risk by selling products with guarantees only valid at contract maturity, i.e. without year-to-year guarantees.

In contrast to many recent studies that focus solely on the effects the current low interest rates pose on European life insurers that sold high guarantees in the past,[5] we investigate a long-term oriented forward-looking product design in form of an adjustable temporary guarantee. Contrary to Reuß et al. (2013), we will maintain the concept of (positive) year-to-year guarantees due to the large demand by policyholders in the past and study a participating endowment life insurance contract with a temporary guaranteed rate of return that adjusts according to the recent development of an interest rate benchmark. This flexible guarantee can mimic the development of life insurers' fixed income portfolios. In other words, selling the temporary guarantee strengthens the co-movement of an insurer's cash flows from the asset and (guaranteed) liability side.

To study the performance of this innovative product design properly, it is vital to incorporate stochastic interest rates, which is of particular relevance for life insurers.[6] For this purpose, many contributions utilize the Cox-Ingersoll-Ross (CIR) model (cf. Reuß et al. (2013) and Eling and Holder (2013b)) or the Vasicek model (cf. Charlier and Kleynen (2005)).[7] However, both models presume a constant mean reverting short-rate level. Thus, in the long-run interest rates become stationary and variation in a temporary guarantee level would only result from the interest rate volatility. In contrast, we intend to analyze the product in different interest rate environments, i.e. different evolutions of the mean interest rates over time. Therefore, we construct a financial market framework that is based on the Hull-White model (see Hull and White (1990)) to generate the following four distinctive interest rate environments: (a) declining (b) rising, (c) u-shaped and (d) hump-shaped interest rates.

Our analysis follows a two-step approach. First, we focus on the German insurance market and develop an asset-liability-model of a stylized German life insurance company. We

[5] See Berdin and Gründl (2015) and Kablau and Weiß (2014).
[6] See Bundesanstalt für Finanzdienstleistungsaufsicht (BaFin) (2011).
[7] An overview of different interest rate models, including the CIR and Vasicek model, and their impact on the Solvency Capital Requirement under Solvency II is given by Martin (2013).

then obtain the behavior of the company's balance sheet within a multi-period setting for two different product designs (traditional and innovative, i.e. fixed and temporary guarantees) within four different interest rate environments. Based on current data from German life insurance companies, we incorporate an existing contract portfolio with traditional endowment life insurance contracts for the insurer's liability side and study the interaction with newly sold products with either fixed or temporary guarantees. In a second step, we will conduct a welfare analysis to study the effects on policyholders' and shareholders' welfare.

The remainder of the paper is organized as follows: Section 2 introduces our model framework as well as the different product designs. In Section 3, we describe the data and calibration adopted. Section 4 studies the interaction effects, the insurer's associated financial risk, and, subsequently, presents a comprehensive welfare analysis from the policyholder's as well as from the shareholder's point of view. Moreover, we assess the robustness of our results. Section 5 concludes.

2 The Model

2.1 The Financial Market Model

The term structure of risk-free interest rates serves as the main driver for the return on securities in the market.[8] Therefore, we employ the Hull-White-short-rate model (also known as Extended Vasicek model, cf. Hull and White (1990)).[9] Then, the short-rate dynamics are given by

$$dr(t) = \alpha_r(\theta(t) - r(t))dt + \sigma_r dW_r(t) \quad \text{with } r(0) = r_0, \tag{1}$$

where $W_r(t)$ is a standard Brownian motion under the real world measure $\mathbb{P}$, $r(t)$ is the instantaneous (short) interest rate at time t, $\alpha_r > 0$ is the speed of reversion, $\sigma_r > 0$ the

[8] A similar approach is used in Maurer et al. (2013).

[9] Although it is usually assumed that short-rates are strictly positive, in reality we can observe negative interbank lending rates (for example EURIBOR in April 2015, source: Datastream). Thus, allowing for negative short-rates by employing the Hull-White-short-rate model is appropriate. However, in our calibration (see Section 3) negative rates are unlikely to occur.

volatility and $\theta(t)$ the (non-constant) level of mean reversion. The stochastic differential equation (1) can be solved, which yields (cf. Brigo and Mercurio (2006))

$$r(t) = r_0 e^{-\alpha_r t} + \alpha_r \int_0^t e^{-\alpha_r (t-u)} \theta(u)\, du + \sigma_r \int_0^t e^{-\alpha_r (t-u)}\, dW_r(u). \tag{2}$$

For the mean reversion level $\theta(t)$, we provide deterministic functions, which set up the interest rate environment:

(a)/(b) *Rising/Declining interest rates:*

$$\theta(t) = f(t) := \gamma + (\beta - \gamma) \left(1 - \frac{1}{1 + e^{-b(t-h)}} \right), \quad \beta, \gamma, b, h > 0.$$

In this setting, b serves as skewness and h as shift parameter. The other parameters can be interpreted as follows:

$$\beta = \lim_{t \to -\infty} f(t) \approx f(0) \quad \text{(mean reversion level in } t = 0\text{)},$$
$$\gamma = \lim_{t \to \infty} f(t) \approx f(T) \quad \text{(mean reversion level in } t = T\text{)}.$$

(c)/(d) *Hump/U-shaped interest rates:*

$$\theta(t) = g(t) := \gamma - a \exp\left(-\frac{(t-c)^2}{2b^2} \right) + a \exp\left(-\frac{c^2}{2b^2} \right), \quad \gamma, c, b > 0,\ a \in \mathbb{R}.$$

It follows that:

$$g(0) = \gamma - a \exp\left(-\frac{c^2}{2b^2} \right) + a \exp\left(-\frac{c^2}{2b^2} \right) = \gamma,$$
$$g(2c) = \gamma - a \exp\left(-\frac{(2c-c)^2}{2b^2} \right) + a \exp\left(-\frac{c^2}{2b^2} \right) = \gamma,$$
$$\text{Minimum/Maximum:} \quad g(c) = \gamma - a + a \exp\left(-\frac{c^2}{2b^2} \right) =: \nu(a).$$

Hence, γ is the short-rate in $t = 0$ and $t = 2c$, c the minimum/maximum point, $\nu(a)$ the minimum/maximum mean reversion level in $t = c$ and b a skewness parameter.

In this interest-rate model, the price $P(t,\tau)$ of a zero-coupon bond at time t with time to maturity τ is given by (cf. Hull and White (1990) and Brigo and Mercurio (2006))

$$P(t,t+\tau) = A(t,t+\tau)e^{-r(t)\,B(\tau)}, \tag{3}$$

where

$$\begin{aligned} & B(\tau) = \frac{1-e^{-\alpha_r \tau}}{\alpha_r} \\ \text{and} \quad & A(t,t+\tau) = \exp\left(\frac{\sigma_r^2}{2\alpha_r^2}\left(\tau - B(\tau)\right) - \frac{\sigma_r^2}{4\alpha_r}B^2(\tau) - \alpha_r \int_t^{t+\tau} \theta(u)B(t+\tau-u)\,du\right). \end{aligned}$$

Hence, the continuously compounded spot rate at time t for time to maturity τ is given by

$$\hat{r}_{f,\tau}(t) = -\frac{1}{\tau - t}\log P(t,t+\tau) = \frac{B(\tau)r(t) - \log A(t,t+\tau)}{\tau - t} \tag{4}$$

and the equivalent annually-compounded spot rate is given by

$$r_{f,\tau}(t) = e^{\hat{r}_{f,\tau}(t)} - 1 = \left(\frac{e^{B(\tau)\,r(t)}}{A(t,t+\tau)}\right)^{1/(\tau-t)} - 1. \tag{5}$$

As stock prices will be influenced by the short-rate, we follow Deelstra et al. (2003) and model the stock dynamics in the following way: The instantaneous drift at time t is given by the risk-free short-rate $r(t)$ modified by a market price of risk λ_S and the diffusion terms of stock and short-rate process are correlated:

$$\frac{dS(t)}{S(t)} = \left(r(t) + \sigma_S \lambda_S\right)dt + \sigma_S dW_S(t) + \rho dW_r(t), \quad \text{with } S(0) = S_0, \tag{6}$$

where $W_S(t)$ is a standard Brownian motion under the real world measure $\mathbb{P}$.

2.2 The Asset Side

We consider a stylized German life insurance company investing its available funds into risk-free and risky assets, which we generally refer to as *bonds* and *stocks*, respectively. To identify the major effects of product design and interest rate environment, we construct our model as simply as possible.

At time $t = 0$, shareholders equip the insurance company with equity capital E_0. The policyholder account L_0 is initialized with the payment of the insurance premiums L_0. Note that L_0 also includes the sum of existing policyholder accounts at time $t = 0$. Thus, the insurer's assets are

$$A_0 = E_0 + L_0. \tag{7}$$

The insurer's asset side consists of risk-free investments (*bonds*) and risky investments (*stocks*). A fixed-coupon bond $B_{t_0,J}$ with maturity date $t_0 + J$ and face value $FV^{(B_{t_0,J})}$ that is purchased and issued at time t_0 will incorporate a yearly coupon rate

$$R^{(B_{t_0,J})} = r_{f,J}(t_0).$$

Hence, at time $t \in [t_0, t_0 + J]$ the market value of $B_{t_0,J}$ is given by

$$MV_t(B_{t_0,J}) = FV^{(B_{t_0,J})} \left(\frac{1}{(1 + r_{f,J+t_0-t}(t))^{J+t_0-t}} + \sum_{k=1}^{J+t_0-t} \frac{r_{f,J}(t_0)}{(1 + r_{f,k}(t))^k} \right), \tag{8}$$

where $r_{f,J}(t)$ is the annually compounded risk-free spot rate at time t with time to maturity J as it is introduced in Section 2.1.

According to the German Insurance Association, in 2013 German life insurers obtained on average the following investment portfolio: 88.7% bonds, 5.7% shares[10], 3.9% real-estate-related assets and 1.8% other investment assets.[11] Since we only consider two types of assets, we use this report as an estimate for the following three static portfolio strategies:

Portfolio No.	**Proportion of Stocks**
1	2.5%
2	5.0%
3	7.5%

Table 1: Investment portfolio.

[10] This includes equities and other shareholdings.
[11] See Gesamtverband der Deutschen Versicherungswirtschaft (2014, p. 15).

