

Moral Hazard in Loss Reduction and the State Dependent Utility

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Abstract: We consider a state dependent utility model with binary states where moral hazard occurs in loss reduction. We find different results depending on the relative sizes of the marginal utilities between the loss state and the no loss state. (i) If the marginal utilities are equal between the two states, the optimal insurance involves full insurance up to a limit and coinsurance above the limit, which corresponds to the case of the state independent utility. (ii) If the marginal utility in the loss state is greater than that in the no loss state, then the optimal insurance includes full insurance, and the moral hazard problem becomes less severe than under the case of the independent utility. (iii) If the marginal utility in the loss state is less than that in the no loss state, then the optimal insurance includes the deductible up to a limit and coinsurance above that limit, and the moral hazard problem becomes more severe. We extend the model into a two period setting, and apply it to the cases of a debt contract of a firm and a wage contract.

Keywords: moral hazard, state dependent utility, loss reduction effort, deductible, coinsurance

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I. Introduction

In the insurance context, moral hazard arises because purchasing insurance lowers the incentives to reduce risks, when insurers cannot observe the policyholders' efforts (actions). Moral hazard incurs costs by distorting the allocation of resources. With no moral hazard, a policyholder can achieve the utility at the (first-best) efficient level by purchasing full insurance. With moral hazard, however, the policyholder purchases partial insurance, failing to achieve the (first-best) efficiency, as shown in Shavell (1979), for example.

Insurance literature often distinguishes between two types of risk reduction: loss prevention and loss reduction. Loss prevention is to lower the probability of loss occurrence (frequency), while loss reduction is to lower the loss size (severity) when a loss occurs (see Ehrlich and Becker (1972) for the earlier economic research).² Examples are fire alarms and medical check-ups for the former, and car airbags and earthquake-resistant construction design for the latter.

Moral hazard may occur in regard both types of risk reduction, although a majority of research is concerned with loss prevention. It is known that moral hazard may lead to different insurance contracts depending on the types of risk reduction. Winter (2000) shows that the optimal insurance is full coverage above a deductible in the case of moral hazard in loss prevention, while it is full coverage up to a limit and partial insurance above the limit in the case of moral hazard in loss reduction.

On the other hand, moral hazard, in most cases, is addressed in monetary terms under the expected utility in literature. This approach presumes that a loss is monetary and the full compensation for the loss can fully recover the level of utility. That is, a loss is a replaceable good in terms of Cook and Graham (1977). While this approach is convenient, there has been a concern that there may be the cases where monetary compensation cannot fully substitute a loss. Cook and Graham (1977) point out that the loss of an irreplaceable good may lower the policyholder's utility more than the monetary loss does. For example, health is an irreplaceable good, so full coverage of medical expenses is not a perfect substitute for good health.

To address this concern, a so-called state dependent utility approach has been utilized in literature. Under the state dependent utility, the loss occurrence may also change the utility function, so that the simple monetary compensation may fail to recover the same level of utility as the pre-loss one. Technically, the change of utility function leads to the change of marginal utility, which produces different outcomes from the standard state independent approach.

Shioshansi (1982), Schlesinger (1984) and Huang and Tzeng (2006) analyze the optimal insurance for the irreplaceable loss under no moral hazard. Shioshansi (1982) suggests a

² Some confusion exists in terminology. For example, Ehrlich and Becker (1972) use self-insurance for loss reduction and self-protection for loss prevention. In the health economics context, Barigozzi (2004) uses primary prevention for loss prevention and secondary prevention for loss reduction.

simple state dependent utility approach to study insurance for irreplaceable commodity. Schlesinger (1984) develops on Shioshansi to allow marginal utilities to vary on the income level as well as states of nature. Both analyses of Shioshansi and Schlesinger are based on the assumption that the insurance contract is of a coinsurance form. On the other hand, Huang and Tzeng (2006) study the optimal form of the insurance contract and find that it includes a deductible and coinsurance above the deductible, which recoups the results of Raviv (1979). Note, however, that the results of Huang and Tzeng are based on the state dependent utility, while those of Raviv (1979) are based on the positive costs and a risk-averse insurer.

Dionne (1982) examines the insurance contract design under state dependent utility and moral hazard in loss prevention. He finds that the optimal coverage is partial due moral hazard and the effort level is affected by the state dependence of utility. He further argues that coverage is higher and effort is lower under state dependence than under state independence if the marginal utility in the loss state is greater than that in the no loss state.

In the similar spirit of Dionne, we also investigate the optimal insurance design under the state dependent utility and moral hazard. Unlike Dionne, however, we study the moral hazard problem in loss reduction. Given that our focus is on loss reduction, we are concerned with the cases of natural disaster outbreaks such as typhoons, earthquakes, and hurricanes whose occurrences are not influenced by human efforts. People can only act to reduce the damage. Recently, we have observed the huge damages from the tsunami in Japan (2011) and the typhoon in the Philippines (2013). We believe that this study can shed light on the optimal insurance contract against catastrophe losses.

Loss reduction and the state dependent utility are also important in the health care and insurance context. First, health is often irreplaceable in that a full reimbursement of medical expenses may not fully recover the quality of life prior to the health loss. Thus, the state dependent utility may be useful in understating health insurance. Second, loss reduction (or secondary prevention) is an important concern for both treatment and prevention of further development of a disease, once the disease is detected (see Ellis and Manning, 2007 and Barigozzi, 2004).

We find that the optimal insurance form depends on the relative sizes of marginal utilities in different states. If the marginal utility in the loss state is greater than that in the no loss state, then the optimal insurance form contains full coverage up to a limit and coinsurance above the limit. Full insurance is possible as well when state dependency is sufficiently large. On the other hand, if the marginal utility in the loss state is smaller than marginal utility in the no loss state, then the optimal insurance includes that of the deductible up to a limit and coinsurance above the limit. This second case is consistent with Huang and Tzeng in obtaining the result of Raviv, even if we do not assume costs and a risk-averse insurer.

We also examine the relative effort level compared to the state independent utility case. If we assume that the coverage level is equal between the two utilities, then the effort level depends on the marginal utility of income in the loss state as well. When the marginal utility of income in the loss state is equal to that of income in the no loss state, the optimal effort level is also equal. Given the indemnity level, if the marginal utility of income in the loss state is greater than that of income in the no loss state, then the relative effort level is higher. Thus, we can state that the moral hazard problem is naturally relieved. In the reverse case, the effort is lower and moral hazard seems to be more severe.

We extend the model into a two period model. Under a state dependent utility, in the

case where marginal utility of income in the loss state is higher than that of income in the no loss state, the loss reduction effort at time 2 can be higher when the loss occurs at time 1 compared to the effort when there is no loss in the first period. In the reverse case, the effort in second period can be lower.

Beyond these results, we also find applications in the real world, such as debt contracts under financial distress situations, and wage contracts. If we assume that production cost function is state dependent because of reputation cost when profit is low, then the effort to increase the operating income can be higher. The reverse case is also possible. We can find the optimal wage contract when employees have a state dependent utility.

This paper proceeds as follows. Section II describes the model. Section III presents the state dependent utility case under no moral hazard. Section IV analyzes the moral hazard problem with state dependent utility and the optimal insurance forms. It also compares the effort level with that under the state independent utility. Section V extends the model into a two period context. Section VI discusses the applications. The last section concludes.

II. Model description

We consider a two state model. Uncertainty is represented by two states regarding loss occurrence, denoted by S : the no loss state ($S = 0$) and the loss state ($S = 1$). Following the state, the utility function is denoted as $u(W, S)$ when W is income. If loss occurs, the utility is lower than the utility without the loss even if the loss is recovered, so $u(W, S = 0) > u(W, S = 1)$ for all W .

The utility function is twice differentiable and strictly concave with respect to an income. That is, $u'(W, S) > 0$ and $u''(W, S) < 0$ for all S . The policyholders' cost function for the effort is $c(e)$. We suppose that $c'(e) \geq 0$ and $c''(e) \geq 0$. We also assume that all policyholders are homogeneous. As a result, an adverse selection problem does not exist in this model.

We assume that the probability of loss occurrence is p and that p is not controlled by effort. The loss size is x and x follows the distribution $f(x; e)$ on a support of $[\underline{x}, \bar{x}]$ given effort e . We impose further restriction on the distribution given e , called the Monotonic Likelihood Ratio Condition, MLRC (Milgrom, 1981). This condition is that

given $e_H > e_L$, $\frac{f(x; e_H)}{f(x; e_L)}$ is decreasing in x . That is, as the effort increases, the likelihood

of occurrence of loss decreases. If the effort is increased, however, we suppose that the support of x does not change. Q is the insurance premium and $I(x)$ is the indemnity. We also suppose that the insurance market is competitive, so the insurer's profit should be zero.

III. No Moral Hazard Case

As a reference, let us note the first best cases with state independent utility and state dependent utility. We suppose that loss size x has the density function $f(x; e)$ given e . The individual maximizes his utility. Then the program is

$$\text{Max}_{e, I(x), Q} [1-p]u(W-Q, S=0) + p \int u(W-Q-x+I(x), S=1)f(x;e)dx - c(e) \quad (1)$$

$$\text{s.t. } Q = p \int I(x)f(x;e)dx \quad (2)$$

The Lagrangian is, where η is the Lagrange multiplier

$$L = [1-p]u(W-Q, S=0) + p \int u(W-Q-x+I(x), S=1)f(x;e)dx - c(e) \\ + \eta [Q - p \int I(x)f(x;e)dx]$$

For notational simplicity, let us write $u_0(W) = u(W, S=0)$ and $u_1(W) = u(W, S=1)$. We also use the notation $W_0 = W - Q$, $W_1 = W - Q - x + I(x)$. Then the first order conditions for optimum with respect to $Q, I(x)$ and e are as follows.

$$L_Q = -[1-p]u'_0(W_0) - p \int u'_1(W_1)f(x;e)dx + \eta = 0 \quad (3)$$

$$L_{I(x)} = pu'_1(W_1)f(x;e) - \eta pf(x;e) = 0, \text{ for each } x \quad (4)$$

$$L_e = p \int u_1(W_1)f_e(x;e)dx - c'(e) - \eta p \int I(x)f_e(x;e)dx = 0 \quad (5)$$

$$L_\eta = Q - p \int I(x)f(x;e)dx = 0 \quad (6)$$

We have followed the first best solution, rearranging first order conditions.

$$u'_0(W_0) = u'_1(W_1) \quad (7)$$

We know that in case of state independent utility, full insurance is optimal. That is, $I(x) = x$, since the expression (7) changes into $u'_0(W_0) = u'_0(W_1)$ in the state independent case. We also obtain the optimal amount of effort by (5). The effort should satisfy equation (8).

$$-u'_0(W_1)p \int xf_e(x;e)dx = c'(e) \quad (8)$$

Recall that the LHS of (8) is transformed to $u'_0(W_1)p \int F_e(x;e)dx$ because $\int xf_e(x;e)dx = xF_e(x;e)|_x^{\bar{x}} - \int F_e(x;e)dx = -\int F_e(x;e)dx$. As a result, (8) is positive. The LHS of (8) implies the marginal benefit of taking care to reduce the loss size and consequently to reduce the premium. The RHS represents the marginal cost of taking such care.

However, if policyholders have state dependent utility, then the optimal insurance coverage and effort level can be different from the coverage under state independent utility depending on the relative size of marginal utilities of income between the loss and the no loss state. Now, let us define that the state dependent utility in the loss state is as below:

$$u_1(W) = u_0(W) - A(W), A(W) > 0, \forall W \quad (9)$$

Then, we have the following relation:

Relation 1. If $u_0'(W) = u_1'(W)$, $\forall W$, then $A'(W) = 0$. That is, $A(W) = A$, A is a constant.

Relation 2. If $u_0'(W) < u_1'(W)$, $\forall W$, then $A'(W) < 0$.

Relation 3. If $u_0'(W) > u_1'(W)$, $\forall W$, then $A'(W) > 0$.

These relations are depicted in figure 1,2, and 3.

According to Schlesinger, relation 2 can be explained by the conjecture that the loss makes people more appreciative of additions to wealth. Relation 3 can be interpreted in the opposite way.

In addition, we define compensation $\theta(W)$ for the loss satisfying following expression.

$$u_0'(W) = u_1'(W + \theta(W)), \forall W \quad (10)$$

This definition differs from that of Cook and Graham regarding of compensation. Cook and Graham define compensation based on utility, but we use marginal utility of income.

By the expression (10) and the Taylor expansion, we obtain the following expression.

$$A'(W) \approx \theta(W)u_1''(W) \quad (11)$$

By the concavity of the utility, we have:

$$A'(W) < 0 \Leftrightarrow \theta(W) > 0 \quad \text{and} \quad A'(W) > 0 \Leftrightarrow \theta(W) < 0 \quad (12)$$

The expression (12) indicates that if the marginal utility in the loss state is greater than that in the no loss state, individuals claim positive compensation even if the wealth level is the same. On the contrary, if the marginal utility in the loss state is less than that in the no loss state, negative compensation is possible. Let us define $|A'(W)|$ as a state dependency.

Then we have $|\theta(W)| \approx \frac{|A'(W)|}{|u_1''(W)|}$ by (11). Thus, we know that the compensation increases when state dependency increases, or the curvature of utility with loss decreases.

Meanwhile, there exists a relationship between compensation $C(W)$ which is defined by Cook and Graham and $\theta(W)$. $C(W)$ satisfies that $u_0(W) = u_1(W + C(W))$, which moves the insured to a utility level without accident state. Using the Taylor expansion, we know that:

$$u_0(W) = u_1(W + C(W)) \approx u_1(W) + C(W)u_1'(W)$$

Thus, we obtain $C(W)u_1'(W) \approx A(W)$ and

$$C'(W) \approx \frac{A'(W)u_1'(W) - A(W)u_1''(W)}{[u_1'(W)]^2} = \frac{A'(W) + A(W)ARA_1(W)}{u_1'(W)} \quad (13)$$

where $ARA_1(W)$ is the absolute risk aversion in the loss state.

By (13), we obtain $C'(W) > 0$ ³ when $A'(W) \geq 0$. In particular, as the absolute risk aversion increases, $C'(W)$ increases as well. On the other hand, if $A'(W) < 0$ then the sign of $C'(W)$ is ambiguous. However, we can say $C'(W) < 0$ when $A'(W)$ or the absolute risk aversion is sufficiently small.

Now, we have Lemma 1.

Lemma 1. (No Moral Hazard Case) Under a state dependent utility, we obtain following results.

- (1) The optimal coverage is full insurance when $u_0'(W) = u_1'(W)$, $\forall W$.
- (2) The optimal coverage is full insurance if over-insurance is not allowed and over-insurance is optimal if possible when $u_0'(W) < u_1'(W)$, $\forall W$.
- (3) The optimal coverage is partial insurance when $u_0'(W) > u_1'(W)$, $\forall W$.

Proof. See the Appendix.//

In the case of $u_0'(W) = u_1'(W)$, the utility after receiving full coverage is lower than the utility without an accident. This is because excess payment for equivalent utility in both states may increase the premium. Hence, the marginal cost of insurance is higher than the marginal benefit of insurance. As a result, full insurance is optimal even if the utility with an accident still lower after paid benefit. In the case of $u_0'(W) < u_1'(W)$, the optimal insurance is over-insurance which is decided at the level that makes marginal utilities identical in the loss state and no loss state, while partial insurance is optimal if $u_0'(W) > u_1'(W)$.