In our framework, the insurer's bond portfolio consists at all times $t \in \{0, ..., T\}$ of J fixed-coupon bonds with maturity dates $t+1, t+2, ..., t+J$. In particular, all past and future bonds exhibit a time to maturity of J years at their purchasing date. Furthermore, we assume that all bonds are held until maturity. At the beginning of each year $t \in \{1, ..., T\}$, the insurer receives coupon payments and one principal payment $FV^{(B_{t-J,J})}$ from the maturing bond. Additionally, in the case of a revolving liability portfolio the insurer sells one additional contract and, thus, receives the up-front premium P_0 and has to pay out the book-value $BV_t\left(C^{x,t_0,t}\right)$ of the maturing contract. The insurer's free cash flow (FCF) is then given by

$$\begin{aligned} FCF_{t-} &= P_0 - BV_t\left(C^{x,t_0,t}\right) + FV^{(B_{t-J,J})} + \sum_{k=1}^{J} FV^{(B_{t-k,J})} R^{(B_{t-k,J})} \\ &= P_0 - BV_t\left(C^{x,t_0,t}\right) + FV^{(B_{t-J,J})} + \sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k). \end{aligned} \tag{9}$$

Note that FCF_{t-} denotes the insurer's realized cash flow at time t prior to the portfolio reallocation which takes place directly after FCF_{t-} is received. If the free cash flow is positive, the insurer uses FCF_{t-} to immediately invest into a new bond $B_{t,J}$ with face value $FV^{(B_{t,J})}$ and to buy or sell stocks. In its investment strategy, the insurer aims for a constant proportion $\alpha^{(i)}$ of stocks relative to the total book value of assets, A_t, where $i = 1, 2, 3$ is the respective target stock ratio as reported in Table 1. Given the stock price $S(t)$, the insurer decides on the number of stocks Δ_t to buy or sell at time t. Hence, the insurer solves the following linear equations with respect to Δ_t and $FV^{(B_t,J)}$, the face value of the bond investment:

$$\overbrace{S(t)\sum_{k=0}^{t}\Delta_k}^{\text{total stock exposure}} = \alpha^{(i)} \overbrace{\left(S(t)\sum_{k=0}^{t}\Delta_k + \sum_{k=1}^{J-1} FV^{(B_{t-k,J})} + FV^{(B_{t,J})}\right)}^{=A_t} \tag{10}$$

and

$$FV^{(B_t,J)} + \Delta_t S(t) = FCF_{t-}. \tag{11}$$

From the budget constraint (11) follows

$$FV^{(B_t,J)} = FCF_{t-} - \Delta_t S(t)$$

which yields

$$S(t)\sum_{k=0}^{t}\Delta_k = \alpha^{(i)}\left(S(t)\sum_{k=0}^{t-1}\Delta_k + \sum_{k=1}^{J-1} FV^{(B_{t-k},J)} + FCF_{t-}\right).$$

Thus, the solutions for $\{(10),(11)\}$ are given by

$$\begin{aligned}\hat{\Delta}_t &= \frac{\alpha^{(i)}}{S(t)}\left(S(t)\sum_{k=0}^{t-1}\Delta_k + \sum_{k=1}^{J-1} FV^{(B_{t-k},J)} + FCF_{t-}\right) - \sum_{k=0}^{t-1}\Delta_k \\ &= \frac{\alpha^{(i)}}{S(t)}\left(\sum_{k=1}^{J-1} FV^{(B_{t-k},J)} + FCF_{t-}\right) - \left(1-\alpha^{(i)}\right)\sum_{k=0}^{t-1}\Delta_k \end{aligned} \tag{12}$$

and

$$\begin{aligned}\widehat{FV}^{B_t,J} &= FCF_{t-} - \frac{\alpha^{(i)}S(t)}{S(t)}\left(\sum_{k=1}^{J-1} FV^{(B_{t-k},J)} + FCF_{t-}\right) + S(t)\left(1-\alpha^{(i)}\right)\sum_{k=0}^{t-1}\Delta_k \\ &= \left(1-\alpha^{(i)}\right)\left(FCF_{t-} + S(t)\sum_{k=0}^{t-1}\Delta_k\right) - \alpha^{(i)}\sum_{k=1}^{J-1} FV^{(B_{t-k},J)}. \end{aligned} \tag{13}$$

However, note that $\hat{\Delta}_t S(t)$ might exceed the free cash flow FCF_{t-} to reach the desired stock proportion. Correspondingly, the insurer would have to "short-sell" the new bond, i.e. $\widehat{FV}^{B_t,J} < 0$. To prevent this case, we constrain $\hat{\Delta}_t S(t)$ by FCF_{t-}. The insurer, thus, accepts a lower stock proportion in favor of keeping the existing bond portfolio. However, for the bond investment it is still permitted to exceed the free cash flow, i.e. $\widehat{FV}^{B_t,J} > FCF_{t-}$. In this case, the insurer has to sell stocks that were bought in previous years, i.e. $\hat{\Delta}_t < 0$. Thus, the (constrained) stock and bond investments are given by

$$\Delta_t = \min\left(\frac{FCF_{t-}}{S(t)},\ \frac{\alpha^{(i)}}{S(t)}\left(\sum_{k=1}^{J-1} FV^{(B_{t-k},J)} + FCF_{t-}\right) - \left(1-\alpha^{(i)}\right)\sum_{k=0}^{t-1}\Delta_k\right) \tag{14}$$

and

$$FV^{B_t,j} = \left(\left(1 - \alpha^{(i)}\right) \left(FCF_{t-} + S(t) \sum_{k=0}^{t-1} \Delta_k \right) - \alpha^{(i)} \sum_{k=1}^{J-1} FV^{(B_{t-k,J})} \right)^+ . \qquad (15)$$

If the free cash flow is negative ($FCF_{t-} < 0$), the insurer first attempts to sell stocks to cover the incurred expenses. If necessary, the insurer then continues to harvest bonds whose market value exceeds the book value up to the amount of FCF_{t-}^+, such that all expenses are being paid, i.e. $|FCF_{t-}| = -\left(FCF_{t-}^+ + S(t) \sum_{k=0}^{t-1} \Delta_t\right)$. This means that the insurer does not buy a new bond. Moreover, the insurer maintains a bond-only strategy in the following year. With positive free cash flows in the future, the insurer then increases its stock proportion back to the target ratio $\alpha^{(i)}$.

As for the starting point $t = 0$, we assume that the stock to asset ratio exactly equals $\alpha^{(i)}$, i.e. $\Delta_0 S(0) = \alpha^{(i)}(L_0 + E_0)$, and the insurer holds J equally weighted bonds with times to maturity $1, ..., J$, i.e. $\sum_{k=0}^{J-1} FV^{(B_{-k,J})} = \left(1 - \alpha^{(i)}\right)(L_0 + E_0)$.

To simplify notations and calculations, we assume that this investment procedure takes place shortly before t, i.e. at $t-$, and that the insurer already incorporates the new bond (if all cash is invested into a stock, we set $FV^{(B_{t_0},J)} = 0$) and the targeted stock proportion at the beginning of each year, i.e. at t. Therefore, at time $t \in \{0, .., T\}$ the total market value of assets is given by

$$MV_t(A_t) = S(t) \sum_{k=0}^{t} \Delta_k + \sum_{j=0}^{J-1} MV_t(B_{t-j,J}). \qquad (16)$$

The return on assets (RoA) between time $t-1$ and t is given by[12]

[12] Due to recent German regulation we do not take any returns from changes of the market value of the bond portfolio into account.

$$
\begin{aligned}
r_t^{Assets} &= \frac{A_t}{A_{t-1}} - 1 \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})}(1 + r_{f,J}(t-k)) + S(t)\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} - 1 \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k) + (S(t) - S(t-1))\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k}. \qquad (17)
\end{aligned}
$$

If funds are sufficient to fulfill the target stock ratio and the resulting face values are approximately equal, the RoA simplifies to:[13]

$$
\begin{aligned}
r_t^{Assets} &= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})}(1 + r_{f,J}(t-k)) + S(t)\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} - 1 \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} + \frac{(S(t) - S(t-1))\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} + \alpha^{(i)}\frac{S(t) - S(t-1)}{S(t-1)} \\
&= \frac{1 - \alpha^{(i)}}{J}\sum_{k=1}^{J} r_{f,J}(t-k) + \alpha^{(i)}\frac{S(t) - S(t-1)}{S(t-1)}. \qquad (18)
\end{aligned}
$$

This simplification allows us to forecast the life insurer's return on assets in future periods without the use of excessive computing capacity, which is necessary to determine the market value of liabilities. In many cases, the latter is replaced by a so-called "best estimate" incorporating only guaranteed payments. However, this "best estimate" only represents a lower bound for the true market value. Therefore, the procedure stated above allows for a substantial improvement for the calculation of market values.

2.3 The Liability Side

In many European countries, endowment life insurance policies incorporate two distinctive features: a yearly ("cliquet-style") minimum guarantee $r^G(t)$ and a yearly surplus mechanism $\phi(t)$. In the following, we consider two types of products: traditional and innovative ones. Both incorporate a surplus participation that is fixed at inception, $\phi(t) \equiv \phi$,

[13] See Appendix A for the derivation.

and the same single upfront premium P_0. However, the two products differ in the guarantee scheme. Note that neither mortality nor surrender risk but only financial risk is considered.

2.3.1 The Traditional Product

Traditional products are defined as endowment life insurance contracts with a yearly minimum guaranteed return that is fixed at the inception of the contract: $r_{t_0}^{G,trad}(t) \equiv r_{t_0}^{G,trad}$. According to the German Directive for the calculation of policy reserves,[14] the maximum yearly investment guarantee $r_{t_0}^{G,trad}$ for a contract that is sold at time t_0 depends on the so-called regulatory interest rate $r_{Z,\zeta}^{G,reg}(t_0)$ at time t_0, which is given by

$$r_{Z,\zeta}^{G,reg}(t_0) = \begin{cases} r_{Z,\zeta}^{G,reg}(t_0-1) - \omega, & \text{if } \zeta \cdot r_Z^{ref}(t_0-1) \leq r_{Z,\zeta}^{G,reg}(t_0-1), \\ r_{Z,\zeta}^{G,reg}(t_0-1) + \omega, & \text{if } \zeta \cdot r_Z^{ref}(t_0-1) \geq r_{Z,\zeta}^{G,reg}(t_0-1) + \omega, \\ r_{Z,\zeta}^{G,reg}(t_0-1) & \text{otherwise,} \end{cases} \tag{19}$$

where $Z = Z^{reg} = 10$ denotes the reference rate moving average, $\zeta = \zeta^{reg} = 0.6$ the reference rate contribution parameter and r_Z^{ref} the reference rate that is essentially defined as the Z-year moving average of the risk-free interest rate with time to maturity Z:

$$r_Z^{ref}(t) = \frac{1}{Z} \sum_{z=0}^{Z-1} r_{f,Z}(t-z). \tag{20}$$

In other words, the regulatory interest rate is adjusted with steps of size ω, if 60% of the 10-year moving average of the 10-year-risk-free interest rate falls below it or exceeds it with at least ω. Consistent with the observed changes in the regulatory interest rate in recent years, we set $\omega = 0.5\%$. Moreover, we assume that a first-mover disadvantage lets insurers set their yearly minimum guarantee equal to $r_{t_0}^{G,trad} = r_{Z^{reg},\zeta^{reg}}^{G,reg}(t_0)$.[15] Finally, to ensure capital preservation, we do not allow for negative guarantees, i.e. $r_{t_0}^{G,trad} = \max\left(r_{Z^{reg},\zeta^{reg}}^{G,reg}(t_0),\, 0\right)$.

[14] According to §2, paragraph (1) of the German Directive for the calculation of policy reserves (Deckungsrückstellungsverordnung - DeckRV), $r_{t_0}^{G,trad}$ is the maximal discount rate for determining the value of the technical provision according to the German GAAP. In the past, this discount rate was used as the investment guarantee rate for life insurance products.

[15] This can easily be observed, for example, in the German life insurance market.