We also obtain Lemma 2 in terms of an optimal effort amount.

Lemma 2. (No moral hazard case) Under a state dependent utility, we have following results.

- (1) The optimal effort is equivalent to a state independent utility case when $u_0'(W) = u_1'(W)$, $\forall W$.
- (2) The optimal effort is greater than that of a state independent utility case when

³ Cook and Graham indicate that if the irreplaceable goods are classified as normal goods, then $C'(W) > 0$. On the contrary, $C'(W) < 0$ when the goods are considered as inferior goods. They also argue that replaceable goods can be viewed as a special case of irreplaceable goods when $C'(W) = 0$.

$u_0'(W) < u_1'(W), \forall W$.

(3) The optimal effort is less than that of a state independent utility case when

$u_0'(W) > u_1'(W), \forall W$.

Proof. See the appendix.//

Although utility is state dependent, since the marginal utility of income is identical in both the loss state and the no loss state, the marginal benefit of putting in an effort to decrease the loss size and premium does not change. Thus, the optimal effort level is unchanged.

On the other hand, if $u_0'(W) < u_1'(W)$, then the marginal benefit of additional effort increases due to disutility. Consequently, the policyholder increases his effort until the marginal benefit of taking care to lower the loss size is equal to the marginal cost of taking such care. The $u_0'(W) > u_1'(W)$ case can be explained in opposite way.

IV. Moral Hazard Case

In this chapter, we develop the loss reduction model with moral hazard based on a state dependent utility. The insured maximizes his utility, so the expected utility is as follows:

$$\text{Max}_{e, I(x), Q} [1-p]u(W-Q, S=0) + p \int u(W-Q-x+I(x), S=1)f(x;e)dx - c(e) \quad (14)$$

$$\text{s.t. } Q = p \int I(x)f(x;e)dx \quad (15)$$

$$e = \arg \max_a [1-p]u(W-Q, S=0) + p \int u(W-Q-x+I(x), S=1)f(x;a)dx - c(a) \quad (16)$$

The second condition is the incentive compatibility constraint under moral hazard. We suppose that the conventional “first order approach” is valid, then the second constraint (16) becomes

$$p \int u(W-Q-x+I(x), S=1)f_e(x;e)dx - c'(e) = 0, \text{ where } \int f_e(x;e)dx = \frac{d}{de} \int f(x;e)dx \quad (17)$$

Let us use the same notation for abbreviation as in the previous section III. The Lagrangian is expressed as:

$$\begin{aligned} L = & (1-p)u_0(W_0) + p \int u_1(W_1)f(x;e)dx - c(e) + \lambda[Q - p \int I(x)f(x;e)dx] \\ & + \gamma[p \int u_1(W_1)f_e(x;e)dx - c'(e)] \end{aligned} \quad (18)$$

The first order conditions are

$$L_Q = -(1-p)u'_0(W_0) - p \int u'_1(W_1)f(x;e)dx + \lambda - \gamma p \int u'_1(W_1)f_e(x;e)dx = 0 \quad (19)$$

$$L_{I(x)} = pu'_1(W_1)f(x;e) - \lambda pf(x;e) + \gamma pu'_1(W_1)f_e(x;e) = 0, \text{ for each } x \quad (20)$$

$$L_e = -\lambda p \int I(x) f_e(x; e) dx + \gamma [p \int u_1(W_1) f_{ee}(x; e) dx - c''(e)] = 0 \quad (21)$$

Rearranging these conditions, we have

$$\frac{1}{u_1'(W_1)} = \frac{1}{u_0'(W_0)} \left(1 + \gamma \frac{f_e(x; e)}{f(x; e)}\right) \quad (22)$$

From (22), we obtain Lemma 3.

Lemma 3. (Holmstrom) Under a state dependent utility, $\gamma > 0$.

Proof. See the Appendix.//

Lemma 3 is consistent with the result of Holmstrom (1979). The condition $\gamma > 0$ means that the insurer also designs the contract to induce more effort from the insured under a state dependent utility.

Before examining the optimal coverage with moral hazard under a state dependent utility, we investigate the coverage under a state independent utility. Winter (2000) remarks that the optimal coverage involves full coverage for small losses and partial insurance for high losses. We obtain the same result as Winter when we suppose that $u_0'(W) = u_1'(W), \forall W$.

As is evident in the no moral hazard case under a state dependent utility, the optimal coverage depends on not only the loss size but also the marginal utility of income. We consider three cases following comparison with the relative size of marginal utilities, and find the optimal coverage in each case as well. We obtain proposition 1.

Proposition 1. (Moral hazard case) Under a state dependent utility, the following results hold.

- (1) The optimal coverage includes full insurance up to a limit and coinsurance above the limit if $u_0'(W) = u_1'(W), \forall W$. In particular, this is equivalent to the optimal coverage level under a state independent utility.
- (2) The optimal coverage includes full insurance up to a limit and coinsurance above the limit if $u_0'(W) < u_1'(W), \forall W$. It also can be full insurance for all loss when a state dependency is sufficiently large.
- (3) The optimal coverage includes a deductible up to a limit and coinsurance above the limit if $u_0'(W) > u_1'(W), \forall W$. It can also be full insurance up to a limit and coinsurance above the limit when a state dependency is sufficiently large.

Proof. See the Appendix.//

Recall that the penalty is given only to the loss size. From (22), we know that insurers penalize the insured for the large loss since the effort affects loss size. On the other hand, insurers do not penalize and give incentive the insured for the small loss to induce the effort as much as possible. As a result, up to a limit, the over-insurance is optimal if it is allowed, and coinsurance is optimal above the limit for all three cases. Assuming that the over-insurance is not allowed, full insurance is optimal up to a limit. In particular, if

$u_0'(W) = u_1'(W)$, $\forall W$ then, the coverage is identical with the state independent utility case. This is because marginal utilities are equal for the same level of income, so the first order condition for the maximization is equivalent to the case with a state independent utility.

We also can see that if $u_0'(W) < u_1'(W)$, $\forall W$, then full insurance is possible when state dependency is sufficiently large. Policyholders need more indemnity than the state independent case, as the state dependency is larger. On the contrary, if $u_0'(W) > u_1'(W)$, $\forall W$ and state dependency is sufficiently larger, then policyholders may require a lower level of indemnity than the case with state independent utility. Therefore, negative insurance can be possible up to a limit. However, negative insurance is not feasible in reality, so the deductible is optimal up to a limit.

We can compare the relative effort level between a state dependent utility case and a state independent utility case as well. This is Proposition 2.

Proposition 2. (Moral Hazard Case) Under a state dependent utility, if the coverage is equal to the coverage with a state independent utility for all coverage level, then we have followings.

- (1) The optimal amount of an effort is equal to an effort under a state independent utility if $u_0'(W) = u_1'(W)$, $\forall W$.
- (2) The optimal amount of an effort is more than an effort under a state independent utility if $u_0'(W) < u_1'(W)$, $\forall W$.
- (3) The optimal amount of an effort is less than an effort under a state independent utility if $u_0'(W) > u_1'(W)$, $\forall W$.

Proof. See the Appendix.//

In Proposition 2, (1) is intuitively clear. The loss does not change individual's attitude to the additional wealth. Therefore, the optimal effort level may be the same. Meanwhile, if $u_0'(W) < u_1'(W)$, then individuals cherish more the additional wealth under a state dependent utility than under a state independent utility. Thus, given indemnity individuals make more effort. Hence, we can say that the moral hazard problem is less severe. In the case of $u_0'(W) > u_1'(W)$, the effort is lower, so the moral hazard appears more severe.

As a result, the moral hazard problem is a still significant problem under the state dependent utility with loss reduction. However, the concern about the moral hazard problem might be excessive or should be higher depending on the marginal utility of income.

V. Extension – Two Period and Moral Hazard Case

In this section, we extend the loss reduction model from one period to a two period model under a state dependent utility. The state regarding loss occurrence is denoted by S_t , where t is period 1 and 2. The loss size and effort are denoted x_t and e_t respectively. Following the loss size and effort level in each period, the indemnity is represented as $I_1(x_1), I_2(x_1, x_2)$. The premium at time t is denoted as Q_t . We suppose that the state variables are independent each other and the effort at time 1 does not affect loss size at time 2. Consequently, we can write the joint density of loss given the effort level as follows.

$$\iint f(x_1, x_2; e_1, e_2) dx_1 dx_2 = \int f(x_1; e_1) dx_1 \int f(x_2; e_2) dx_2 \quad (23)$$

We also suppose that the loss size in each period has the same distribution and that the distribution of loss size given e still follows MLRC. The effort in period 2 depends on the loss size at time 1. That is, $e_2 = e_2(x_1)$.

We assume that both the insurer and the insured commit to the long term insurance contract. We suppose the ‘‘First order approach’’ on effort is also valid in a two period model as well. Thus, the problem is,

$$\begin{aligned} \underset{Q_1, Q_2, I_1(x_1), I_2(x_1, x_2), e_1, e_2}{Max} \quad & (1-p)u(W - Q_1, S_1 = 0) + p \int u(W - Q_1 - x_1 + I_1(x_1), S_1 = 1) f(x_1; e_1) dx_1 - c(e_1) \\ & + (1-p)[(1-p)u(W - Q_2(x_1 = 0), S_1 = 0, S_2 = 0) \\ & + p \int u(W - Q_2(x_1 = 0) - x_2 + I_2(x_1 = 0, x_2), S_1 = 0, S_2 = 1) f(x_2; e_2(x_1 = 0)) dx_2 \\ & - c(e_2(x_1 = 0))] \\ & + p[(1-p) \int u(W - Q_2(x_1), S_1 = 1, S_2 = 0) f(x_1, e_1) dx_1 \\ & + p \int \int u(W - Q_2(x_1) - x_2 + I_2(x_1, x_2), S_1 = 1, S_2 = 1) f(x_1; e_1) f(x_2, e_2(x_1)) dx_1 dx_2 \\ & - \int c(e_2(x_1)) f(x_1; e_1) dx_1] \end{aligned} \quad (24)$$

$$s.t. \quad Q_1 = p \int I_1(x_1) f(x_1; e_1) dx_1 \quad (25)$$

$$Q_2(x_1) = p \int I_2(x_1, x_2) f(x_2; e_2(x_1)) dx_2, \text{ where } x_1 \neq 0 \quad (26)$$

$$Q_2(x_1 = 0) = p \int I_2(x_1 = 0, x_2) f(x_2; e_2(x_1 = 0)) dx_2 \quad (27)$$

$$\begin{aligned} & p \int u(W - Q_1 - x_1 + I_1(x_1), S_1 = 1) f(x_1; e_1) dx_1 \\ & + p(1-p) \int u(W - Q_2(x_1), S_1 = 1, S_2 = 0) f_{e_1}(x_1, e_1) dx_1 \\ & + p[p \int \int u(W - Q_2(x_1) - x_2 + I_2(x_1, x_2), S_1 = 1, S_2 = 1) f(x_2; e_2(x_1)) dx_2 f_{e_1}(x_1; e_1) dx_1 \\ & - \int c(e_2(x_1)) f_{e_1}(x_1; e_1) dx_1] = 0 \end{aligned} \quad (28)$$

$$p \int u(W - Q_2(x_1 = 0) - x_2 + I_2(x_1 = 0, x_2), S_1 = 0, S_2 = 1) f_{e_2}(x_2; e_2(x_1 = 0)) dx_2 - c'(e_2(x_1 = 0)) = 0 \quad (29)$$

$$\begin{aligned} & p \int u(W - Q_2(x_1) - x_2 + I_2(x_1, x_2), S_1 = 1, S_2 = 1) f_{e_2}(x_2; e_2(x_1)) dx_2 - c'(e_2(x_1)) = 0 \\ & \text{for each } x_1, x_1 \neq 0 \end{aligned} \quad (30)$$

Conditions (28), (29), and (30) are the incentive compatibility constraints. From the expression (28), we know that the insured should consider the effect of the effort at time 1 on the indemnity at time 2.

Like the preceding case, we simply denote utility function as $u_i(W) = u(W, S_1 = i)$ and $u_{ij}(W) = u(W, S_1 = i, S_2 = j)$, where $i, j = 0$ or 1 . In addition, we write wealth of each period as $W_0^t = W - Q_t$, where $t = 1, 2$, $W_1^1 = W - Q_1 - x_1 + I_1(x_1)$, $W_1^2(x_1 = 0) = W - Q_2(x_1 = 0) - x_2 + I_2(x_1 = 0, x_2)$, and $W_1^2(x_1) = W - Q_2(x_1) - x_2 + I_2(x_1, x_2)$ where $x_1 \neq 0$. Let us define the state dependent utility for period 2 as below.

$$u_{i1}(W) = u_{i0}(W) - A(W) \quad \text{where } i = 0, 1 \quad (31)$$

This is similar to (9). By (31), we know that $u_{11}(W) = u_{00}(W) - 2A(W)$. Using these notations, we obtain Lemma 4 from above problem.

Lemma 4. (Lambert)

(1) The optimal risk sharing rule is

$$\begin{aligned} \text{a. } & \frac{1}{u_1'(W_1^1)} = \frac{1}{u_0'(W_0^1)} \left(1 + \mu_1 \frac{f_{e_1}(x_1; e_1)}{f(x_1; e_1)}\right) \\ \text{b. } & \frac{1}{u_{00}'(W_0^2)} \left[1 + \mu_2(x_1 = 0) \frac{f_{e_2}(x_2; e_2(x_1 = 0))}{f(x_2; e_2(x_1 = 0))}\right] = \frac{1}{u_{01}'(W_1^2(x_1 = 0))} \\ \text{c. } & \frac{1}{u_{10}'(W_0^2)} \left[1 + \mu_1 \frac{f_{e_1}(x_1; e_1)}{f(x_1; e_1)} + \mu_2(x_1) \frac{1}{f(x_1; e_1)} \frac{f_{e_2}(x_2; e_2(x_1))}{f(x_2; e_2)}\right] = \frac{1}{u_{11}'(W_1^2(x_1))} \end{aligned}$$

(2) $\mu_2(x_1) > 0$ for all x_1

(3) $\mu_1 > 0$

Proof. See the Appendix.//

We know that the indemnity for the second period still depends on the loss size for the first period from $\mu_1 > 0$. In addition, $\mu_2(x_1) > 0$ implies that the higher loss size in the first period brings out the lower indemnity of the second period for each effort.

In the second period, if $x_1 = 0$ then the insured can fully hedge the risk. That is,

$$E_{x_2} \left[\frac{u_{00}'(W_0^2(x_1 = 0))}{u_{01}'(W_1^2(x_1 = 0))} \right] = 1. \quad \text{On the contrary, } E_{x_2} \left[\frac{u_{10}'(W_0^2(x_1))}{u_{11}'(W_1^2(x_1))} \mid x_1 \neq 0 \right] = 1 + \mu_1 \frac{f_{e_1}(x_1; e_1)}{f(x_1; e_1)}, \text{ so}$$

if $x_1 > 0$, then the insured may take the risk as $\mu_1 \frac{f_{e_1}(x_1; e_1)}{f(x_1; e_1)}$. This means that if the over-

insurance is not allowed, the insurer does not penalize the insured by some level of loss which occurs in the first period. However, the loss at time 1 exceeds the critical value, then, the insured is given the penalty in the second period. That is, the insurer can disperse the risk of the first period into the second period.