The calculation of the market value $MV_t(C^{(trad,t_0,T_0)})$ at time t for a traditional contract $C_{t_0}^{trad,T_0}$ that is sold at time t_0 and matures at time T_0 is given by the discounted cumulative return under the risk-neutral measure $\mathbb{Q}$:[16]

$$MV_t(C^{(trad,t_0,T_0)}) = \frac{P_0}{(1+r^{\mathbb{Q}}_{f,t_0T_0-t}(t))^{t_0+T_0-t}}\mathbb{E}_t^{\mathbb{Q}}\left[\prod_{m=1}^{T_0-t_0}\left(1+\max\left(r_{t_0}^{G,trad^{\mathbb{Q}}}, \phi\, r^{Assets^{\mathbb{Q}}}(t_0+m)\right)\right)\right]. \tag{21}$$

To perform the risk-neutral valuation, we firstly forecast bond and stock returns in Equation (21) by setting the market price of risk equal to zero (cf. Section 2.1) and, secondly, calculate the return on assets by employing Equation (18) which simplifies the numerical procedure. We are thus able to incorporate future surplus participation benefits into the market valuation of the current liabilities.[17]

2.3.2 The Innovative Product

The innovative product exhibits a flexible temporary guarantee: After every T^{innov} years the guarantee is adjusted to equal $r^{G,reg}_{Z^{innov},\zeta^{innov}}(t)$, i.e. the current guarantee for newly sold contracts. In our analysis we also allow for values $Z^{innov} \neq Z^{reg}$ and $\zeta^{innov} \neq \zeta^{reg}$, i.e. different reference rate moving averages as well as different reference rate contribution parameters, respectively. At contract inception, t_0, the guaranteed rate $r_{t_0}^{G,innov}(1)$ for an innovative contract C^{innov,t_0,T_0} is initialized by

$$r_{t_0}^{G,innov}(1) = r^{G,reg}_{Z^{innov},\zeta^{innov}}(t_0). \tag{22}$$

The guarantee is adjusted after every T^{innov} years and is bounded below by 0%, thus, for every period $k = 0, ..., \lceil T_0/T^{innov} - 1\rceil$ and inner-period development year $z = 0, ..., T^{innov} - 1$ the guarantee is given by

$$r_{t_0}^{G,innov}\left(kT^{innov}+z+1\right) = \max\left(r^{G,reg}_{Z^{innov},\zeta^{innov}}\left(t_0+kT^{innov}\right); 0\right). \tag{23}$$

[16] Cf. e.g.. Grosen and Jorgensen (2000, p.47), who refer to this kind of contract as "European contract".
[17] This approximates the procedure developed in Grosen and Jorgensen (2000, p. 49).

Note that while the guarantee for the traditional product $r_{t_0}^{G,trad}$ is time-independent and known at any time $t \geq t_0$, the guarantee for the innovative product is known only for the next T^{innov} years at every change point $t_0 + kT^{innov}$, and unknown for $t \geq t_0 + (k+1)T^{innov}$. Figure 1 illustrates the temporary guarantee adjustment mechanism for $T^{innov} = 2$.

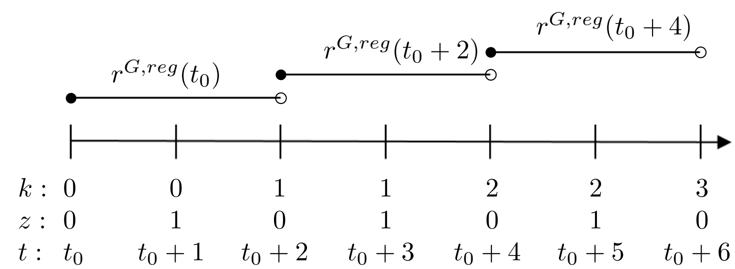


Figure 1: Temporary guarantee adjustment for $T^{innov} = 2$.

Similar to Equation (21), the market value of the innovative product $MV_t(C^{(innov,t_0,T_0)})$ is given by

$$MV_t(C^{(innov,t_0,T_0)}) = \frac{P_0}{(1 + r_{f,t_0+T_0-t}^{\mathbb{Q}}(t))^{t_0+T_0-t}} \mathbb{E}_t^{\mathbb{Q}} \left[\prod_{m=1}^{T_0-t_0} \left(1 + \max\left(r_{t_0}^{G,innov^{\mathbb{Q}}}(m), \phi\, r^{Assets^{\mathbb{Q}}}(t_0 + m)\right)\right) \right]. \tag{24}$$

As for the traditional product, we use the simplified Equation (18) to forecast the insurer's return on assets for the calculation of the market value.

2.3.3 The Liability Structure

At time $t = 0$ the insurer's liability side incorporates M *prior* contracts (all of type *traditional*). The contracts all had the same up-front premium P_0 and times to maturity J^{CMat}. The contracts mature at times $1, ..., M$. Every year $t = 0, ..., T$, the insurer sells one additional contract with up-front premium P_0 and time to maturity J^{CMat}. All newly sold contracts are either of type *traditional* or *innovative*. Thus, at time $M + 1$ all *prior* contracts have matured and only new contracts are in place.

The aggregated market value of the liability side is given by

$$MV_t(L_t) = \underbrace{\sum_{k=t+1}^{M} MV_t\left(C^{\left(trad,k-J^{CMat},k\right)}\right)}_{prior\text{ contracts}} + \underbrace{\sum_{k=0}^{t} MV_t\left(C^{\left(x,k,k+J^{CMat}\right)}\right)}_{new\text{ contracts}}, \tag{25}$$

where $x \in \{trad, innov\}$ determines the type of the *new* contracts.

2.4 Contract Valuation

Following Gatzert et al. (2012), we assume that policyholders have mean-variance preferences (see also Mayers and Smith (1983)). Consequently, the policyholders' certainty equivalent under the real-world measure $\mathbb{P}$ is given by the difference between expected wealth and the variance of the wealth multiplied by half of the policyholder's individual risk aversion coefficient a (see, e.g. Doherty and Richter (2002)):

$$\mathbb{CE} = \mathbb{E}(w) - \frac{a}{2} \cdot var(w),$$

with w denoting the policyholder's wealth at maturity. For simplicity reasons, we will not consider the total wealth of the policyholder but only the contract value (e.g. all other assets of the policyholder are considered to be non-stochastic). Therefore, we will base a policyholder's utility from a contract $C^{(x,t_0,T_0)}$ of type $x \in \{trad, innov\}$ on the policyholder payoff at maturity T_0. In case the insurer does not default until or at T_0, the payoff is given by the contract's book value at T_0:

$$P_{T_0}^{PH}\left(C^{(x,t_0,T_0)}\right) := P_0 \prod_{k=1}^{T_0-t_0}\left(1 + \max\left(r_{t_0}^{G,x}(k), \phi\, r^{Assets}(t_0+k)\right)\right). \tag{26}$$

However, if the insurer defaults at time $\tau \leq T_0$ due to overindebtedness, policyholders receive the insurer's remaining assets relative to their book values. The recovery value $\delta_\tau\left(C^{(x,t_0,T_0)}\right)$ is:

$$\delta_\tau\left(C^{(x,t_0,T_0)}\right) = MV_\tau(A_\tau)\frac{BV_\tau\left(C^{(x,t_0,T_0)}\right)}{BV_\tau(L_\tau)}. \tag{27}$$

To obtain comparability, we assume that afterwards $\delta_\tau\left(C^{(x,t_0,T_0)}\right)$ is invested in risk-free bonds with maturity T_0. Thus, the total policyholder payoff is given by

$$P^{PH}\left(C^{(x,t_0,T_0)}\right) = \mathbb{1}_{\tau>T_0} P^{PH}_{T_0}\left(C^{(x,t_0,T_0)}\right) + \mathbb{1}_{\tau\leq T_0}(1+r_{f,T_0-\tau}(\tau))^{T_0-\tau}\delta_\tau\left(C^{(x,t_0,T_0)}\right), \tag{28}$$

where τ is the (stopping) time of default,[18]

$$\tau = \inf\left\{t \in \{0,1,...,T\} : MV_t(A_t) < MV_t(L_t)\right\}. \tag{29}$$

The higher the value of the certainty equivalent, the higher the contract value from the policyholder's perspective.

2.5 Shareholder Return and Probability of Default

Following Bohnert et al. (2015), we base the evaluation of the compensation for the company's shareholders on the final value of their initial equity capital contribution at time T. Consequently, the cumulative return on shareholders' equity capital, i.e. their total return on equity (RoE) at time T is given by[19]

$$r^E_T = \frac{(MV_T(A_T) - MV_T(L_T))^+}{MV_0(A_0) - MV_0(L_0)} - 1. \tag{30}$$

Moreover, to determine the impact of different product designs on the riskiness of the life insurance company, we calculate the insurer's cumulative probability of default, which is defined as

$$\Pi_t = \mathbb{P}\left(MV_k(A_k) < MV_k(L_k) \text{ for any } k \in \{0,1,...,t\}\right). \tag{31}$$

Note, that under the Solvency II regulation regime, the insurer's one-year probability of default is intended to be constrained by 0.5%. Thus, we can easily interpret Π_t from the perspective of Solvency II. In particular, for $T = 40$ (cf. Section 3) Solvency II restricts

[18] Note that we define $\inf(\emptyset) = \infty$.
[19] We denote $(x)^+ := \max(x, 0)$.

$\Pi_{40} \leq 18.17\%$.[20]

3 Calibration

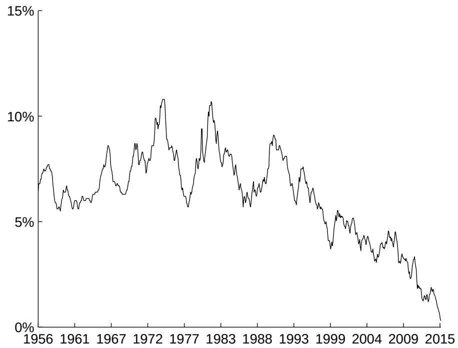
The baseline financial market calibration is shown in Table 2. The short-rate volatility and speed of mean reversion are based on parameters for the Vasicek model estimated by historical data from the Euro OverNight Index Average (EONIA).[21] However, we choose a slightly larger speed of mean reversion α_r and smaller volatility σ_r to increase the influence of the mean reversion level $\theta(t)$.[22]

Our fitting procedure for the interest rate environments is as follows: At first, we identify specific interest rate environments from historical data (see Figure 2) and adjust the evolution and skewness of the deterministic functions f and g to evolve in a similar way.

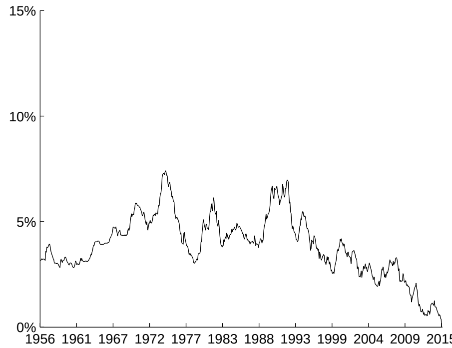
[20] However, one should note that the Solvency II-restriction applies to the one-year probability of default, i.e. $\mathbb{P}_{\mathbf{k}}\left(MV_{k+1}(A_{k+1}) < MV_{k+1}(L_{k+1})\right)$, whereas our analysis is based on $t = 0$, i.e. $\Pi_{k+1} = \mathbb{P}_{\mathbf{0}}\left(MV_{k+1}(A_{k+1}) < MV_{k+1}(L_{k+1})\right)$.

[21] Time series from January 1999 until December 2010 (end of month). Source: Datastream.