Now, we can compare the effort level at time 2 depending on the state dependency of

utility. We show that Proposition 3 is satisfied.

Proposition 3. (Moral Hazard in Two Period Case) Under a state dependent utility, suppose that the coverage of the second period when loss does not occur in the first period is equal to the coverage with a loss in the first period. Given coverage level, the following results hold.

- (1) The optimal amount of effort at time 2 with loss at time 1 is equal to an effort with no loss at time 1, if $u_0'(W) = u_1'(W), \forall W$.
- (2) The optimal amount of effort at time 2 with loss at time 1 is more than an effort with no loss at time 1, if $u_0'(W) < u_1'(W), \forall W$.
- (3) The optimal amount of effort at time 2 with loss at time 1 is less than an effort with no loss at time 1, if $u_0'(W) > u_1'(W), \forall W$.

Proof. See the Appendix.//

Due to the state dependency, the effort level under a state dependent utility in the second period can be higher or lower than that under a state independent utility. In addition, the results of Proposition 3 show that under a state dependent utility, when the accident occurs in the first period, the moral hazard problem of the second period may be alleviated or deepened, compared to the case with no loss of the first period depending on the marginal utilities of income between loss and no loss states.

Generally, it is known that the moral hazard problem is mitigated in a long term contract. However, when the loss is fortunately low in the first period, the strategic behavior where the insured slackens effort in the second period still exists in a long term contract. Proposition 3 shows that under a state dependent utility, the possibility of this type of strategic behavior may be lower in the case of $u_0'(W) < u_1'(W), \forall W$. In case of $u_0'(W) > u_1'(W), \forall W$, the possibility is higher as well.

VI. Applications

Models of loss reduction with moral hazard can explain some cases in the real market world. First of all, as previously stated, our model can explain the optimal insurance form for catastrophes such as floods or earthquakes. Since this kind of insurance is usually offered by government agencies such as FEMA (Federal Emergency Management Agency) of the U.S., because the loss size is usually very huge and it is not easy to estimate the probability of the natural disaster, our results may hold some policy implications. To be a little more specific, if $u_0'(W) < u_1'(W), \forall W$, the government indemnity includes full insurance to recover the insured's utility. In addition, the effort to reduce the loss can be higher than what people may have thought. That is, there exists the possibility that the moral hazard problem can be overemphasized. The reverse case can also be true.

Second, in health insurance, the x in this study can be considered as the expense for the health loss expressed as a monetary term. We can regard that illness as the accident as well. If the health expense includes an expense to reduce the risk of complications, then the loss reduction effort plays an essential role in the insurance payment. In addition, after the onset of illness, quality of life cannot be recovered. In the case where the utility depends on the loss occurrence, the moral hazard problem may not be critical. The effort to lower the

severity of the potential illness may be higher, and more indemnity compared to the state independent case can enhance efficiency when $u_0'(W) < u_1'(W), \forall W$. Note that the moral hazard with a loss reduction model can be interpreted as an ex-post moral hazard problem in this case.

Part of the health economics literature, such as Nyman (1999a, 1999b), Newhouse (2006), Ellis and Manning (2007), and Seog (2012) suggests that more generous insurance coverage can improve the welfare as well. Nyman indicates that the conventional approach overstates the welfare loss due to the moral hazard. Newhouse argues that lower cost sharing would reduce total cost, including future cost, by spending more on health services. Seog also claims that the optimal coverage should be higher when treatment is not preventative. Although Ellis and Manning focus on the loss preventive effort, the argument that more coverage can be optimal is consistent with our results.

We can explain other examples using our model. Let us discuss the optimal debt contract. There is an enterprise with two states denoted by S : output is lower than debt ($S = 1$) and profit is higher than the debt ($S = 0$). For simplicity, we assume that the probability with state S is p and $1 - p$. In a low profit state, the profit π given effort e follows the distribution $f(\pi; e)$, and the profit follows the distribution $g(\pi; e)$ in a high profit state. We can interpret π as operating income. These distributions follow MLRC. The cost is divided into two parts. The cost function for effort is denoted by c . The cost function v means the production cost function, and v is convex. We suppose that $v(\pi, S = 0) = v(\pi, S = 1) - A(\pi)$. If the profit is low, then the reputation cost is increased and the value of the brand is decreased, so we can assume that $A(\pi) > 0$. The debt principal is I and the interest rate is r_1 . We denote the debt repayment schedule as $B(\pi)$ and $B(\pi)$ is a non-decreasing function. We first note that the production cost function is state independent. That is, $v(\pi, S = 0) = v(\pi, S = 1)$. The model is as follows:

$$\begin{aligned} & \underset{B(\pi), e}{\text{Max}} \quad p \int_0^{\pi_1} [\pi - B(\pi) - v(\pi)] f(\pi; e) d\pi + (1 - p) \int_{\pi_1}^{\infty} [\pi - B(\pi) - v(\pi)] g(\pi; e) d\pi - c(e) \\ \text{s.t.} \quad & p \int_0^{\pi} B(\pi) f(\pi; e) d\pi + (1 - p) \int_{\pi}^{\infty} B(\pi) g(\pi; e) d\pi = (1 + r_1) I \\ & p \int_0^{\pi_1} [\pi - B(\pi) - v(\pi)] f_e(\pi; e) d\pi + (1 - p) \int_{\pi_1}^{\infty} [\pi - B(\pi) - v(\pi)] g_e(\pi; e) d\pi = c'(e) \end{aligned}$$

$$\begin{aligned} \text{where} \quad & 0 \leq B(\pi) \leq \min(\pi - v(\pi), (1 + r_1) I), \quad \pi_1 = (1 + r_1) I \\ & 0 \leq v'(\pi) \leq \pi \end{aligned}$$

Solving this problem, we obtain Result 1.

Result 1. Under a state dependent production function, if debt interest is equal, then we have following results.

- (1) The effort is equal to the effort under a state independent production cost function when $v'(\pi, S = 0) = v'(\pi, S = 1)$.
- (2) The effort is higher than the effort under a state independent production cost function when $v'(\pi, S = 0) > v'(\pi, S = 1)$.

- (3) The effort is lower than the effort under a state independent production cost function when $v'(\pi, S=0) < v'(\pi, S=1)$.

Proof. See the Appendix.//

Result 1 is consistent with Proposition 3. If the production cost function is state dependent, then the moral hazard problem of a borrower may be less severe than the problem under a state independent cost function. This case can be interpreted that the extra cost $A(\pi)$, which is brand value or reputation cost, decreases as π increases. In the reverse case, the moral hazard problem is more significant.

Lastly, we investigate the moral hazard problem in the wage contract when the employee has a state dependent utility. We denote the salary and the reservation utility as $S(\pi)$ and \bar{u} , respectively. Like the preceding examples, we assume that there exist two states. State 0 indicates that the profit is higher than the limit value π^* , and state 1 presents the profit as being lower than π^* . In each state, the profit π given effort e has the density functions $g(\pi; e)$ and $f(\pi; e)$. Both $g(\pi; e)$ and $f(\pi; e)$ satisfy MLRC. The cost function for employee effort is c . We also suppose that $u(S(\pi), S=1) = u(S(\pi), S=0) - A(S(\pi))$ and $A(S(\pi)) > 0$. We can regard this $A(S(\pi))$ as a sentiment value or the reputation of an employee. This means that if performance is below some critical value, then the employee's utility is decreased additionally. The model is

$$\begin{aligned} & \text{Max}_{S(\pi), e} p \int_0^{\pi^*} [\pi - S(\pi)] f(\pi; e) d\pi + (1-p) \int_{\pi^*}^{\infty} [\pi - S(\pi)] g(\pi; e) d\pi \\ \text{s.t. } & p \int_0^{\pi^*} u(S(\pi), S=1) f(\pi; e) d\pi + (1-p) \int_{\pi^*}^{\infty} u(S(\pi), S=0) g(\pi; e) d\pi - c(e) = \bar{u} \\ & p \int_0^{\pi^*} u(S(\pi), S=1) f_e(\pi; e) d\pi + (1-p) \int_{\pi^*}^{\infty} u(S(\pi), S=0) g_e(\pi; e) d\pi = c'(e) \end{aligned}$$

The solution of this problem exhibits the following Result 2. Result 2 shows that the moral hazard problem should be considered cautiously as well.

Result 2. Under a state dependent utility, if the salary is given then we obtain following results.

- (1) The effort of an employee is equal to the effort under a state independent utility when $u'(S(\pi), S=1) = u'(S(\pi), S=0)$.
- (2) The effort of an employee is higher than the effort under a state independent utility when $u'(S(\pi), S=1) > u'(S(\pi), S=0)$.
- (3) The effort of an employee is lower than the effort under a state independent utility when $u'(S(\pi), S=1) < u'(S(\pi), S=0)$.

Proof. See the Appendix.//

VII. Conclusion

This study considers a state dependent utility under moral hazard, focusing on the loss

reduction effort. We assume a two state-model: loss occurrence state and no loss occurrence state. According to the marginal utility of income on the states, we analyze the optimal insurance coverage. If the marginal utility of income is equal between the two states, the optimal indemnity and effort level are identical with the state independent utility case. Optimal insurance involves full insurance up to a limit and coinsurance above the limit. If the marginal utility of income in the loss state is greater than that of income in no loss state, then the optimal insurance includes full insurance. On the contrary, if the relation is the reverse between the marginal utilities, then the optimal insurance includes the deductible up to a limit and coinsurance above that limit. In this case, full insurance also can be involved. This paper also investigates whether the moral hazard is more or less severe under a state dependent utility as well. As a result, if the indemnity level is equal to the indemnity under a state independent utility, the effort is higher than that of a state independent utility when the marginal utility of income in the loss state is larger than that of income in the no loss state. That is, the moral hazard problem may be relieved. In contrast, moral hazard can be more severe when the marginal utility of income in the loss state is lower than that of income in the no loss state. We extend the state dependent model into a two period model. The effort level at time 2 when the loss occurs in the first period can be higher than the effort when the loss did not occur at time 1, if the marginal utility of income in the loss state is greater than that of income in the no loss state. This model can be applied in a debt contract and a wage contract models.

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Appendix

1. Proof of Lemma 1.

- (1) This case is equivalent to the state independent case, so is omitted.
- (2) By (7), we know that $u_0'(W_0) = u_1'(W_1)$. Thus, if $u_0'(W) < u_1'(W), \forall W$, then we can write $u_0'(W_0) = u_1'(W_0 + \theta(W_0))$ and $\theta(W_0) > 0$ by (10) and (12). As a result, $I(x) = x + \theta(W_0)$.
- (3) This proof is similar to the (2), so we would skip. //

2. Proof of Lemma 2.

- (1) In equation (8), $u_1'(W_1) = u_0'(W_1)$ and indemnity is full insurance, which is equal to the indemnity under a state dependent utility. Thus, the optimal effort level is identical with the effort under a state independent utility.
- (2) Recall that the optimal indemnity is greater than the indemnity under state independent utility by lemma1. (8) is changed as follows.

$$-u_1'(W_0 + \theta(W_0))p \int (x + \theta(W_0))f_e(x; e)dx = -u_0'(W_0)p \int xf_e(x; e)dx = c'(e) \quad (\text{A.1})$$

Since the premium is increased as $I(x)$ increases in (8), following expression is satisfied with respect to e^* which is the optimal effort level under a state independent utility.

$$-u_0'(W_0)p \int xf_e(x; e^*)dx - c'(e^*) > 0$$

As a result, the effort level is increased. Furthermore, if over insurance is not allowed, then

$u_1'(W_1) > u_0'(W_1)$ in LHS of (8) so the effort has to be increased as well. Thus, the effort level under a state dependent utility increases when $u_0'(W_0) < u_1'(W_1), \forall W$.

(3) The proof is similar to the proof of (2), so is omitted.//

Proof of Lemma 3.

Case 1) $u_0'(W) = u_1'(W), \forall W$

At first, we suppose that $\gamma = 0$. To satisfy equation (22), $I(x) = x$. However, with the constraint that $p \int u_0(W_1) f_e(x; e) dx - c'(e) = 0$, we know that the following condition is satisfied and this is a contradiction..

$$0 = p \int u_0(W_1) f_e(x; e) dx - c'(e) = p u_0(W_1) \int f_e(x; e) dx = c'(e) > 0. \quad (\text{A.2})$$

On the other hand, suppose $\gamma < 0$. Let us define that $x^+ \in \{x | f_e(x; e) > 0\}$ and $x^- \in \{x | f_e(x; e) < 0\}$. We also define $I^-(x)$ as the indemnity for x^- and $I^+(x)$ for x^+ . Thus we have

$$\begin{aligned} \frac{1}{u_0'(W_1(I^-(x)))} &= \frac{1}{u_1'(W_0)} (1 + \gamma \frac{f_e(x; e)}{f(x; e)}) \geq \frac{1}{u_0'(W_1(I = x))} \\ \Leftrightarrow u_0'(W_1(I^-(x))) &\leq u_0'(W_1(I)) \\ \Leftrightarrow I^-(x) &\geq I = x \end{aligned} \quad (\text{A.3})$$

In a similar way, we obtain, $I^+(x) \leq I(x) = x$

We obtain the following relation as well:

$$\begin{aligned} \int I(x) f_e(x; e) dx &\leq \int [I(x) - x] f_e(x; e) dx = \int_{x^-} [I(x) - x] f_e(x; e) dx + \int_{x^+} [I(x) - x] f_e(x; e) dx < 0 \\ (\text{A.4}) \\ (\because \int x f_e(x; e) dx &= x F_e(x; e) \Big|_{x^-}^{\bar{x}} - \int F_e(x; e) dx = - \int F_e(x; e) dx \leq 0) \quad (\text{by First Order Stochastic} \\ \text{Dominance}) \end{aligned}$$

This is a contradiction. As a result, $\gamma > 0$

Case 2) $u_0'(W) < u_1'(W), \forall W$

Likewise Case 1, by (23), $u_1'(W_1) > u_0'(W_0)$ for x with $f_e(x; e) > 0$, so we have $I(x) < x + \theta$, where $u_1' = u_0'$ when $I(x) = x + \theta, \theta > 0$. In this case, by (10) and (12), where $\theta = \theta(W_0)$. We also know $u_1'(W_1) < u_0'(W_0)$ for x with $f_e(x; e) < 0$, so

$I(x) > x + \theta$. If $\gamma = 0$ then, $I(x) = x + \theta$ so it is a contradiction that.

$$0 = pu_0(W_1) \int f_e(x; e) dx = c'(e) > 0.$$

In addition, $\gamma < 0$. This is also a contradiction, because $\int \theta f_e(x; e) dx = 0$.