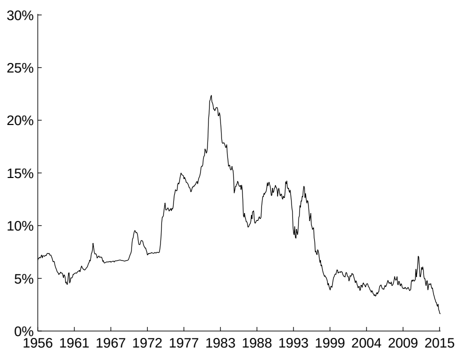
[22] The Vasicek model is an appropriate reference since it equals our model if setting $\theta(t) \equiv \theta \in \mathbb{R}$. Parameter estimates for the Vasicek models can be found e.g. in Martin (2013), Graf et al. (2011) or Zeytun and Gupta (2007).



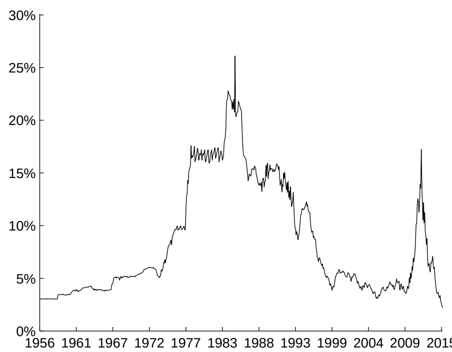
(a) Germany



(b) Switzerland



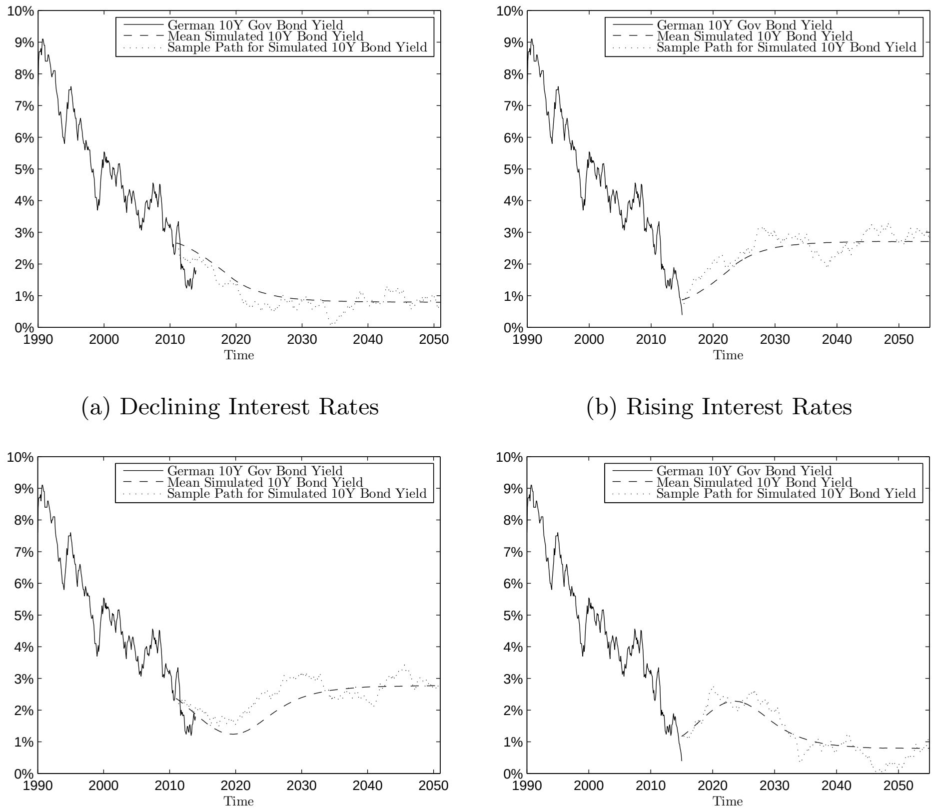
(c) Italy



(d) Portugal

Figure 2: Selected historical interest rates of 10-year government bonds.

For the initial 10Y spot rate of each environment, we use two different benchmarks: In environments (b) and (d) (rising and hump-shaped), the reference point for time $t = 0$ is the beginning of the year 2014 with a regulatory maximum yearly guaranteed rate in Germany of $r_0^{G,reg} = 1.75\%$. For environments (a) and (c) (declining and u-shaped), which start at a higher level, we start in the year 2011 with $r_0^{G,reg} = 2.25\%$. Whereas the resulting parameters are reported in Table 3, Figure 3 illustrates the resulting environments.



(a) Declining Interest Rates

(b) Rising Interest Rates

(c) U-shaped Interest Rates

(d) Hump-shaped Interest Rates

Figure 3: Evolution of the 10Y interest rates under the four interest rate environments.

Parameter	**Notation**	**Value**
Short-rate volatility	σ_r	0.005
Speed of mean reversion	α_r	0.2
GBM: stock motion drift	$\overline{\mu}_S$	0.072
GBM: stock motion volatility	$\overline{\sigma}_S$	0.16
Target stock ratio	α	5%

Table 2: Baseline financial market calibration.

To calibrate the risky asset process we start with estimates for the drift $\overline{\mu}_S$ and volatility $\overline{\sigma}_S$ of a standard Geometric Brownian Motion based on the DAX (the main German stock index), as reported in Table 2.[23] Based on these, we derive the matching parameters σ_S, λ_S and ρ for every interest rate environment, such that the drift and variance in our

[23] The parameters are based on Berdin and Gründl (2015).

model approximately fit those of the standard GBM model.[24] Moreover, empirical evidence suggests a negative correlation between the evolution of the stock market and the short-rate. With the calibration reported in Table 3 we obtain an average correlation coefficient value of -52% (driven by the value for ρ) which corresponds to empirical results (cf. Berdin and Gründl (2015)).

Parameter	**Notation**	**Environment**			
		(a)	(b)	(c)	(d)
Short-rate starting value	r_0	0.0268	0.008	0.0272	0.008
Market price of risk	λ_S	0.2816	0.3991	0.2791	0.3991
Calibration of the mean reversion level					
Skewness	b	4.5	4.5	3.2	3.2
Shift parameter	h	8	8	–	–
Mean reversion level in $t = 0$	β	0.0268	0.008	0.0272	0.008
Mean reversion level in $t = T$	γ	0.008	0.0268	0.0272	0.008
Maximum/Minimum point	c	–	–	10	10
Maximum/Minimum	ν	–	–	0	0.035
Calibration of the insurance company					
Initial equity capital ratio	E_0/A_0	8.0%	10.5%	7.5%	9.0%
Initial liability portfolio guarantee	$\overline{r}_0^G$	3.46%	3.13%	3.46%	3.13%

Table 3: Interest rate environment calibration.

The equity capital ratio of the upper quartile of the equity capital ratios of roughly 90% of the largest German life insurers in 2011 and 2014 (according to market share) amounts to approximately 8%. Note that we adjust the initial equity capital ratios in each environment in such a way that they represent a threshold level that allows us to observe some defaults, i.e. as under Solvency II, we allow for a small yearly default probability larger than 0%.[25]

Badorff et al. (2014) report that in 2014 the average guaranteed rate of German life insurance contracts was 3.1%. In our model, we reproduce this guarantee level by starting the model in the year 1993 and adding one contract (with equal up-front premium) every year (including 2013) to the insurer's contract portfolio. Consequently, the average guarantee in the year 2014 equals 3.13%. Assuming that the contract sold in 1993 already matured in 2014, for 2015 the overall guarantee equals 3.01% (i.e. the average (modeled) regulatory interest rate of the years 1994,...,2014). Based on the same procedure, the corresponding

[24] Cf. Corollary B.0.1.

[25] This calibration also illustrates that in environment (b) and (d), which start later than (a) and (c), the insurer already requires a higher initial equity capital ratio since interest rates are lower.

average guarantee in 2011 equals 3.46% (i.e. the average (modeled) regulatory interest rate of the years 1990,...,2010). From there on, at the end of every year $t = 0, ..., T$, one of the 21 prior contracts matures and afterwards a new contract is sold.

The policyholder's risk aversion coefficient is set to $a = 3$, which represents a low degree of risk aversion.[26] We take P_0 exogenously given as it is done e.g. in Grosen and Jorgensen (2000) and set it to $P_0 = 1$ for all contracts.[27]

To allow for a more realistic time horizon for the interest rate environment, we choose a fixed contract length of 20 years and a total time horizon of 40 years. Furthermore, the insurer only invests in sovereign bonds with 10 years to maturity. Table 4 summarizes the parameters.

Parameter	**Notation**	**Value**
Single contract length (in years)	$J^{CMat} = T_0 - t_0$	20
Number of prior contracts	M	21
Sovereign bond term	J	10
Time horizon (in years)	T	40
Risk aversion coefficient	a	3
Reference rate contribution parameter	ζ^{innov}	0.7
Reference rate moving average (in years)	Z^{innov}	10
Temporary guarantee adjustment (in years)	T^{innov}	2

Table 4: Baseline model calibrations.

To assess the sensitivity of our results, we will vary the insurer's equity capital ratios ($E_0/A_0 \pm 1\%$), the reference rate ($\zeta^{innov} \pm 0.1$), the temporary guarantee adjustment time horizon ($\hat{T}^{innov} = 5$) as well as the insurer's investment strategy ($\alpha^{(i)}$).

4 Results

We discuss our results in two steps. Firstly, we present the baseline results from our calibration in Section 3 and show how the different product designs affect the average and

[26] Schmeiser and Wagner (2013, p. 18) use the same calibration in a similar setting. In Section 4.2.4, we consider a sensitivity analysis for this parameter.

[27] Note that size does not matter as the insurer's balance sheet may be scaled up or down.

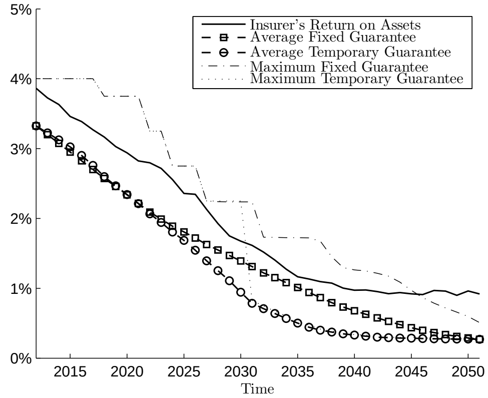
maximum guarantee level of the insurer's contract portfolio, the insurer's default risk as well as the payoffs to policyholders and shareholders. Secondly, we assess the sensitivity of our results towards changes in the input parameters, especially with respect to the insurer's initial equity capital ratio, the product design as well as the insurer's investment strategy.

4.1 Baseline Results

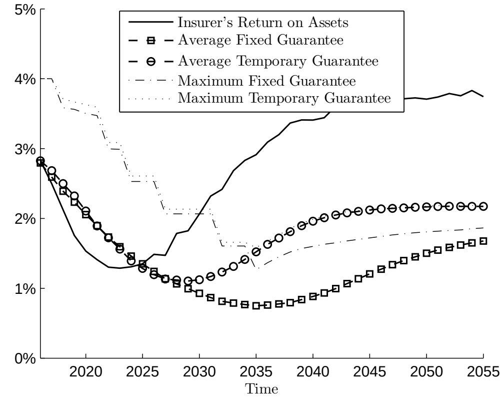
Figure 4 illustrates the development of the average contract guarantee as well as the maximum guarantee in the insurer's liability portfolio when either the traditional fixed guarantee product or the temporary guarantee product (with $T^{innov} = 2$)[28] is sold in each interest rate environment.