Case 3) $u_0'(W) > u_1'(W), \forall W$

By (18), $u_1'(W_1) < u_0'(W_0)$ for x with $f_e(x; e) < 0$, so we know $I(x) > x - \theta$ and by (10) and (12), where $\theta = \theta(W_0)$. In addition, it should be satisfied $u_1'(W_1) > u_0'(W_0)$ for $f_e(x; e) > 0$, we have $I(x) < x - \theta$. At this time, $I(x) = x - \theta$ to fulfill $u_1'(W_1) = u_0'(W_0)$. In a way similar to Case 2, $\gamma \leq 0$ is a contradiction. //

Proof of Proposition 1.

We first prove that all three cases involve full insurance up to a limit and coinsurance above that limit. By (23) and the proof of Lemma 1, we observe that optimal insurance includes full insurance up to a limit. Now we prove that the coinsurance above the limit is included in the optimum. Let us assume that $x_L < x^1 < x^2$ for x_L such that $f_e(x_L, e) = 0$. Then by MLRC, $f_e(x^1; e) > f_e(x^2; e)$ is satisfied, so we can derive the following relation.

$$\begin{aligned} \frac{1}{u_1'(W - Q - x^1 + I(x^1))} &> \frac{1}{u_1'(W - Q - x^2 + I(x^2))} \\ \Leftrightarrow u_1'(W - Q - x^1 + I(x^1)) &< u_1'(W - Q - x^2 + I(x^2)) \\ \Leftrightarrow -x^1 + I(x^1) &> -x^2 + I(x^2) \\ \frac{I(x^2) - I(x^1)}{x^2 - x^1} &< 1 \end{aligned} \quad (\text{A.5})$$

As a result, coinsurance can be possible.

(1) We know that $u_0(W) = u_1(W) + A$, where A is a constant. Then, $u_0'(W) = u_1'(W)$. We also have, by (17),

$$\begin{aligned} 0 &= p \int u_1(W_1) f_e(x; e) dx - c'(e) = p \int [u_0(W_1) - A] f_e(x; e) dx - c'(e) \\ &= p \int u_0(W_1) f_e(x; e) dx - c'(e) \end{aligned}$$

because $\int A f_e(x; e) dx = 0$. That is, two utilities have an identical coverage and effort level in optimum.

(2) By proof of case 2 in Lemma 1, for $f_e(x_L, e) = 0$ we know that the

$u'_0(W_1) = u'_1(W - Q - x_L + I(x_L))$ and $I(x_L) = x_L + \theta(W - Q)$. If the level of $\theta(W - Q) = \theta$ is sufficiently large, then full coverage is possible.

(3) In similar ways with (3), we see that $I(x_L) = x_L - \theta$ for $f_e(x_L, e) = 0$ by proof of Case 3 in Lemma 1. Thus, if $\theta(W - Q) = \theta$ is sufficiently large, then optimal insurance can be deductible up to a limit and coinsurance above the limit. On the other hand, $\theta(W - Q) = \theta$ is sufficiently small, then optimal insurance can be full insurance as well. //

Proof of Proposition 2.

(1) The proof is similar to the proof of (1) of Proposition 1, so is omitted here.

(2) Under a state dependent utility, equation (17) is transformed as

$$\begin{aligned} p \int [u_0(W_1) - A(W_1)] f_e(x; e) dx - c'(e) &= 0 \\ \Rightarrow p \int u_0(W_1) f_e(x; e) dx - c'(e) &= p \int A(W_1) f_e(x; e) dx \end{aligned} \quad (\text{A.6})$$

For x_L where $f_e(x_L; e) = 0$, we can write

$$\int A(W_1) f_e(x; e) dx = \int^{x_L} A(W_1) f_e(x; e) dx + \int_{x_L} A(W_1) f_e(x; e) dx \quad (\text{A.7})$$

We already know $-x^1 + I(x^1) > -x^2 + I(x^2)$ for $x^1 < x^2$. Thus by (10), we have following relation.

$$\int^{x_L} A(W_1) f_e(x; e) dx < A(W - Q - x_L + I(x_L)) \int^{x_L} f_e(x; e) dx \quad (\text{A.8})$$

$$\begin{aligned} \int_{x_L} A(W_1) f_e(x; e) dx &< A(W - Q - x_L + I(x_L)) \int_{x_L} f_e(x; e) dx \\ &= -A(W - Q - x_L + I(x_L)) \int^{x_L} f_e(x; e) dx \quad (\because \int f_e(x; e) dx = 0) \end{aligned} \quad (\text{A.9})$$

Hence, we obtain $\int A(W - Q - x + I(x)) f_e(x; e) dx < 0$ when $u'_0(W) < u'_1(W)$, $\forall W$. As a result, the effort should be increased if the indemnity is equal in a state dependent case and a state independent case for all indemnity levels.

(3) The proof is similar to the proof of (2), so is omitted. //

Proof of lemma 4.

(1) From (24) to (30), we have following first order conditions.

$$L_{Q_1} = -(1-p)u'_0(W_0^1) - p \int u'_1(W_1^1) f(x_1; e_1) dx_1 + \lambda_1 - \mu_1 p \int u'_1(W_1^1) f_{e_1}(x_1; e_1) dx = 0 \quad (\text{A.10})$$

$$L_{I(x_1)} = pu_1'(W_1^1)f(x_1;e_1) - \lambda_1 pf(x_1;e_1) + \mu_1 pu_1'(W_1^1)f_{e_1}(x_1;e_1) = 0 \quad (\text{A.11})$$

$$L_{Q_2(x_1=0)} = -(1-p)[(1-p)u_{00}'(W_0^2) + p\int u_{01}'(W_1^2(x_1=0))f(x_2;e_2(x_1=0))dx_2] \\ + \lambda_2(x_1=0)(1-p) - \mu_2(x_1=0)p\int u_{01}'(W_1^2(x_1=0))f_{e_2}(x_2;e_2(x_1=0))dx_2 = 0 \quad (\text{A.12})$$

$$L_{I_2(x_1=0,x_2)} = (1-p)pu_{01}'(W_1^2(x_1=0))f(x_2;e_2(x_1=0)) - \lambda_2(x_1=0)(1-p)pf(x_2;e_2(x_1=0)) \\ + \mu_2(x_1=0)pu_{01}'(W_1^2(x_1=0))f_{e_2}(x_2;e_2(x_1=0)) = 0 \quad (\text{A.13})$$

$$L_{Q_2(x_1)} = -p[(1-p)\int u_{10}'(W_0^2)f(x_1,e_1)dx_1 + p\int\int u_{11}'(W_1^2(x_1))f(x_1,x_2;e_1,e_2(x_1))dx_2dx_1] \\ + \lambda_2(x_1)p - \mu_1p(1-p)\int u_{10}'(W_0^2)f_{e_1}(x_1,e_1)dx_1 \\ - \mu_1pp\int\int u_{11}'(W_1^2(x_1))f(x_2;e_2(x_1))dx_2f_{e_1}(x_1;e_1)dx_1 \\ - p\int \mu_2(x_1)p\int u_{11}'(W_1^2(x_1))f_{e_2}(x_2;e_2(x_1))dx_2f(x_1;e_1)dx_1 \quad (\text{A.14})$$

$$L_{I_2(x_1,x_2)} = pp u_{11}'(W_1^2(x_1))f(x_2;e_2(x_1))f(x_1;e_1) - \lambda_2(x_1)ppf(x_2;e_2(x_1))f(x_1;e_1) \\ + \mu_1pp u_{11}'(W_1^2(x_1))f(x_2;e_2(x_1))f_{e_1}(x_1;e_1) \\ + \mu_2(x_1)pp u_{11}'(W_1^2(x_1))f_{e_2}(x_2;e_2(x_1))f(x_1;e_1) = 0 \quad (\text{A.15})$$

Rearranging these conditions, we have followings

$$\lambda_1 = u_1'(W_0^1) \\ \lambda_2(x_1=0) = u_{00}'(W_0^2) \\ \lambda_2(x_1) = u_{10}'(W_0^2)$$

We have the following risk sharing rules for period 1.

$$\frac{1}{u_1'(W_1^1)} = \frac{1}{u_0'(W_0^1)} \left(1 + \mu_1 \frac{f_{e_1}(x_1;e_1)}{f(x_1;e_1)}\right) \quad (\text{A.16})$$

We also have the rules for period 2 in case of no accident in $t = 1$ as below:

$$[u_{01}'(W_1^2(x_1=0)) - u_{00}'(W_0^2)]f(x_2;e_2(x_1=0)) + \mu_2(x_1=0)u_{01}'(W_1^2(x_1=0))f_{e_2}(x_2;e_2(x_1=0)) = 0 \\ \Rightarrow \frac{1}{u_{00}'(W_0^2)} \left[1 + \mu_2(x_1=0) \frac{f_{e_2}(x_2;e_2(x_1=0))}{f(x_2;e_2(x_1=0))}\right] = \frac{1}{u_{01}'(W_1^2(x_1=0))} \quad (\text{A.17})$$

When accidents occur in $t = 1$:

$$\begin{aligned}
& [u_{11}'(W_1^2(x_1)) - u_{10}'(W_0^2)]f(x_2; e_2)f(x_1; e_1) + \mu_1 u_{11}'(W_1^2(x_1))f(x_2; e_2(x_1))f_e(x_1; e_1) \\
& + \mu_2(x_1)u_{11}'(W_1^2(x_1))f_{e_2}(x_2; e_2(x_1)) = 0 \\
\Rightarrow & \frac{1}{u_{10}'(W_0^2)} \left[1 + \mu_1 \frac{f_e(x_1; e_1)}{f(x_1; e_1)} + \mu_2(x_1) \frac{1}{f(x_1; e_1)} \frac{f_{e_2}(x_2; e_2(x_1))}{f(x_2; e_2)} \right] = \frac{1}{u_{11}'(W_1^2(x_1))} \quad (\text{A.18})
\end{aligned}$$

(2) The proof (2) and (3) is similar to Lemma 3 and Lambert, so it is omitted. //

Proof of Proposition 3.

We first prove (2). (1) & (3) are proved in a similar way with (2). Under a state dependent utility, we can transform equations (29) and (30) as below.

$$p \int [u_{00}(W_1^2(x_1 = 0)) - A(W_1^2(x_1 = 0))]f_{e_2}(x_2; e_2(x_1 = 0))dx_2 - c'(e_2(x_1 = 0)) = 0 \quad (\text{A.19})$$

$$p \int [u_{00}(W_1^2(x_1)) - 2A(W_1^2(x_1))]f_{e_2}(x_2; e_2(x_1))dx_2 - c'(e_2(x_1)) = 0 \quad (\text{A.20})$$

The remainder of the proof is similar with the proof of proposition 2, so we omitted. As a result, the second period effort is higher when accident occurs if $u'_0 < u'_1$. //

Proof of Result 1.

(1) We first prove result 1 (2). We have the following first order condition.

$$\begin{aligned}
& p \int_0^{\pi_1} [\pi - B(\pi) - v(\pi, S = 1)]f_e(\pi; e)d\pi + (1 - p) \int_{\pi_1}^{\infty} [\pi - B(\pi) - v(\pi, S = 0)]g_e(\pi; e)d\pi = c'(e) \\
\Rightarrow & p \int_0^{\pi_1} [\pi - B(\pi) - v(\pi, S = 0) - A(\pi)]f_e(\pi; e)d\pi + (1 - p) \int_{\pi_1}^{\infty} [\pi - B(\pi) - v(\pi, S = 0)]g_e(\pi; e)d\pi \\
& = c'(e) \quad (\text{A.21})
\end{aligned}$$

If $v'(\pi, S = 0) > v'(\pi, S = 1)$, then $A'(\pi) < 0$ and $\int_{\pi_1}^{\pi^*} A(\pi)f_e(\pi; e)d\pi < 0$, so we obtain that the effort level would be higher.

(2) The proof of result 1(1) and 1(3) are similar to result 1 (2), so it is omitted. //

Proof of Result 2.

(1) We first prove result 2(2).

$$p \int_0^{\pi^*} u(S(\pi), S = 1)f_e(\pi; e)d\pi + (1 - p) \int_{\pi^*}^{\infty} u(S(\pi), S = 0)g_e(\pi; e)d\pi = c'(e)$$

$$\Rightarrow p \int_0^{\pi^*} [u(S(\pi), S=0) - A(S(\pi))] f_e(\pi; e) d\pi + (1-p) \int_{\pi^*}^{\infty} u(S(\pi), S=0) g_e(\pi; e) d\pi = c'(e)$$

(A.22)

If $u'(S(\pi), S=1) > u'(S(\pi), S=0)$, then $A'(S(\pi)) < 0$ and $\int^{\pi^*} A(S(\pi)) f_e(\pi; e) d\pi < 0$, so we know that the effort level would be higher.

(2) The proof of result 2 (1) and 2 (3) are similar to result 2 (2), so is omitted. //

Figure 1. State dependent utility function when marginal utility with loss is identical with marginal utility without loss

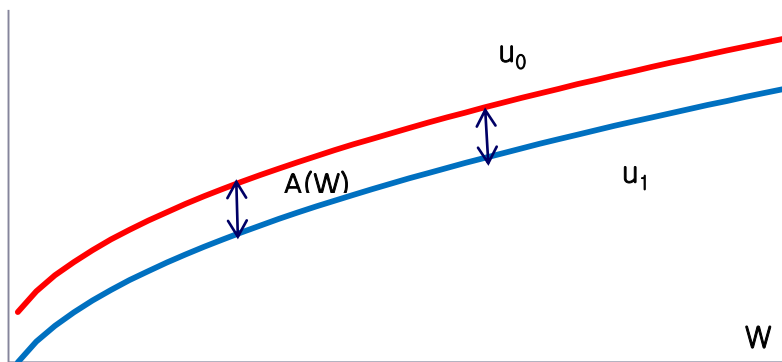


Figure 2. State dependent utility function when marginal utility with loss is greater than marginal utility without loss

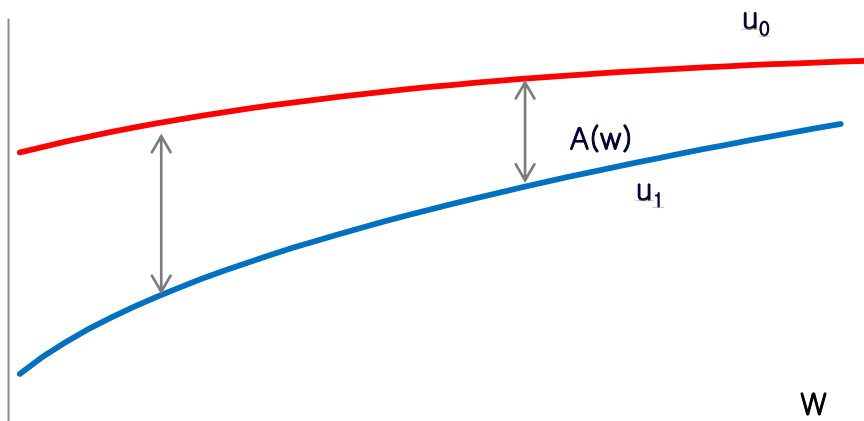


Figure 3. State dependent utility function when marginal utility with loss is less than marginal utility without loss