Since the adjustable temporary guarantee contract portfolio moves together with the insurer's return on assets, the probability that the guarantee exceeds the return on assets ($\mathbb{P}(r^G > r^A)$) decreases. This effect intensifies over time when old traditional contracts mature. As a result, the insurance company exhibits a lower default risk. Particularly, Figures 4a and 4d demonstrate the benefits of a temporary guarantee. While the insurer's return on assets exceeds the maximum guarantee of the temporary guarantee contract portfolio, it is below the maximum fixed guarantee ($\max r_{t_0}^{G,innov}(t) < r^A(t) < \max r_{t_0}^{G,trad}(t)$). Both findings emphasize the slow and limited adaptability of a life insurer's fixed guarantee contract portfolio towards different interest rate levels. As the insurer continuously invests in new bonds, the moving average of its guarantee portfolio moves significantly slower than that of its bond portfolio. Even worse, Figure 4d illustrates how an insurer selling the traditional product lurches from crisis (2015-2025) to crisis (2045-2055) in which the maximum guarantee always exceeds the return on assets. In contrast, the evolution of the temporary guarantee portfolio mimics the development of the insurer's asset portfolio and, thereby, successfully decreases the interest rate dependency of the insurer's solvency situation.

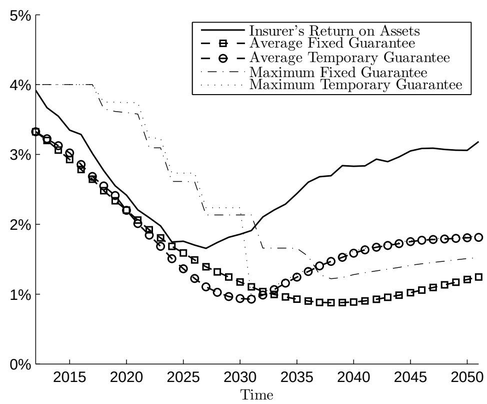
[28] In Section 4.2.2, we consider a sensitivity analysis for this parameter.



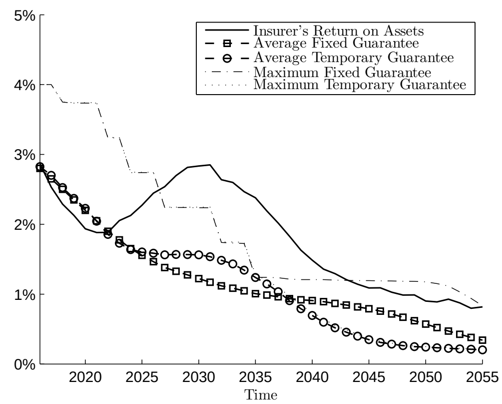
(a) Declining Interest Rates



(b) Rising Interest Rates



(c) U-shaped Interest Rates



(d) Hump-shaped Interest Rates

Figure 4: Fixed versus temporary guarantee scheme under the four interest rate environments.

Both the development of the average and the maximum portfolio guarantee affect the life insurer's default risk. Figure 5 demonstrates that selling the temporary guarantee product predominantly achieves a lower (cumulative) default probability than selling the fixed guarantee product. To be specific, selling the traditional product results in several stress situations: As the guarantees become "too expensive", the insurer's default probability increases dramatically (cf. Figures 4d and 5d from the year 2040 onwards). Consequently, the temporary guarantee is able to reduce a life insurer's default risk to a large degree, which we acknowledge as its major advantage.

Figure 5 also highlights the importance of implementing different interest rate environments as a change in the evolution of the interest rate can result in a fundamentally different cumulative default probability function. For example, since the interest rates are particularly low in the beginning and increase in the long-run in environments (b) and (c), the insurers' (one-year) default risk is especially high within the first years and later converges to zero.

Note that although the maximum guarantee in the portfolio exceeds the insurer's return on assets in each environment in the beginning, the default risk is quite low since the insurer uses equity capital to settle the high guarantee commitments.

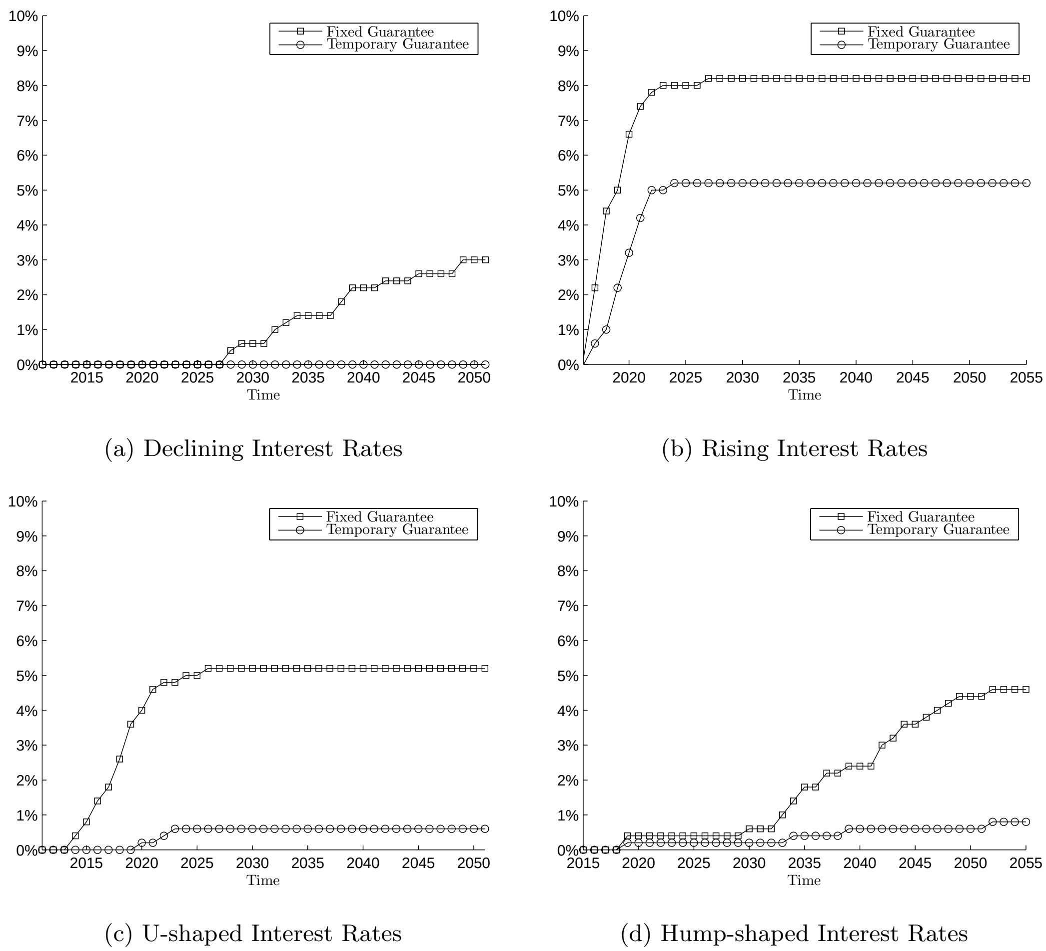
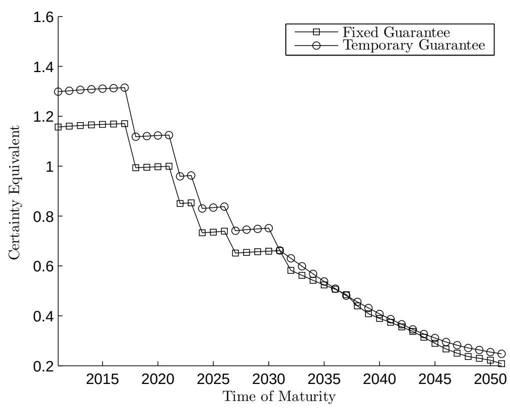


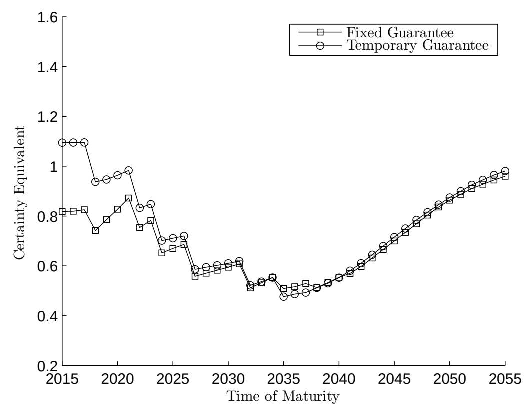
Figure 5: Cumulative default probability for fixed versus temporary guarantee scheme under the four interest rate environments.

To study the policyholders' welfare in different interest rate environments, Figure 6 investigates the certainty equivalents for different contracts depending on their time of maturity. Note that the certainty equivalent in year 2015 in Figure 6 corresponds to the first contract cohort sold in 1995 as the contract length is set to $T = 20$ years. Consequently, the certainty equivalent for the first contract sold in 2015 corresponds to the value in year 2035.

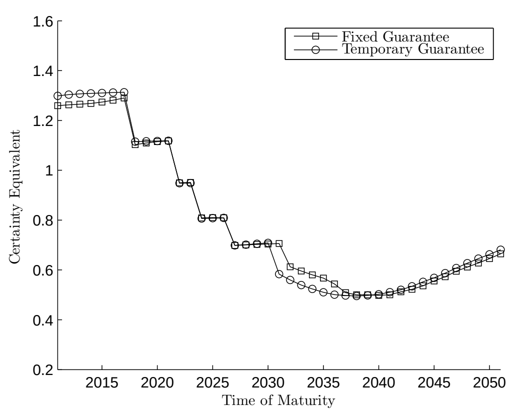
We find that for the policyholder signing a traditional contract in the year 2015, a (high) fixed guarantee product can lead to a higher certainty equivalent in environment (b) and (c) (c.f. year 2035 in Figure 6b and year 2030 in Figure 6c). In this case, despite the higher default probability, policyholders benefit from high expected payoffs resulting from the high fixed guarantee of 1.75% and 2.25% with the traditional product. Nevertheless, this changes for later cohorts, where the certainty equivalents from the innovative product prevail in each interest rate environment.



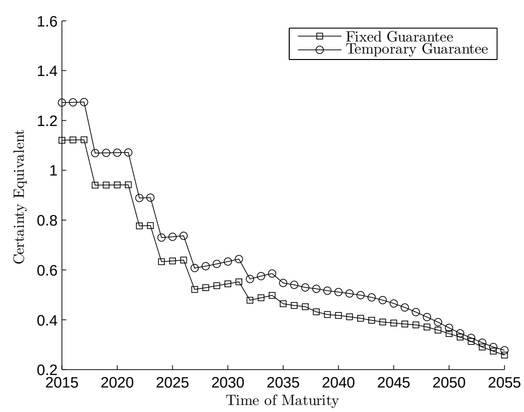
(a) Declining Interest Rates



(b) Rising Interest Rates



(c) U-shaped Interest Rates



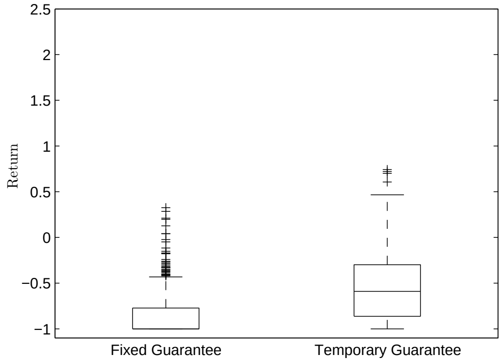
(d) Hump-shaped Interest Rates

Figure 6: Policyholder's certainty equivalents for fixed versus temporary guarantee scheme under the four interest rate environments.

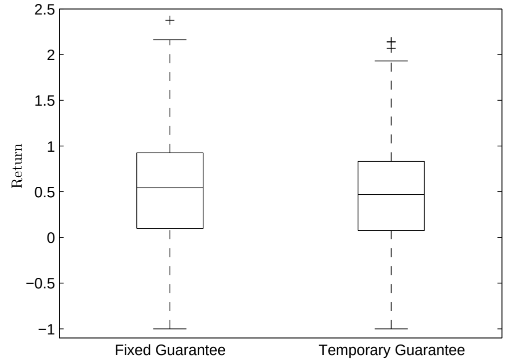
The resulting payouts to shareholders are presented in Figure 7 (distribution of the total return on equity)[29] and Table 5 (expected total return on equity). The figures show that the temporary guarantee product is able to increase the payoff to shareholders within three of the four interest rate environments. To be specific, in environments (a), (c) and (d), the expected shareholder return is on average 13% larger for the temporary guarantee. In the rising interest rate environment (b), the shareholders' payoff from selling the traditional product dominates the other since the insurer's equity capital base is high enough to keep the company solvent (in market values) within the first years. In the following years, interest rates increase but the average level of fixed guarantees is below that of the temporary guarantees (see Figure 4b). Thus, the smaller fixed guarantees increase the payout to shareholders in comparison to the temporary ones.[30]

[29] Note that in Figure 6, the central rectangle spans the first quartile to the third quartile (the interquartile range or IQR). The segment inside the rectangle shows the median and "whiskers" above and below the box show the locations of $1.5 \cdot IQR$ above the third quartile and $1.5 \cdot IQR$ below the first quartile, respectively. Values beyond these points are considered outliers.

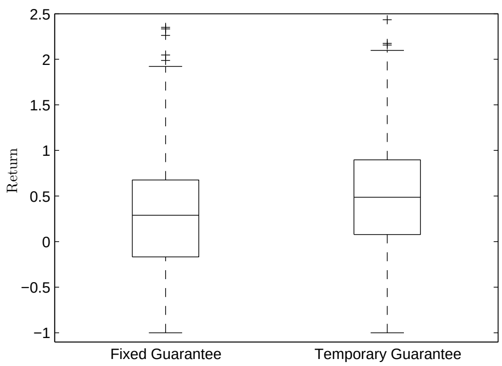
[30] Note that this will change by varying the insurer's initial equity capital ratio, see Section 4.2.1.



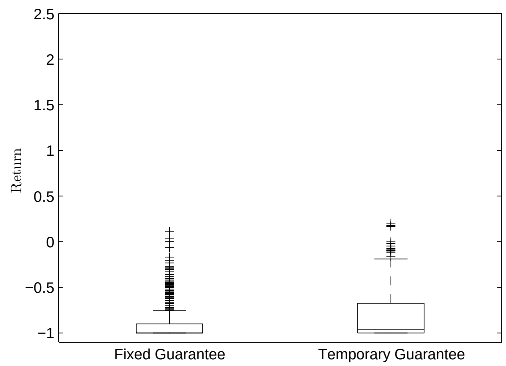
(a) Declining Interest Rates



(b) Rising Interest Rates



(c) U-shaped Interest Rates



(d) Hump-shaped Interest Rates

Figure 7: Shareholder's total returns under the four interest rate environments.

Expected Total Return on Equity	**Environment**			
	(a)	(b)	(c)	(d)
Fixed Guarantee	-0.8499	0.4735	0.2840	-0.9018
Temporary Guarantee	-0.5353	0.4391	0.4970	-0.8150

Table 5: Shareholders' expected total returns under the four interest rate environments.

4.2 Sensitivity Analysis

In order to assess the robustness of the results presented in the previous section, we investigate the sensitivity towards changes in the input parameters. Thereby, we will focus on individual interest rate environments.[31]

4.2.1 Equity Capital

We first vary the insurer's initial equity capital ratio. In general, we find that increasing (decreasing) the equity capital ratio increases (decreases) the return to policyholders due to a lower (higher) default risk. Figure 8 illustrates the change in the insurer's cumulative default probability under rising interest rates. Consequently, also the payout to shareholders increases with the initial equity capital ratio (see Table 6). At the same time, increasing (decreasing) the equity capital leads to an increase (decrease) in the certainty equivalents from both products.

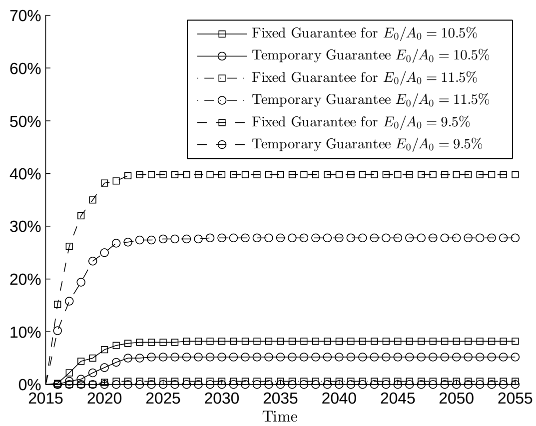


Figure 8: Cumulative default probability for fixed versus temporary guarantee scheme under rising interest rates for different equity capital ratios.

Expected Total Return on Equity	**Initial Equity Capital Ratio**		
	9.5%	10.5%	11.5%
Fixed Guarantee	0.0272	0.4735	0.5986
Temporary Guarantee	0.0693	0.4391	0.5164

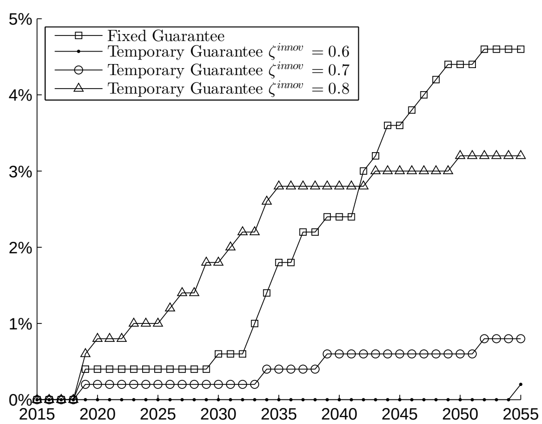
Table 6: Shareholders' expected total returns under rising interest rates for different equity capital ratios.

[31] Further results can be obtained at the authors request.

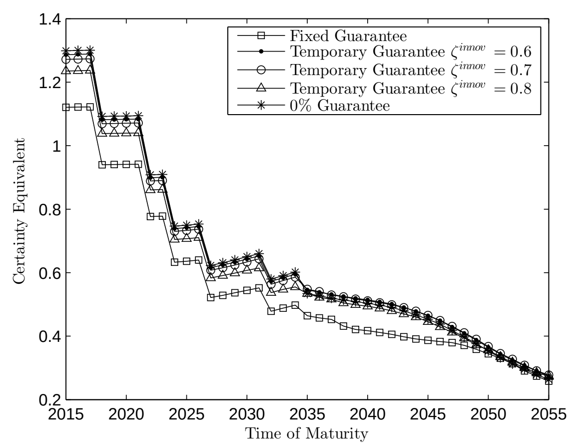
4.2.2 Guarantee Mechanism

The level of the temporary guarantee at any point in time depends on two factors: the reference rate contribution parameter ζ^{innov}, which determines the level of the guarantee in percent of the reference rate (see Equation 19) and the temporary guarantee adjustment time horizon T^{innov}. In the following, we first vary the reference rate contribution parameter ζ^{innov}.

Increasing ζ^{innov} leads to a higher guarantee which then increases the insurer's default risk (see Figure 9a) and eventually lowers the shareholders' return. As a result, the certainty equivalents decline (see Figure 9b). Note that although lowering ζ^{innov} decreases the insurer's default risk, the policyholders' certainty equivalents do not change by much. Even though the guarantee declines, the resulting payout is similar due to the profit participation. Note that we introduced the certainty equivalents for a fixed 0% guarantee product in Figure 9b, which are very close to the CE's from the innovative temporary guarantee product.



(a) Cumulative Default Probability



(b) Certainty Equivalents

Figure 9: Cumulative default probability and certainty equivalents for fixed versus temporary guarantee scheme under hump-shaped interest rates for different ζ^{innov}.

Increasing the temporary guarantee adjustment time horizon to $T^{innov} = 5$ years leads to a conversion of the results from the traditional and innovative product. With a longer adjustment horizon, a guarantee level is maintained for a longer time period and is thus less able to react to a sudden change in the interest rate environment.

Consequently, increasing ζ^{innov} and T^{innov} also increases interest rate dependency and default risk, whereas policyholders' certainty equivalents and shareholders' total return decrease (see Table 7).

Expected Total Return on Equity	**Temporary Guarantee Parameters**			
	$\zeta^{innov} = 0.6$	$\zeta^{innov} = 0.7$	$\zeta^{innov} = 0.8$	$T^{innov} = 5$
Fixed Guarantee	-0.9018	-0.9018	-0.9018	-0.9018
Temporary Guarantee	-0.7732	-0.8150	-0.8687	-0.8528
0% Guarantee	-0.6136	-0.6136	-0.6136	-0.6136

Table 7: Shareholders' expected total returns under hump-shaped interest rates for different ζ^{innov} and T^{innov}.

4.2.3 Investment Strategy

EIOPA's 2014 "EU/EEA (re)insurance statistics" show significant differences in the stock ratios across the European life insurance market.[32] Whereas the stock ratio is around 5% in German speaking countries, other jurisdictions exhibit a much higher (f.e. Denmark and Netherlands) or lower (f.e. France and Portugal) stock exposure. We thus vary the stock ratio from our baseline calibration to study the sensitivity of our results with respect to an insurer's default risk as well as the payouts to policyholders and shareholders.

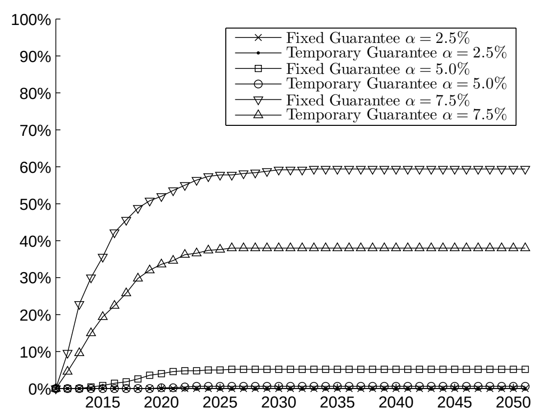
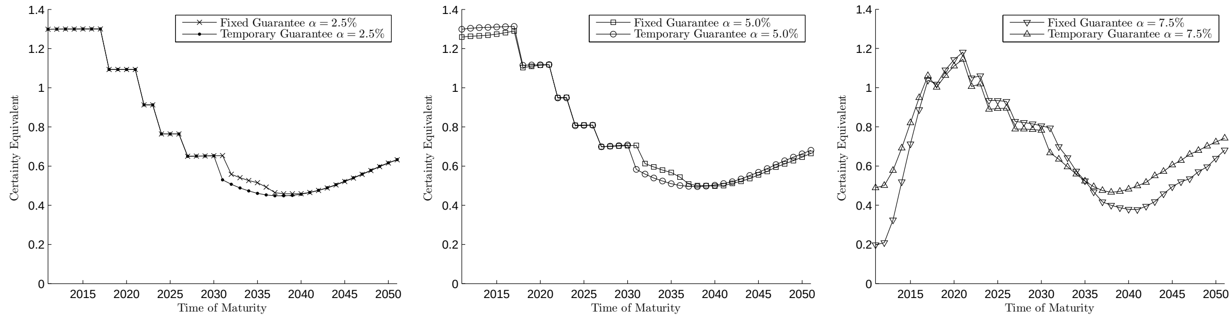


Figure 10: Cumulative default probability for fixed versus temporary guarantee scheme under u-shaped interest rates for different stock ratios.

[32] See European Insurance and Occupational Pensions Authority (EIOPA) (2013).



(a) $\alpha = 2.5\%$ (b) $\alpha = 5.0\%$ (c) $\alpha = 7.5\%$

Figure 11: Certainty equivalents for fixed versus temporary guarantee scheme under u-shaped interest rates for different stock ratios.

As Figure 10 illustrates, increasing the stock ratio results in a higher (cumulative) default probability due to the additional volatility of the insurer's asset portfolio. This, in turn, reduces policyholders' certainty equivalents (Figure 11c) and the return on equity (Table 8). To be specific, increasing the stock ratio from 5.0% to 7.5% reduces the shareholders' expected total return from 28.4% to -51.35% for the traditional product and from 49.7% to -36.75% for the innovative product in the u-shaped interest rate environment (Table 8). Moreover, the higher the default risk of the company, the larger the difference between the traditional and the innovative product. In contrast, lowering the stock ratio reduces the default risk and increases the return on equity. At the same time the differences between the products diminish (see Figures 11a and 11b).

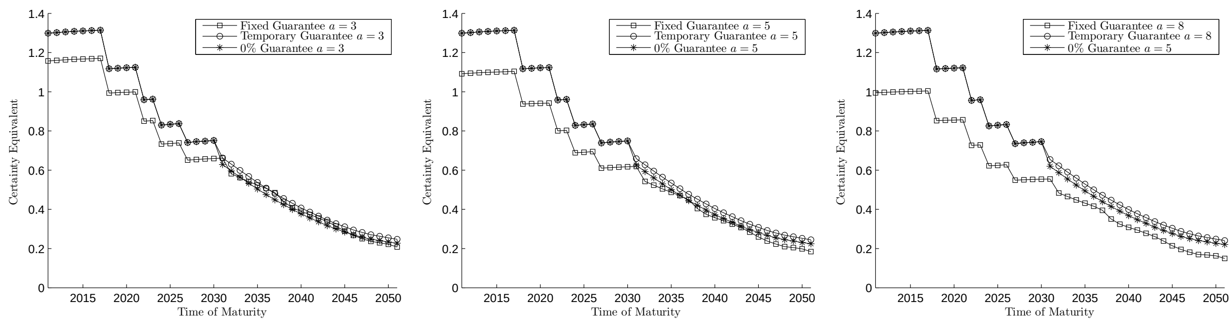
Expected Total Return on Equity		**Stock Ratio**	
	$\alpha = 2.5\%$	$\alpha = 5.0\%$	$\alpha = 7.5\%$
Fixed Guarantee	0.6990	0.2840	-0.5136
Temporary Guarantee	0.9633	0.4970	-0.3675

Table 8: Shareholders' expected total returns under u-shaped interest rates for different stock ratios.

4.2.4 Risk Aversion

To discuss the results from the point of view of a policyholder with different risk preferences, Figure 12 illustrates three different levels of risk aversion. Therefore, Figure 12a shows policyholders' certainty equivalents for a low degree of risk aversion (a = 3), whereas Figures 12b and 12c present certainty equivalents for customers with higher degrees of risk aversion (a = 5 and a = 8).[33]

Due to the high volatility of the return of a (high) fixed guarantee policy, the certainty equivalents decrease with increasing risk aversion. In contrast, the policyholder's payoffs from the innovatove and 0% guarantee product are less volatile and, thus, the certainty equivalents do not by much. This is due to the reduced financial risk these products impose on the life insurance company.



(a) $a = 3$ (b) $a = 5$ (c) $a = 8$

Figure 12: Certainty equivalents for fixed versus temporary guarantee scheme under declining interest rates for different risk aversion parameters.

5 Summary and Conclusion

Our study highlights the financial risk that traditional endowment life insurance policies pose on many European life insurance companies. As these policies feature a fixed guaranteed rate, many companies have to cope with very slowly moving average guarantee levels throughout different interest rate environments, a phenomenon that we refer to as the *guarantee trap*. Particularly due to the current low interest rates, this phenomenon leads to an increased default risk among companies that have sold traditional endowment life insurance contracts with high guarantees in the past.

To reduce interest rate dependency of an insurer's solvency situation and to increase

[33] See Schmeiser and Wagner (2013, p. 18).

the financial stability of life insurers in the long-run, we study a more flexible guarantee mechanism with an adaptable shorter-term (temporary) guarantee. This product design successfully reduces the financial risk associated with interest guarantees since an insurer's duration gap diminishes. Thus, an adjustable temporary guarantee mechanism constitutes a way for life insurers to release themselves from the guarantee trap.

Our results show that if life insurers continue to sell traditional fixed guarantee policies, they risk lurching from one crisis into another due to the weak co-movement of the cash flows from their asset and liability side. In contrast, the temporary guarantee adjusts to different economic conditions.

We illustrate that with an appropriate product design both policyholders and shareholders benefit from the innovative short-term guarantee product design, essentially since the default risk is substantially lower. Put differently, the temporary guarantee product increases the co-movement between the evolutions of a life insurer's asset and liability side and, thus, decreases the company's duration gap.

Especially risk managers and regulators will benefit from our analysis since a reduction of the default risk coincides with a reduction of the capital requirement under the upcoming Solvency II framework. We thus expect the importance of sustainable life insurance product design to increase.

The results of our analysis depend on both the calibration of the model and on the necessary simplifications adopted. In general, the model is a reduced version of a life insurance company without product line diversification, group diversification, a buffer or reinsurance activities. Nevertheless, we are confident that a more complex model would not change the overall results.

Appendix

A Computation of the insurer's return on assets

The return on assets (RoA) between time $t-1$ and t is given by

$$\begin{aligned}
r_t^{Assets} &= \frac{A_t}{A_{t-1}} - 1 \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})}(1 + r_{f,J}(t-k)) + S(t)\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} - 1 \\
&= \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + (S(t) \pm S(t-1))\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} - 1 \\
&= 1 + \frac{(S(t) - S(t-1))\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} - 1 \\
&= \left(-\alpha^{(i)}\right) + \frac{S(t-1)}{S(t-1)} \frac{S(t)\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \\
&\quad + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \\
&= \left(-\alpha^{(i)}\right) + \frac{S(t)}{S(t-1)}\alpha^{(i)} + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \\
&= \alpha^{(i)}\left(\frac{S(t)}{S(t-1)} - 1\right) + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \\
&= \alpha^{(i)}\frac{S(t) - S(t-1)}{S(t-1)} + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} \quad ,
\end{aligned}$$

with

$$\frac{S(t-1)\sum_{k=0}^{t-1}\Delta_k}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} = \alpha^{(i)}$$

and

$$\frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})}}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1)\sum_{k=0}^{t-1}\Delta_k} = 1 - \alpha^{(i)}.$$

Under the assumption that funds are sufficient to fulfill the target stock ratio and the resulting face values are approximately equal ($\sum_{k=1}^{J} FV^{(B_{t-k,J})}$ simplifies to $J \cdot FV^{(B_{t-k,J})}$), the RoA takes the following form:

$$\begin{aligned} r_t^{Assets} &= \alpha^{(i)} \frac{S(t) - S(t-1)}{S(t-1)} + \frac{\sum_{k=1}^{J} FV^{(B_{t-k,J})} r_{f,J}(t-k)}{\sum_{k=1}^{J} FV^{(B_{t-k,J})} + S(t-1) \sum_{k=0}^{t-1} \Delta_k} \\ &= \alpha^{(i)} \frac{S(t) - S(t-1)}{S(t-1)} + \frac{1 - \alpha^{(i)}}{J} \sum_{k=1}^{J} r_{f,J}(t-k). \end{aligned}$$

B Distributional properties of the stock process

Lemma B.0.1 (Closed Form for the Stock Process)**.**

a) The stock process evolution is given by

$$S(t) = S(0) \ \exp \left(\int_0^t r(u)\, du + \left(\sigma_S \lambda_S - \frac{\sigma_S^2 + \rho^2}{2} \right) t + \sigma_S W_S(t) + \rho W_r(t) \right). \quad (32)$$

b) Assuming $S(0) = 1$, the log-stock-process is

$$\begin{aligned} \log S(t) = r_0 \frac{1 - e^{-\alpha_r t}}{\alpha_r} &+ \int_0^t \theta(s) \left(1 - e^{-\alpha_r (t-s)} \right) ds + \left(\sigma_S \lambda_S - \frac{\sigma_S^2 + \rho^2}{2} \right) t \\ &+ \sigma_S W_S(t) + \int_0^t \rho + \frac{\sigma_r}{\alpha_r} \left(1 - e^{-\alpha_r (t-s)} \right) dW_r(s), \end{aligned} \quad (33)$$

which is normally distributed. Thus, $S(t)$ is log-normally distributed.

Proof.

a) The stock process is given by

$$\frac{dS(t)}{S(t)} = (r(t) + \sigma_S \lambda_S)\, dt + \sigma_S\, dW_S(t) + \rho\, dW_r(t).$$

Applying Ito's multidimensional Formula (cf. Sondermann (2006, p. 29)) for $\log S(t)$ yields

$$\begin{aligned} d \log(S(t)) &= \left(r(t) + \sigma_S \lambda_S - \frac{1}{2} \left(\sigma_S^2 + \rho^2 \right) \right) dt + \sigma_S\, dW_S(t) + \rho\, dW_r(t) \\ &= \frac{dS(t)}{S(t)} - \frac{1}{2} \left(\sigma_S^2 + \rho^2 \right) dt, \end{aligned}$$

which is equivalent to

$$\begin{aligned}\log\left(\frac{S(t)}{S(0)}\right) &= \int_0^t (r(u)+\sigma_S\lambda_S)\,du + \sigma_S W_S(t) + \rho W_r(t) - \frac{1}{2}\left(\sigma_S^2+\rho^2\right)t \\ \Leftrightarrow \quad S(t) &= S(0)\,\exp\left(\int_0^t r(u)\,du + \left(\sigma_S\lambda_S - \frac{\sigma_S^2+\rho^2}{2}\right)t + \sigma_S W_S(t) + \rho W_r(t)\right).\end{aligned} \tag{34}$$

b) With integration by parts[34] we obtain

$$\begin{aligned}\int_0^t r(s)\,ds &= \int_0^t \left(r_0 e^{-\alpha_r s} + \alpha_r \int_0^s e^{-\alpha_r(s-u)}\theta(u)\,du + \sigma_r \int_t^s e^{-\alpha_r(s-u)}\,dW_r(u)\right)ds \\ &= \int_0^t \left(r_0 e^{-\alpha_r s} + \alpha_r \int_0^s e^{-\alpha_r(s-u)}\theta(u)\,du\right)ds + \frac{\sigma_r}{\alpha_r}\int_0^t 1 - e^{-\alpha_r(t-s)} dW_r(s).\end{aligned}$$

Again with integration by parts, the second integral is given by

$$\begin{aligned}&\alpha_r \int_0^t e^{-\alpha_r s}\int_0^s e^{\alpha_r u}\theta(u)du\;ds = \left[-e^{-\alpha_r s}\int_0^s e^{\alpha_r u}\theta(u)du\right]_0^t + \int_0^t e^{-\alpha_r s}e^{\alpha_r s}\theta(s)ds \\ &= -e^{-\alpha_r t}\int_0^t e^{\alpha_r s}\theta(s)ds + \int_0^t \theta(s)ds = \int_0^t \theta(s)\left(1-e^{-\alpha_r(t-s)}\right)ds\end{aligned}$$

Adopting this in (34) yields

$$\begin{aligned}\log S(t) = r_0\frac{1-e^{-\alpha_r t}}{\alpha_r} &+ \int_0^t \theta(s)\left(1-e^{-\alpha_r(t-s)}\right)ds + \left(\sigma_S\lambda_S - \frac{\sigma_S^2+\rho^2}{2}\right)t \\ &+ \sigma_S W_S(t) + \int_0^t \rho + \frac{\sigma_r}{\alpha_r}\left(1-e^{-\alpha_r(t-s)}\right)dW_r(s).\end{aligned}$$

Since $\int_0^t \rho + \frac{\sigma_r}{\alpha_r}\left(1-e^{-\alpha_r(t-s)}\right)dW_r(s)$ is normally distributed with mean 0 and variance $\int_0^t \left(\rho + \frac{\sigma_r}{\alpha_r}\left(1-e^{-\alpha_r(t-s)}\right)\right)^2 ds$, $\log S(t)$ is normally distributed as well.

□

Lemma B.0.2 (Distributional Properties of the Stock Process)**.** *The first moment, the*

[34] See Sondermann (2006).

variance and their first-order Taylor polynomial of the log-Stock Process are given by

$$
\begin{aligned}
\mathbb{E}[\log S(t)] &= r_0 \frac{1-e^{-\alpha_r t}}{\alpha_r} + \int_0^t \theta(s)\left(1-e^{-\alpha_r(t-s)}\right) ds + \left(\sigma_S\lambda_S - \frac{\sigma_S^2+\rho^2}{2}\right) t, && (35)\\
&\approx \left(r_0 + \sigma_S\lambda_S - \frac{\sigma_S^2+\rho^2}{2}\right) t && (36)
\end{aligned}
$$

$$
\begin{aligned}
var(\log S(t)) &= \sigma_S^2\, t + \int_0^t \left(\rho + \frac{\sigma_r}{\alpha_r}\left(1-e^{-\alpha_r(t-s)}\right)\right)^2 ds && (37)\\
&= \sigma_S^2\, t + \frac{e^{-2\alpha_r t}}{2\alpha_r^3}\left(-\sigma_r^2 + 4\sigma_r e^{\alpha_r t}(\alpha_r\rho+\sigma_r) + e^{2\alpha_r t}\left(2\alpha_r t(\alpha_r\rho+\sigma_r)^2 - \sigma_r(4\alpha_r\rho+3\sigma_r)\right)\right)\\
&\approx \left(\sigma_S^2+\rho^2\right) t. && (38)
\end{aligned}
$$

Moreover, the covariance between $\log S(t)$ *and* $r(t)$ *and its second-order Taylor polynomial is given by*[35]

$$
\begin{aligned}
cov(\log S(t), r(t)) &= \frac{\sigma_r}{2\alpha_r^2}\left(e^{-\alpha_r t} - e^{-2\alpha_r t}\right)\left(e^{\alpha_r t}(2\alpha_r\rho+\sigma_r) - \sigma_r\right) && (39)\\
&\approx \rho\sigma_r t + \frac{1}{2}\sigma_r t^2(\sigma_r - \alpha_r\rho). && (40)
\end{aligned}
$$

Proof.

By employing Lemma B.0.1 the computation of the mean, variance and their Taylor polynomials is straightforward.

The covariance between $\log S(t)$ and $r(t)$ is given by

$$
\begin{aligned}
cov(\log S(t), r(t)) &= cov\left(\int_0^t \rho + \frac{\sigma_r}{\alpha_r}(1-e^{-\alpha_r(t-s)})\, dW_r(s),\ \sigma_r \int_0^t e^{-\alpha_r(t-s)}\, dW_r(s)\right)\\
&= \sigma_r\, \mathbb{E}\left[\left(\int_0^t \rho + \frac{\sigma_r}{\alpha_r}(1-e^{-\alpha_r(t-s)})\, dW_r(s)\right)\cdot\left(\int_0^t e^{-\alpha_r(t-s)}\, dW_r(s)\right)\right]\\
&= \sigma_r \int_0^t e^{-\alpha_r(t-s)}\left(\rho + \frac{\sigma_r}{\alpha_r}(1-e^{-\alpha_r(t-s)})\right) ds\\
&= \frac{\sigma_r}{2\alpha_r^2}\left(e^{-\alpha_r t} - e^{-2\alpha_r t}\right)\left(e^{\alpha_r t}(2\alpha_r\rho+\sigma_r) - \sigma_r\right).
\end{aligned}
$$

□

Corollary B.0.1 (Calibration and Correlation)**.** *To match the stock drift and volatility*

[35] This is also an approximation for $cov(S(t)-1, r(t)) = cov(S(t), r(t))$ since $\log(S(t)) \approx S(t)-1$ for $S(t)$ near 1.

between estimated GBM parameters $\overline{\mu}_S$ *and* $\overline{\sigma}_S$ *and the model parameters* r_0, σ_S, λ_S *and* ρ *we obtain the following conditions* [36]:

$$\begin{aligned} &A) \quad r_0 + \sigma_S \lambda_S - \frac{\sigma_S^2 + \rho^2}{2} = \overline{\mu}_S - \frac{\overline{\sigma}_S{}^2}{2} \\ &B) \quad (\overline{\sigma}_S)^2 = \sigma_S^2 + \rho^2, \end{aligned}$$

where $\overline{\mu}$ *and* $\overline{\sigma}_S$ *are the stock drift and volatility in step I, respectively. Besides, the correlation between* $S(t)$ *and* $r(t)$ *can be approximated with*

$$corr\left(S(t),\, r(t)\right) \approx \frac{\rho\sigma_r t + \frac{1}{2}\sigma_r t^2(\sigma_r - \alpha_r\rho)}{\sqrt{\frac{\sigma_r^2}{2\alpha_r}\left(1 - e^{-2\alpha_r t}\right)}\sqrt{(\sigma_S^2 + \rho^2)t}}. \tag{41}$$

Lemma B.0.3 (Distributional Properties of the log-Stock-Return)**.** *The log-Stock-Return*

$$\hat{r}^{stocks}_{t_0,T_0} := \log\left(\frac{S_{T_0}}{S_{t_0}}\right) = \log\left(1 + r^{stocks}_{t_0,T_0}\right)$$

is conditionally on time t_0 *normally distributed,* $\hat{r}^{stocks}_{t_0,T_0} \sim \mathcal{N}\left(\hat{\mu}^{stocks}_{t_0,T_0}, \left(\hat{\sigma}^{stocks}_{t_0,T_0}\right)^2\right)$, *with*

$$\hat{\mu}^{stocks}_{t_0,T_0} = r_0 \frac{e^{-\alpha_r t_0} - e^{-\alpha_r T_0}}{\alpha_r} + \int_{t_0}^{T_0} \theta(s)\left(1 - e^{-\alpha_r(T_0 - s)}\right) ds + \left(\sigma_S\lambda_S - \frac{\sigma_S^2 + \rho^2}{2}\right)(T_0 - t_0)$$

and $$\left(\hat{\sigma}^{stocks}_{t+1}\right)^2 = \sigma_S^2(T_0 - t_0) + \frac{1}{2\alpha_r^3}\Big(2\alpha_r(T_0 - t_0)(\alpha_r\rho + \sigma_r)^2 + \sigma_r\left(e^{\alpha_r(t_0 - T_0)} - 1\right) \times \left(4\alpha_r\rho - \sigma_r\left(e^{-\alpha_r(T_0 - t_0)} - 3\right)\right)\Big).$$

[36] Since both the drift and variance in the standard GBM are linear dependent on t, we use first-order Taylor polynomials to obtain linear approximations for the drift and variance in our model and set them equal to the estimated GBM drift and variance.

Thus, the annually compounded stock-return obtains mean and variance

$$\begin{aligned}\mu_{t_0,T_0}^{stocks} &:= \mathbb{E}_{t_0}\left[r_{t_0,T_0}^{stocks}\right] = \mathbb{E}_{t_0}\left[\exp\left(\hat{r}_{t_0,T_0}^{stocks}\right)\right] - 1 \\ &= \exp\left(\hat{\mu}_{t_0,T_0}^{stocks} + \left(\hat{\sigma}_{t_0,T_0}^{stocks}\right)^2/2\right) - 1 \\ \textit{and } \left(\sigma_{t_0,T_0}^{stocks}\right)^2 &:= var_{t_0}\left(r_{t_0,T_0}^{stocks}\right) = var_{t_0}\left(\exp\left(\hat{r}_{t_0,T_0}^{stocks}\right)\right) \\ &= \exp\left(2\hat{\mu}_{t_0,T_0}^{stocks} + \left(\hat{\sigma}_{t_0,T_0}^{stocks}\right)^2\right)\left(\exp\left(\left(\hat{\sigma}_{t_0,T_0}^{stocks}\right)^2\right) - 1\right)\end{aligned}$$

Proof. We employ Lemma (B.0.1) to obtain

$$\begin{aligned}\hat{r}_{t_0,T_0}^{stocks} &= \int_{t_0}^{T_0} r(u)\,du + \left(\sigma_S\lambda_S - \frac{\sigma_S^2 + \rho^2}{2}\right)(T_0 - t_0) + \sigma_S\left(W_S(T_0) - W_S(t_0)\right) + \rho\left(W_r(T_0) - W_r(t_0)\right) \\ &= r_0\frac{e^{-\alpha_r t_0} - e^{-\alpha_r T_0}}{\alpha_r} + \int_{t_0}^{T_0}\theta(s)\left(1 - e^{-\alpha_r(T_0 - s)}\right)ds + \left(\sigma_S\lambda_S - \frac{\sigma_S^2 + \rho^2}{2}\right)(T_0 - t_0) \\ &\quad + \sigma_S\left(W_S(T_0) - W_S(t_0)\right) + \int_{t_0}^{T_0}\rho + \frac{\sigma_r}{\alpha_r}\left(1 - e^{-\alpha_r(T_0 - s)}\right)dW_r(s).\end{aligned}$$

□

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